1

Forelest fra foiler

$\boxed{\text{Nelson-Aalen:}}$

~~Assume no~~

NONPARAMETRIC ESTIMATION

Order all failure times as

$$t_1 < t_2 < \dots < t_n \quad (\text{"pooled failures"})$$

Let $d_j(t_i)$ = # failing in system j at time t_i

Let $Y_j(t) = \begin{cases} 1 & \text{if system } j \text{ is under obs. at time } t \\ 0 & \text{otherwise} \end{cases}$

Thus $Y(t)$ = # systems under obs at t $= \sum_{j=1}^{m} Y_j(t)$

$$d(t_i) = \sum_{j=1}^{m} d_j(t_i) = \text{\# failing at } t_i$$

Then $\hat{W}(t) = \sum_{t_i \le t} \frac{d(t_i)}{Y(t_i)}$ is 1 if no multiple failures

For NHPP $\widehat{\mathrm{Var}\,\hat{W}(t)} = \sum_{t_i \le t} \frac{d(t_i)}{\{Y(t_i)\}^2}$

For general processes (Lawless & Nadeau)

$$\widehat{\mathrm{Var}\,\hat{W}(t)} = \sum_{j=1}^{m} \left\{ \sum_{t_i \le t} \frac{Y_j(t_i)}{Y(t_i)} \left[d_j(t_i) - \frac{d(t_i)}{Y(t_i)} \right] \right\}^2$$

(One term for each system)

(2)

Simple example: t_1:

$$\operatorname{Var} \hat{W}(t_1) = \begin{cases} \text{Sys 1: } \left\{ \frac{1}{3} \left[0 - \frac{1}{3} \right] \right\}^2 = \frac{1}{81} \\ \text{Sys 2: } \left\{ \frac{1}{3} \left[0 - \frac{1}{3} \right] \right\}^2 = \frac{1}{81} \\ \text{Sys 3: } \left\{ \frac{1}{3} \left[1 - \frac{1}{3} \right] \right\}^2 = \frac{4}{81} \end{cases}$$

4

~~Sum~~: $\frac{2}{3} \cdot \frac{1}{3} = \frac{2}{9}$ $\qquad \frac{6}{81}$

$$\Rightarrow SD = \frac{\sqrt{6}}{9} = 0.2722$$

~~Stan~~ Conf int:

Recall Standard for θ: $\hat{\theta} \pm 1.96\, \widehat{SD(\hat{\theta})}$

Standard for $g(\theta)$: $g(\hat{\theta}) \pm 1.96 \cdot \widehat{SD(g(\hat{\theta}))}$

i.e. $g(\hat{\theta}) \pm 1.96 \, |g'(\hat{\theta})| \, \widehat{SD(\hat{\theta})}$

$g = \ln$: For $\ln\theta$: $\ln\hat{\theta} \pm 1.96 \cdot \frac{\widehat{SD(\hat{\theta})}}{\hat{\theta}}$

$\Rightarrow$ For θ: $e^{\ln\hat{\theta} \pm 1.96 \cdot \frac{\widehat{SD(\hat{\theta})}}{\hat{\theta}}}$

i.e. $\hat{\theta} \cdot e^{\pm 1.96 \cdot \frac{\widehat{SD(\hat{\theta})}}{\hat{\theta}}}$

Here: $\frac{1}{3} \cdot e^{\pm 1.96 \cdot \frac{\sqrt{6}}{9 \cdot \frac{1}{3}}}$

~~$\frac{1}{3} \cdot e^{\pm 1}$~~ $\Rightarrow$ Med $-$: 0.0673
Med $+$: 1.6516

③

With NHPP - estimate we could also use this last confidence interval method:

$$\frac{1}{3}\cdot e^{\pm 1.96 \cdot \frac{\frac{1}{3}}{\frac{1}{3}}} = \frac{1}{3}\cdot e^{\pm 1.96}$$

$$(0.0470,\ 2.3664)$$

What about $t_3 = 9$

Var: Syst. 1: $\left\{ \underset{t_1=4}{\frac{1}{3}\left[0-\frac{1}{3}\right]} + \underset{t_2=5}{\frac{1}{3}\left[1-\frac{1}{3}\right]} + \underset{t_3=9}{\frac{1}{3}\left[0-\frac{1}{3}\right]} \right\}^2$

$$= \left\{-\frac{1}{9}+\frac{1}{3}-\frac{1}{9}-\frac{1}{9}\right\}^2 = 0.$$

Syst 2: $\left\{ \frac{1}{3} \right\}$ Same thing!

$$\cancel{\frac{1}{3}} \left\{-\frac{1}{9}+\frac{2}{9}\right\}^2 = \frac{1}{81}$$

$$\frac{1}{3}\left[0-\frac{1}{3}\right]+\frac{1}{3}\left[1-\frac{1}{3}\right] + \left\{-\frac{1}{9}-\frac{1}{9}\right\}^2 + \frac{4}{81}$$

$$-\frac{1}{9}+\frac{2}{9} \quad \frac{1}{81} + \left\{-\frac{1}{9}-\frac{1}{9}\right\}^2 + \frac{4}{81}$$

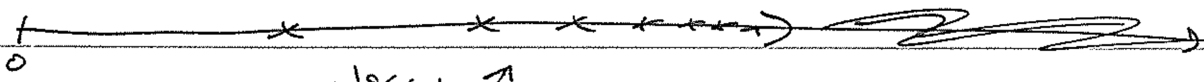
$$= \frac{9}{81} = \frac{1}{9}$$

~~FORELESNING 12 22/4-2004~~

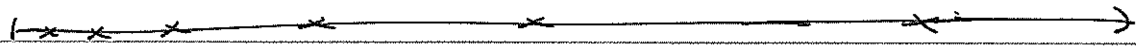
NHPP PROCESSES

TESTER FOR TREND I ~~REKURRENTE HENDELSER~~

~~Anta NHPP med intensity w(t)~~



$w(t) \nearrow$



$w(t) \downarrow$

$w(t) =$ ~~α~~ constant betyr HPP.

Trend-testen er laget for å teste

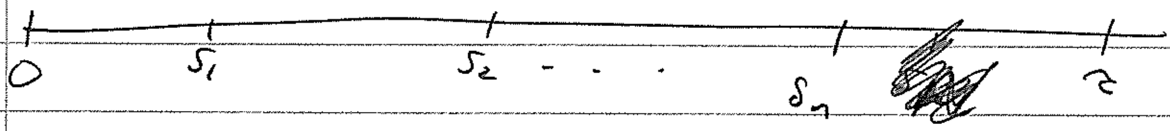
H_0: $w(t)$ er konstant i t mot $\begin{cases} H_1: w(t) \nearrow \\ \text{eller } H_1: w(t) \searrow \\ \text{eller } H_1 \text{ ikke-konstant.} \end{cases}$

To berømte tester:

Laplace - testen
MIL-HDBk-testen }

Baserer seg på følgende:

Hvis H_0 gjelder har vi HPP:
Anta tidstrunkering ved τ, og at $S_1\ S_2 \ldots S_n$ observeres



Note: We have conditional tests - ~~...~~ determines what to do on the basis of n.

Gitt $N=n$ feil, vil disse være ~~uniformt~~ fordelt som n uniforme variable

dvs. hver av dem har forvent. $\frac{\tau}{2}$, Varians $\frac{\tau^2}{12}$

$$\sum_{i=1}^{n} S_i \approx N\left(n\frac{\tau}{2},\ n\frac{\tau^2}{12}\right)$$ ved sentralgrenseteoremet.

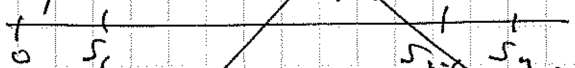
dvs
$$W = \frac{\sum_{i=1}^{n} S_i - \frac{n\tau}{2}}{\tau\sqrt{\frac{n}{12}}} \approx N(0,1)$$

dvs
$$W = \frac{\sum_{i=1}^{n}\left(S_i - \frac{\tau}{2}\right)}{\tau\sqrt{\frac{n}{12}}}$$

Dette er observatoren i LAPLACE-TESTEN

~~Men dette definerer en testobservator for H_0: HPP mot H_1: NHPP som kalles Laplace-testen.~~

~~Hvis forsøket stoppes ved n feil,~~

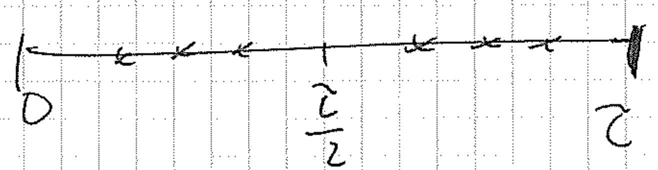


~~brukes at $S_1, \ldots, S_{n-1}$ gitt S_n er uniforme på $(0, S_n)$~~

~~Erstatt da $\underline{\underline{\tau}}$ med $\underline{\underline{S_n}}$ og $\underline{\underline{n}}$ med $\underline{\underline{n-1}}$~~

~~Se etter~~ Grampus: 1.22 (ikke forkast)

Ide: Hvis ingen trend, vil tiene være jevnt fordelt på hver side av $\frac{\tau}{2}$.



Ved monoton trend, vil de hope seg opp på en side.

Dermed W liten (negativ) hvis mange svikt tidlig avtagende trend $\lambda(t) \downarrow$

W stor (positiv) hvis mange svikt sent voksende trend $\lambda(t) \nearrow$

Grampus: 1,22 (ikke forkast, men nesten)

Enkelt eksempel:

Har $m=3$ systemer.

~~Kan regne ut W for hver av dem~~ teste for hver av dem først: og ~~bruke at de er $W_1 + W_2 + \cdots + W_m \approx N(0, m)$ hvis H_0 gjelder~~.

$$\text{Sys 1:} \quad W_1 = \frac{(5-10)+(12-10)+(17-10)}{20\sqrt{\frac{3}{12}}}$$

$$= \frac{4}{20\sqrt{\frac{3}{12}}} = 0.4$$

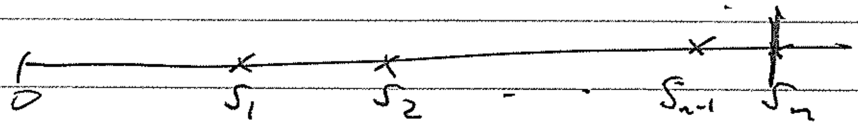
$$\text{Sys 2:} \quad W_2 = \frac{(9-15)+(23-15)}{30\sqrt{\frac{2}{12}}} = \frac{2}{30\sqrt{\frac{2}{12}}} = 0.1633$$

$$\text{Sys 3:} \quad W_3 = \frac{4-5}{10\sqrt{\frac{1}{12}}} = -0.3464$$

Ingen av dem er nær noen kritiske verdi.

Hvis vi vil teste H_0: alle er HPP
kan vi evt addere $W_1 + W_2 + W_3 \approx N(0, m)$
~~hvis alle er HPP~~

Feiltrunkering: Anta observere til n te feil



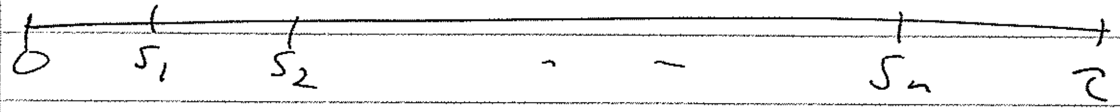
Da er $S_1, S_2, \cdots, S_{n-1}$, gitt $S_n = s_n$, uniform

Testobs. blir da

$$W = \frac{\sum_{i=1}^{n-1}\left(S_i - \frac{S_n}{2}\right)}{S_n\sqrt{\frac{n-1}{12}}} \approx N(0,1)$$

hvis H_0 gjelder

Military Handbook test



Betinger at $S_1 \cdots S_n$ ordn av $U[0, \tau]$.

Resultat: Hvis $S \sim U[0, \tau]$ så er

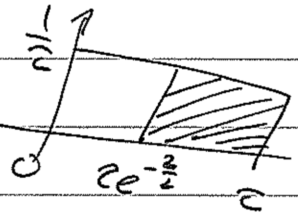
$$Z = 2\ln\frac{\tau}{S} \sim \chi^2_2$$

Bevis: $P(Z \leq z) = P(2\ln\frac{\tau}{S} \leq z)$

$\cancel{= P\left(\frac{S}{c} \le e^{z}\right)}$

$\cancel{= P\left(S \le c e^{z}\right)}$

$$= P\left(\ln \frac{c}{S} \le \frac{z}{2}\right)$$



$$= P\left(\frac{c}{S} \le e^{\frac{z}{2}}\right)$$

$$= P\left(S \ge c e^{-\frac{z}{2}}\right) = 1 - e^{-\frac{z}{2}}$$

Dermed $f_Z(z) = \frac{1}{2} e^{-\frac{z}{2}} \sim \chi^2_2$.

Dermed foreslås testobservator

$$Z = \sum_{i=1}^{n} 2 \ln \frac{c}{S_i} = 2 \sum_{i=1}^{n} \ln \frac{c}{S_i} \sim \chi^2_{2n}$$

hvis H_0 gjelder

Ser igjen at hvis

er det mange store verdi, dvs Z blir stor

ved $w(t) \downarrow$

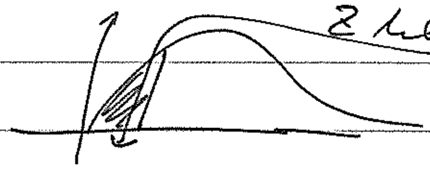
Hvis

blir $\ln \frac{c}{S_i}$ nær $\ln 1$ for mange verdi, og

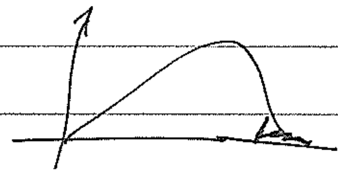
Z blir liten [illegible]

Dermed rimelig:

~~Forkast~~ H_0: $w(t)$ const mot $w(t) \nearrow$: Forkast for Z lille



mot $w(t) \searrow$



Feks i vårt system 1

$$Z = 2\left(\ln \frac{20}{5} + \ln \frac{20}{12} + \ln \frac{20}{17}\right)$$

$$= 5.1409$$

Hvis H_0 gjelder er

$$Z \sim \chi^2_6$$

(dvs $E(Z) = 6$)

altså har vi fått en "liten" verdi

Grampus: $Z = 82$