Løysingsforslag TMA 4315 H-2005

a) $$\pi(x) = \frac{e^{\beta_0 + \beta_1 x}}{1 + e^{\beta_0 + \beta_1 x}}$$

$\chi^2_{0.1, 18} = 25.99 \Rightarrow$ ingen grunn til å forkaste modellen (Heller ikkje null modellen utan forklaringsvariabel).

β_0 og β_1 er begge signifikante på 10% nivå.

$$\hat{\pi}(25) = \frac{e^{-14.76 + 0.656 \cdot 25}}{1 + e^{-14.76 + 0.656 \cdot 25}} = 0.84$$

b) $$\frac{\hat{\pi}(25)}{1 - \hat{\pi}(25)} = \frac{0.84}{0.16} = 5.25$$

Estimert multiplikativ forandring i odds $= e^{0.656} = \underline{1.93}$

$$Z = e^{\hat{\beta}_1} \Rightarrow \operatorname{Var} Z \approx \left(e^{\hat{\beta}_1}\right)^2 \cdot \operatorname{Var}(\hat{\beta}_1)$$

$$\Rightarrow SD(e^{\hat{\beta}_1}) \approx e^{\hat{\beta}_1} \cdot SD(\hat{\beta}_1) = 1.93 \cdot 0.352 = 0.68$$

$$0.9 = \frac{1}{1 + e^{14.76 - 0.656x}} \Rightarrow 0.9 e^{14.76 - 0.656x} = 0.1$$

$$\Rightarrow e^{14.76 - 0.656x} = \frac{1}{9} \Rightarrow 14.76 - 0.656x = -\ln 9$$

$$\Rightarrow x = \frac{14.76 + \ln 9}{0.656} = 25.9$$

②

c)

$\operatorname{Var} Y_i = \sigma_i^2 = w_{ii}$ i logistisk regresjon,

$$(\underline{X}'\underline{W}\underline{X})^{-1} = \left(\begin{bmatrix} 1 & 1 & \cdots & 1 \\ x_{11} & x_{12} & \cdots & x_{120} \end{bmatrix} \begin{bmatrix} \sigma_1^2 & & & \\ & \sigma_2^2 & & \\ & & \ddots & \\ & & & \sigma_{20}^2 \end{bmatrix} \begin{bmatrix} 1 & x_{11} \\ 1 & x_{12} \\ \vdots & \vdots \\ 1 & x_{120} \end{bmatrix} \right)^{-1}$$

$$= \left(\begin{bmatrix} 1 & 1 & \cdots & 1 \\ x_{11} & x_{12} & \cdots & x_{120} \end{bmatrix} \begin{bmatrix} \sigma_1^2 & \sigma_1^2 x_{11} \\ \sigma_2^2 & \sigma_2^2 x_{12} \\ \vdots & \vdots \\ \sigma_{20}^2 & \sigma_{20}^2 x_{120} \end{bmatrix} \right)^{-1} = \begin{bmatrix} \sum_{i=1}^{20} \sigma_i^2 & \sum_1^{20} \sigma_i^2 x_{1i} \\ \sum_1^{20} \sigma_i^2 x_{1i} & \sum_1^{20} \sigma_i^2 x_{1i}^2 \end{bmatrix}^{-1}$$

$$\operatorname{Var}(\hat{\beta}_0 + \hat{\beta}_1 x_1) = \operatorname{Var}\hat{\beta}_0 + x_1^2 \operatorname{Var}\hat{\beta}_1 + 2x_1 \operatorname{Cov}(\hat{\beta}_0, \hat{\beta}_1)$$

$$\approx 74.42 + 25^2 \cdot 0.124 - 50 \cdot 3.02 \approx 0.92$$

$$\Rightarrow SD(\hat{\beta}_0 + \hat{\beta}_1 x_1) \approx 0.96.$$

Tilnærmet konfidensintervall for oddsen

$$\left(5.25 \cdot e^{-1.96 \cdot 0.96},\ 5.25 \cdot e^{1.96 \cdot 0.96} \right) = (0.8,\ 34.46)$$

d)

$$\hat{\mu}(\underline{x}) = e^{-3.44 + 0.13x_1 + 0.97x_2 + 1.49x_3 + 0.78x_4}$$

β_0, β_1 og β_3 er signifikante på 5% nivå.

$\chi^2_{0.05,15} = 25 > 11.99 \Rightarrow$ ingen grunn til å forkaste modellen på 5% nivå.

e) $\hat{\mu}(\underline{x}) = e^{-3.45 + 0.134 X_1 + 1.49 X_3}$

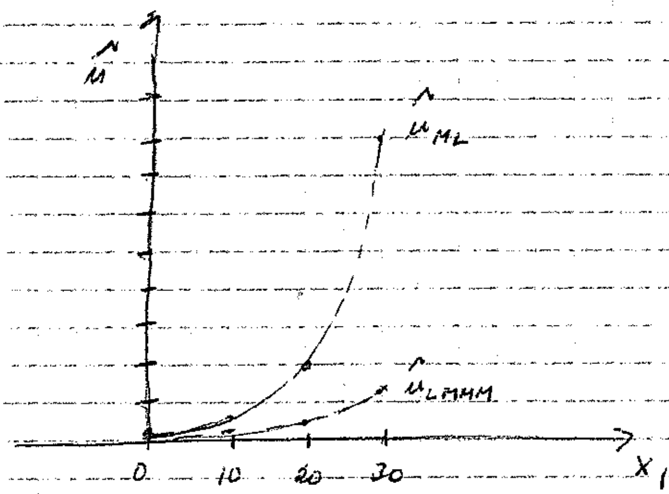
For lyse, middels mørke og mørke krabber er

$\hat{\mu}_{LMMM}(x) = e^{-3.45 + 0.134 x_1}$

For middels lyse krabber er

$\hat{\mu}_{ML}(x) = e^{-1.96 + 0.134 x_1}$

X_1	0	10	20	30
$\hat{\mu}_{LMMM}(x_1)$	$e^{-3.45}$	$e^{-2.1}$	$e^{-0.77}$	$e^{0.57}$
$\hat{\mu}_{ML}(x_1)$	$e^{-1.96}$	$e^{-0.62}$	$e^{0.72}$	$e^{2.06}$



Det er ingen kryssledd i modellen.

$e^{0.134 \pm 1.96 \cdot 0.057} = (1.02, 1.28)$

f) $e^{\beta_0 + \beta_1 x} = 5 \Rightarrow \beta_1 x = \ln 5 - \beta_0 \Rightarrow x = \dfrac{\ln 5 - \beta_0}{\beta_1}$

For middels lyse krabber: $x = \dfrac{\ln 5 + 1.96}{0.134} = 26.64$

Konfidensintervall: $\dfrac{\hat{\beta}_0 + \hat{\beta}_1 x - \ln 5}{SD(\hat{\beta}_0 + \hat{\beta}_1 x)} \approx N(0,1)$

Finn dei x som oppfyller $\left| \dfrac{\hat{\beta}_0 + \hat{\beta}_1 x - \ln 5}{SD(\hat{\beta}_0 + \hat{\beta}_1 x)} \right| \le z_{0.025}$

Oppgåve 2

a) $$f(y) = P(Y=y) = \binom{n}{y} p^y (1-p)^{n-y}$$

$$\ln f = \ln\binom{n}{y} + y \ln p + (n-y)\ln(1-p)$$

$$= \ln\binom{n}{y} + y \ln\left(\frac{p}{1-p}\right) + n \ln(1-p)$$

$$D^*(\underline{y}, \hat{\mu}) = 2l(y; y) - 2l(\hat{\mu}, y)$$

$$= 2\sum\left(y_i \ln\left(\frac{y_i}{n_i - y_i}\right) + n_i \ln\left(\frac{n_i - y_i}{n_i}\right) - y_i \ln\left(\frac{\frac{\hat{\mu}_i}{n_i}}{\frac{n_i - \hat{\mu}_i}{n_i}}\right) - n_i \ln\left(\frac{n_i - \hat{\mu}_i}{n_i}\right)\right)$$

$$= 2\sum\left(y_i \ln\left(\frac{y_i}{\hat{\mu}_i}\right) + n_i \ln\left(\frac{n_i - y_i}{n_i - \hat{\mu}_i}\right) - y_i \ln\left(\frac{n_i - y_i}{n_i - \hat{\mu}_i}\right)\right)$$

b) $$w_{ii} = \left(\frac{\partial \mu_i}{\partial \eta_i}\right)^2 \Big/ \operatorname{Var}(y_i)$$

Poisson: $\mu = \eta \Rightarrow \frac{\partial \mu}{\partial \eta} = 1 \Rightarrow w_{ii} = \frac{1}{\mu_i}$

$$\mu = e^{\eta} \Rightarrow \frac{\partial \mu}{\partial \eta} = e^{\eta} \Rightarrow w_{ii} = \frac{e^{2\eta_i}}{e^{\eta_i}} = e^{\eta_i} = \mu_i$$

Binomisk

$$\mu = \eta \Rightarrow \frac{\partial \mu}{\partial \eta} = 1 \Rightarrow w_{ii} = \frac{1}{n_i p_i (1-p_i)}$$

$$\mu = \frac{n e^{\eta}}{1+e^{\eta}} \Rightarrow \frac{\partial \mu}{\partial \eta} = \frac{n e^{\eta}}{1+e^{\eta}} - \frac{n e^{2\eta}}{(1+e^{\eta})^2} = \frac{n e^{\eta}}{(1+e^{\eta})^2} = np(1-p)$$

$$\Rightarrow w_{ii} = \frac{(n p_i (1-p_i))^2}{n_i p_i (1-p_i)} = n_i p_i (1-p_i)$$