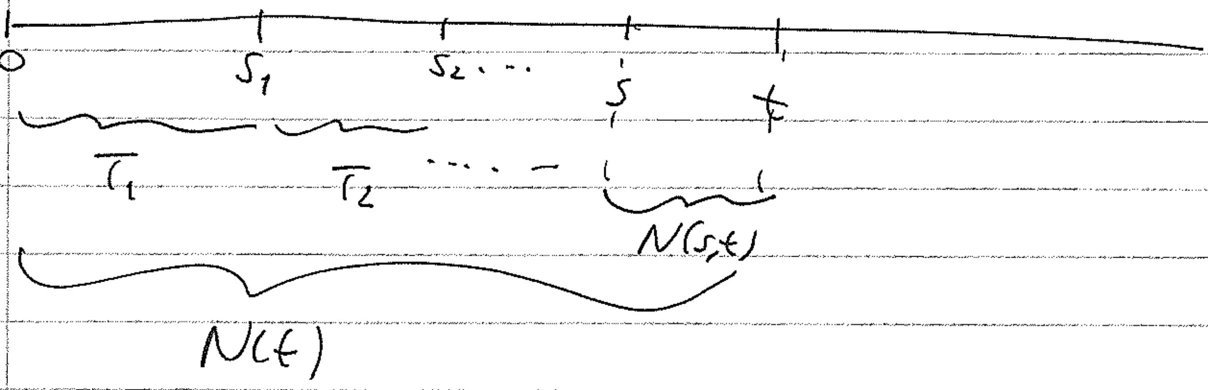
LECTURE WEEK 11
18/4-05

Counting processes



$W(t) = E[N(t)]$ CROCOF

$w(t) = W'(t)$ ROCOF

Regular: $P(\text{1 event in } (t, t+h)) = w(t)\,h + o(h)$

Only one event at a time

NHPP: ROCOF = intensity function = $w(t)$

(1) $N(s,t) \sim \text{Poisson}\left[\int_s^t w(u)\,du\right]$

dvs $\text{Poisson}\left[W(t) - W(s)\right]$

where $W(t) = \int_0^t w(u)\,du = E[N(t)]$

(2) $N(s_1,t_1), N(s_2,t_2), \ldots$ independent if intervals $(s_1,t_1], (s_2,t_2] \ldots$ are pairwise disjoint

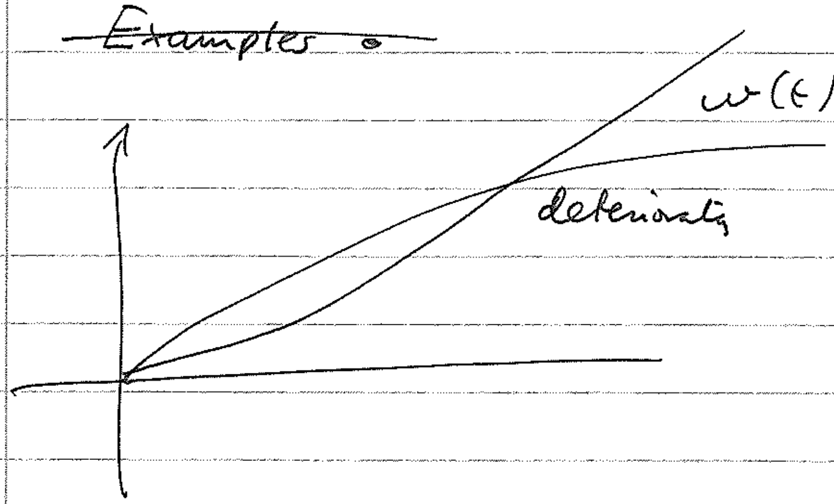
$w(t) \nearrow$ deteriorating system ("sad system")

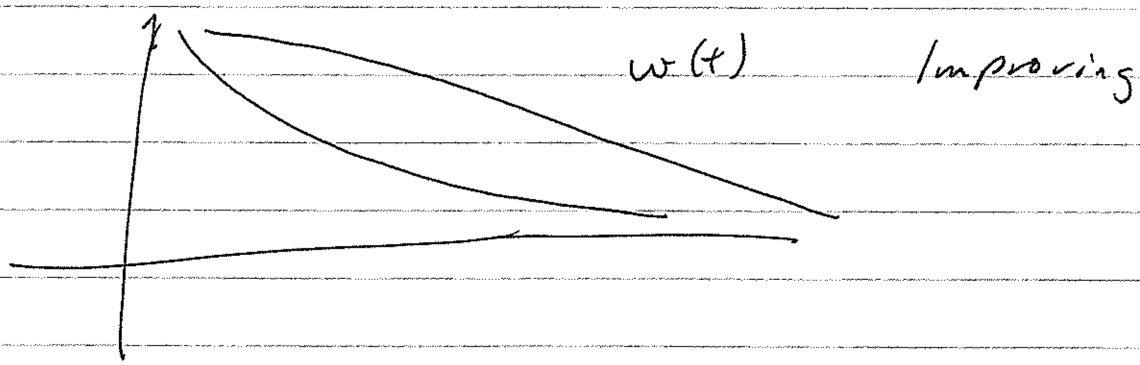
$w(t) \searrow$ improving system ("happy system")

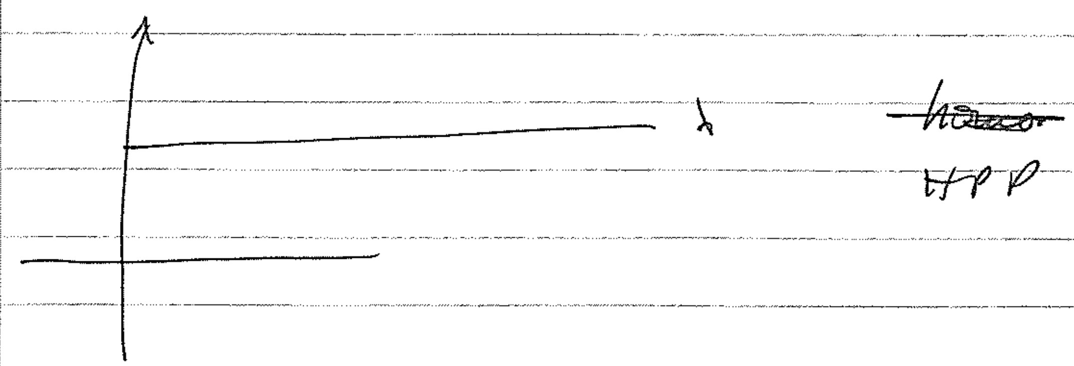
e.g. Software reliability.

$w(t) = \lambda$ (constant) : HPP

~~Examples~~







Parametric models:

Power law ~~proces~~ NHPP:

$$w(t) = \lambda \beta t^{\beta - 1} \quad \text{for } \beta > 0.$$

$\searrow$ if $\beta < 1$

$\nearrow$ if $\beta > 1$

NHPP if $\beta = 1$

[Similar to Weibull intensity], $\dfrac{\alpha}{\theta}\left(\dfrac{t}{\theta}\right)^{\alpha - 1}$

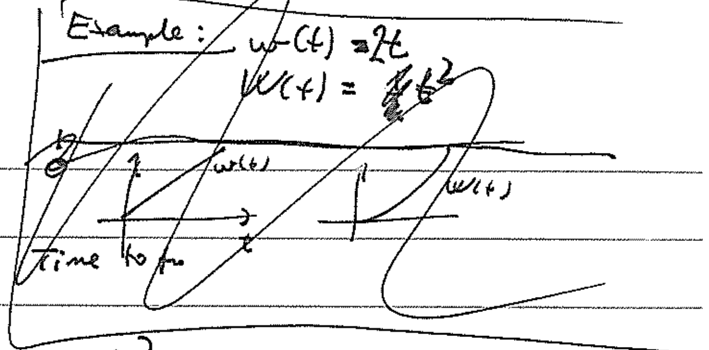
Log linear law NHPP:

$$w(t) = e^{\alpha + \beta t} \quad ;$$

$\searrow$ if $\beta < 0$

$\nearrow$ if $\beta > 0$

$\rightarrow$ if $\beta = 0$.

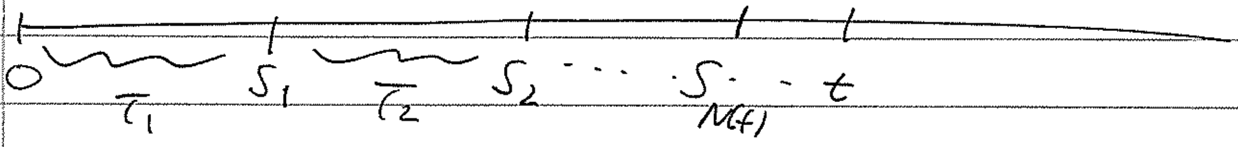


~~Simuleringer~~

Modellering av

Reparasjon:

[Perfect repair & Minimal repair]



Anta vi har ~~ett~~ ~~enhet~~ komponent ~~system~~ (e.l.) med levetid T, ~~og~~ og tilhørende hasardrate $z(t)$ etc.

Perfekt reparasjon:

Anta ~~systemet~~ komponenten repareres til så god som ny (eller evt. skiftes ut) ved hver feil.

Rimelig å anta at vi da kan betrakte

$T_1, T_2, \ldots$ som uavh. realisasjoner av T

Dermed er feilprosessen $S_1, S_2, \ldots$ en fornyelsesprosess.

Det betyr at hvis jeg går inn på et bestemt tidspunkt t, vil sannsynl. for en feil i $(t, t+h)$ være ~~z(aktuell~~

$\approx z($~~alder~~ tid siden siste fornyelse$) \cdot h$

$= \cancel{z(t - S_{N(t)}) \cdot h}$ (Se figur!)

We call this a conditional ROCOF since ~~it is~~ we use ~~given~~ information of the ~~pa~~ history

~~Hva kalles dette en~~

dvs $P(\text{feil i } (t, t+h) \mid \text{historien opp til tid } t)$

$$= z(t - S_{N(t)}) \cdot h$$

~~Hva~~ dvs $\lim_{h \to 0} \frac{P(\text{feil i } (t, t+h) \mid \text{historien opp til } t)}{h} = z(t - S_{N(t)})$

Dette kalles en betinget ROCOF ~~fordi~~

(den betinges m.h.p. historien).

~~Hvis jeg ikke kjenner historien, ville jeg~~

(Til forskjell fra ROCOF $= \lim_{h \to 0} \frac{P(\text{feil i } (t, t+h))}{h}$

~~Minimc~~

Minimal reparasjon. $0 \quad t, t+h \quad S_1 \quad S_2 \quad t \quad t+h$

Anta at systemet ved feil repareres bare til samme tilstand som det var i (Eksempel: Bytt en tennplugg i en bil).

~~Hva er $P(\text{feil i } (t, t+h) \mid \text{historien fra til tid } t)$~~ ?

Da er sannsynl. for å feile i $(t, t+h)$ alltid det samme som den ville vært

~~fra et nytt system~~
fra et system som hadde gått siden tid 0, dvs $\approx z(t)\,h$ (SE FIGUR!)
Thus - rate of occurrence of failures is independent of ~~time~~ the history.
~~= UAVHENGIG AV HISTORIEN =~~

~~Altså: P(feil i (t, t+h) | historien opp til t)~~

~~= P(feil i (t, t+h) for et system som ikke har feilet opp til t)~~

~~Kan ses på som def. av MINIMAL REP-PROSESS~~

~~= z(t)~~

~~Dermed blir også ROCOF = P(feil i (t, t+h))~~

~~= z(t)~~

~~Husk at hvis A, B uavh.~~
~~P(A) = P(A|B)~~

FIGUR (foil).

Can be shown that the definition of minimal repair implies that we have a NHPP with $w(t) = z(t)$.

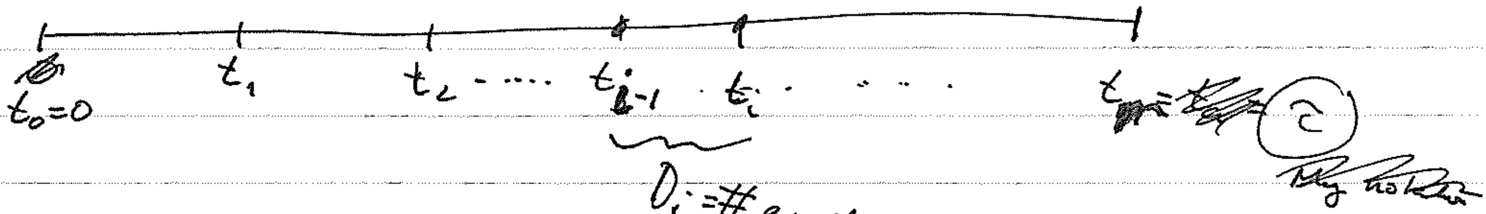
Week 11, 22/4-05

Start by foils from "Lecture Week 11, Supplement"

Estimation in NHPP, Parametric. General: $w(t;\theta)$

e.g. $w(t;\theta) = \lambda\beta t^{\beta-1}$

(1) Single Unit, Interval recurrence data. Note: These t_i are not failure times.



$$D_j = \#\text{ events in } (t_{j-1}, t_j] \sim \text{Poisson}\Big(\underbrace{W(t_{j-1}, t_j; \theta)}_{\int_{t_{j-1}}^{t_j} w(u;\theta)\,du}\Big)$$

$$P(D_j = d_j) = \frac{W(t_{j-1}, t_j; \theta)^{d_j}}{d_j!} e^{-W(t_{j-1}, t_j; \theta)}$$

$$L(\theta) = \text{product of these} = \left\{\prod_{j=1}^{r} \frac{W(t_{j-1}, t_j; \theta)^{d_j}}{d_j!}\right\} \cdot e^{-W(0, \tau; \theta)}$$

(2) Single unit, exact failure times $S_1, S_2, \dots, S_n$
Let grid $t_1, t_2, \dots$ be tighter and tighter so that max 1 event in each. Then $d_i = 0$ in most cases

and $d_j = 1$ in intervals containing failure times s_i.
For those: $W(t_{j-1}, t_j; \theta) = \int_{t_{i-1}}^{t_i} w(u;\theta)\,du \approx w(s_i)(t_i - t_{i-1})$

the failure time in the interval

can be assumed the same.

$\Downarrow$

$$L(\theta) = \left\{ \prod_{i=1}^{n} w(s_i;\theta) \right\} e^{-W(0,\tau;\theta)}$$

If several systems (see ford!)
Take product of likelihoods for each

$$L(\theta) = \prod_{j=1}^{m} \left[\left\{ \prod_{i=1}^{N_j} w(s_{ij};\theta) \right\} e^{-W(0,\tau_j;\theta)} \right]$$

so log-likelihood

$$\ell(\theta) = \sum_{j=1}^{m} \left[\sum_{i=1}^{N_j} \ln w(s_{ij};\theta) - W(0,\tau_j;\theta) \right]$$

~~6~~ - ~~13~~ - ~~18~~ - 15 - NY PAG.

Parametriske NHPP-modeller:

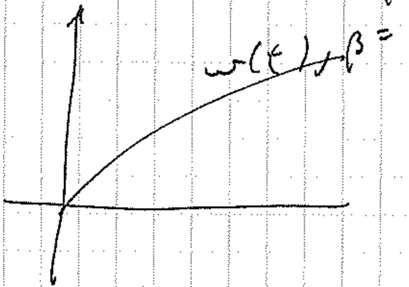
Har data av typen gitt tidligere
- enten én prosess
- flere prosesser

~~$w(t)$~~

Power law: $w(t|\lambda,\beta) = \lambda \beta t^{\beta-1}$

$W(t|\lambda,\beta) = \lambda t^{\beta}$

Only one in MINITAB.
$\boxed{w(t) = \frac{\alpha}{\theta}\left(\frac{t}{\theta}\right)^{\alpha-1}}$



Brukes til å modellere både ~~voksende / avtagende~~ $\nearrow$ og $\searrow$ trend.

Andre modeller: **Lineær:** $w(t) = \alpha + \beta t$
$W(t) = \alpha t + \frac{1}{2}\beta t^2$
(Problem med negativ ROCOF)

Loglineær modell:
(Cox-Lewis modellen)

$$w(t|\alpha,\beta) = e^{\alpha+\beta t}$$

Estimering:

Ønsker å estimere dette fra data.