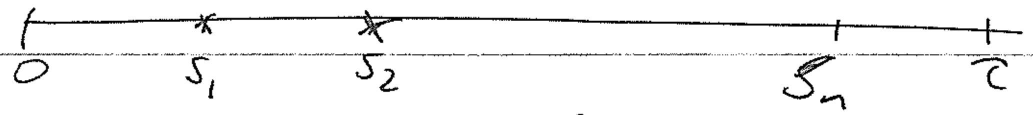
Week 12, 29/4-05

Trend testing (continued.)

~~Several processes:~~

One process, time truncation

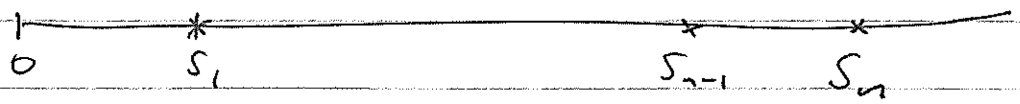


H_0: HPP vs H_1: not HPP

Laplace: $$W = \frac{\sum_{i=1}^{n} S_i - \frac{n\tau}{2}}{\sqrt{\frac{n\tau^2}{12}}} \approx N(0,1)$$ under H_0.

Mil Hbk: $$Z = 2\sum_{i=1}^{n} \ln \frac{\tau}{S_i} \sim \chi^2_{2n}$$ under H_0.

One process, failure truncation at S_n



Then $S_1, \ldots, S_{n-1}$ are uniform on $(0, S_n)$.

So: ~~Put~~ Let $\tau \to S_n$, $n \to n-1$

$$W = \frac{\sum_{i=1}^{n-1} S_i - \frac{(n-1)S_n}{2}}{\sqrt{\frac{(n-1)S_n^2}{12}}}$$

Trend tests for $m > 1$ systems

Suppose m processes, that are NHPP ~~with~~

H_0: the m processes are all HPP

vs H_1: ~~at least one of the~~ ~~the~~ m processes have ~~at least~~
- increasing trend
- decreasing trend
- ~~non-constant trend~~
- not all processes are HPP

For $j = 1, 2, \ldots, m$

$0 \quad S_{1j} \quad \ldots \quad S_{n_j j} \quad \tau_j$

~~Define~~ Now for each fixed j we use that the

$S_{1j}, \ldots, S_{n_j j}$

~~$W_{pooled} =$~~ are orderings of n_j uniforms on $(0, \tau_j)$

thus $E(S_{ij}) = \dfrac{\tau_j}{2}$, $Var(S_{ij}) = \dfrac{\tau_j^2}{12}$

Define

$$W_{pooled} = \frac{\sum_{j=1}^{m} \sum_{i=1}^{n_j} S_{ij} - E\left[\sum\sum S_{ij}\right]}{\sqrt{Var \sum\sum S_{ij}}}$$

Here $E\left[\sum_{j=1}^{m} \sum_{i=1}^{n_j} S_{ij}\right] = \sum_{j=1}^{m} \sum_{i=1}^{n_j} \dfrac{\tau_j}{2} = \sum_{j=1}^{m} \dfrac{n_j \tau_j}{2}$

$Var\left[\qquad\right] = \sum_{j=1}^{m} \sum_{i=1}^{n_j} \dfrac{\tau_j^2}{12} = \sum_{j=1}^{m} \dfrac{n_j \tau_j^2}{12}$

$$\Rightarrow \quad W_{pooled} = \frac{\sum_j \sum_i s_{ij} - \frac{1}{2}\sum_j n_j \tau_j}{\sqrt{\frac{1}{12}\sum_j n_j \tau_j^2}} \approx N(0,1) \text{ under } H_0 \,!$$

For simple NHPP:

$$W_{pooled} = \frac{5+12+17+9+23+4 - \frac{1}{2}(3\cdot 20 + 2\cdot 30 + 1\cdot 10)}{\sqrt{\frac{1}{12}\left[3\cdot 20^2 + 2\cdot 30^2 + 1\cdot 10^2\right]}}$$

$$= \frac{70 - 65}{\sqrt{\frac{1}{12}[3100]}} \qquad 1200 + 1800 + 100$$

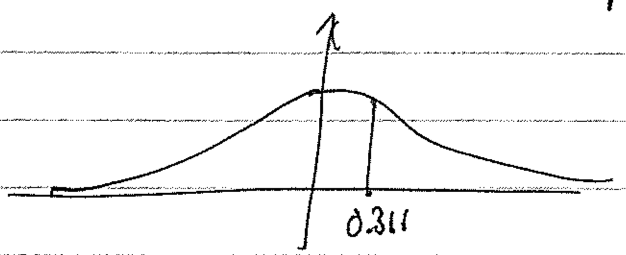
$$= \frac{5}{\sqrt{\frac{3100}{12}}} = 0.3111 \qquad [\text{SE MINITAB}]$$

P-value in MINITAB is for test of "all HPP" vs "not all HPP" so p-value

$$= P(|W| \geq 0.311)$$

$$= 2\cdot P(W \geq 0.311)$$

$$= 2\cdot 0.378 = 0.756.$$



<u>Note</u>: This test is in fact a ~~of~~ test of the null hypothesis that processes are all HPP's, but possibly with different individual hazards.

Note - there is also something called TTT-based test, W_{TTT}, ~~but~~ which is for the null hypothesis that <u>all</u> processes have the <u>same</u> intensity.

Note that W_{pooled} can be written as a weighted sum of individual statistics

$$W_j = \frac{\sum_{i=1}^{n_j} S_{ij} - \frac{n_j \tau_j}{2}}{\sqrt{\frac{n_j \tau_j^2}{12}}} \qquad (\approx N(0,1))$$

~~Thus~~ $\cancel{W_{pooled} = \sum_{j=1}^{m} \frac{\sqrt{\frac{n_j \tau_j^2}{12}}}{\sqrt{\frac{1}{12}\sum_{j=1}^{m} n_j \tau_j^2}} W_j}$

See that ~~Second~~ $\sum_{i=1}^{n_j} S_{ij} - \frac{n_j \tau_j}{2} = \sqrt{\frac{n_j \tau_j^2}{12}}\, W_j$

Thus $W_{pooled} = \dfrac{\sum_{j=1}^{m} \sqrt{\frac{n_j \tau_j^2}{12}}\, W_j}{\sqrt{\frac{1}{12} \sum n_j \tau_j^2}}$

What about the pooled Mil Hbk?

~~Bi~~ Note that all the $2 \ln \frac{\tau_j}{s_{ij}}$ are $\sim \chi^2_2$

Thus $Z_{pooled} = 2 \sum_{j=1}^{m} \sum_{i=1}^{n_j} \ln \frac{\tau_j}{s_{ij}} \sim \chi^2_{2n}$

where $n = \sum_{j=1}^{m} n_j$

Can write simply

$$Z_{pooled} = \sum_{j=1}^{m} Z_j$$

where $Z_j = 2 \sum_{i=1}^{n_j} \ln \frac{\tau_j}{s_{ij}}$ is the test-statistic for process # j.

~~B~~

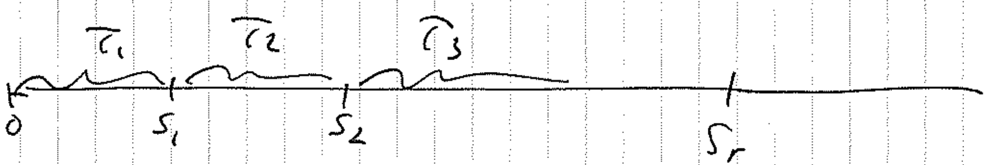
RP

1/4 - 03

1

Hva du skal vite om renewal prosesser:

(Kap. 7.3 s 295)



__Def:__ $T_1, T_2, \ldots$ er ~~uavh~~ u.i.f. med fordeling F_T

1. Tid til r-te hendelse: $S_r = \underbrace{T_1 + \ldots + T_r}$ har en ford som kalles r-te konvolusjon av T.

   Feks. Hvis T-ene er eksp. fordelte, er S_r gamma-fordelt.

2. ~~N~~ Som før:

$$N(t) = \text{ant. hendelser i } (0,t]$$
$$= \max\{r : S_r \le t\}$$

3. $W(t) = E(N(t))$ = "renewal function" (= EROCOF)

4. $w(t) = W'(t)$ ("renewal density" = ROCOF)

Ofte vanskelig å finne pene uttrykk for disse. ~~vanskelig~~ unntak var HPP da ~~de~~ $N(t) \sim \text{Poisson}(\lambda t)$ og $W(t) = \lambda t$, $w(t) = \lambda$

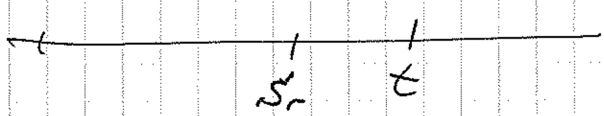
Viktige resultater :

La $\mu = E(T)$
$\sigma^2 = Var(T)$

$\cancel{N(t) \geq r \iff S_r \leq t}$

Klart at $S_r \approx N(r\mu, r\sigma^2)$

Har nå



$N(t) \geq r \iff S_r \leq t$

dvs $P(N(t) \geq r) = P(S_r \leq t)$

$$\approx \Phi\left(\frac{t - r\mu}{\sigma\sqrt{r}}\right)$$

& $P(N(t) = r) = P(N(t) \geq r) - P(N(t) \geq r+1)$

$$\approx \Phi\left(\frac{t - r\mu}{\sigma\sqrt{r}}\right) - \Phi\left(\frac{t - (r+1)\mu}{\sigma\sqrt{r+1}}\right)$$

~~Taleas (Ross)~~ $\cancel{P(N(t) \geq r) \approx \Phi\left(\frac{r - \frac{t}{\mu}}{\sqrt{t\frac{\sigma^2}{\mu^3}}}\right)}$

Grenseresultater :

$$\lim_{t\to\infty} \frac{N(t)}{t} = \frac{1}{\mu}$$ (The Elementary Renewal Theorem)

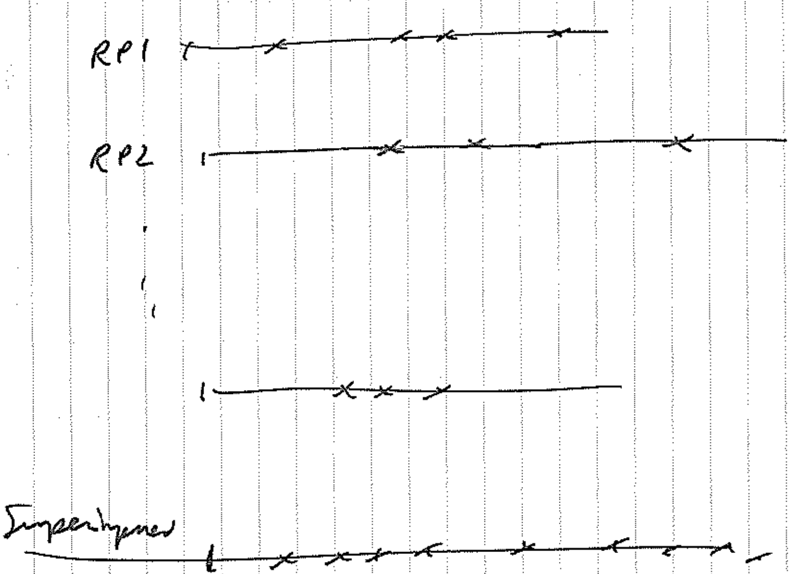
(hvis $\mu < \infty$. $= 0$ hvis $\mu = \infty$)

Blackwell's Theorem: Hvis F_T er ~~non-lattice~~ en kont. fordeling:

$$\lim_{t\to\infty} \frac{W(t+\alpha)-W(t)}{\mu} = \frac{\alpha}{\mu} \quad \text{for } \alpha > 0$$

I et intervall av lengde α, uansett lagt ute, vil det forventet være $\frac{\alpha}{\mu}$ hendelser

5.301: Superimposed Renewal Process:



Drenick (1960): Superimposed av mange blir $\approx$ HPP.