Lecture week 10: 15/4-05

Chapter 7: Counting processes

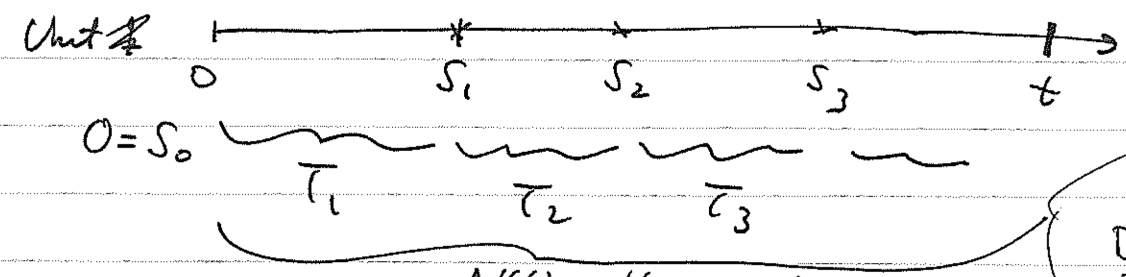
Repairable systems
Recurrent events

Typical: More than one event for each unit.

For example:

1. System is repaired and put into use
2. ~~Motorel~~ Machine part is replaced
3. Relapse from disease (epileptic seizures, reccurence of tumors)

Notation: Events on a time axis



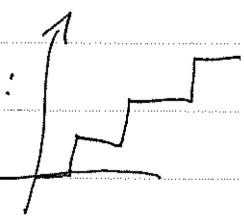
$N(t) = \#$ events have in $(0,t]$

Also: $N(s,t) = \#$events in $(s,t] = N(t) - N(s)$.

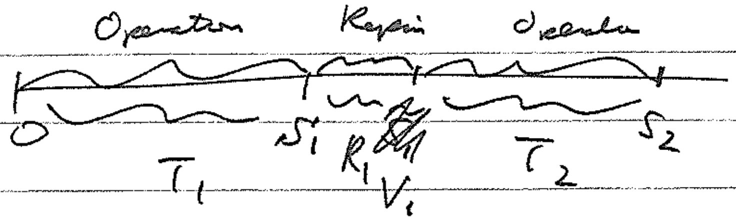
$S_1, S_2, \cdots$ are times of events

$T_1, T_2, \cdots$ are times between events

Draw $N(t)$: This is a counting process



Note: Common to disregard repair time, but could have situation like



An alternating process.
(see Q.7.3.11

Simplest type: $T_1, T_2, \cdots$ are independent with same distribution.
This is a <u>Renewal Process</u>

One such is Homogeneous Poisson Process.
when distrib of interfailure time is <u>exponential</u>.

But - usually $T_1, T_2, \cdots$ are neither ~~&~~ indep. nor identically dist.

Ascher & Feingold (book, 1984) distinguish between
Happy systems
Sad systems
+ (None)

- Valve seat data - -
- Grampus data
- Proschan's data
- Aalen - Husebye's data

More definitions -3-
and notation

<u>Def</u>.

$$W(t) =_{def} E(N(t)) = \text{Cumulative}\ \overset{\text{Rate of}}{\text{Occurrence}} = \text{forventet antall hendelser i } (0, t]$$

$w(t) =_{def} W'(t)$ kalles Rate of Occurence of Failures (ROCOF) for prosessen.

~~Vi ser at~~

Vi ser at

$$w(t) = \lim_{h \to 0} \frac{W(t+h) - W(t)}{h}$$

$$= \cancel{\lim_{h \to 0} E(N+h) - E}$$

$$= \lim_{h \to 0} \frac{E(N(t+h)) - E(N(t))}{h}$$

$$= \lim_{h \to 0} \frac{E(N(t+h) - N(t))}{h} = \lim_{h \to 0} \frac{E[N(t, t+h)]}{h}$$

$$= \lim_{h \to 0} \frac{\text{Forv. ant. feil i } (t, t+h]}{h}$$

dvs $w(t) =$ "Forv. ant. feil per tidsenhet ved tid t"
"Raten av feil ved tid t"

So $E[N(t, t+h)] \approx w(t)\, h$

Definér en <u>Regulær</u> <u>prosess</u> ved

at $P(N(t+h)-N(t) \geq 2) = o(h)$

dvs <u>liten</u> ifht
selv i forhold til h.

(I praksis: Regulær prosess betyr i praksis at
vi ikke kan ha to hendelser samtidig).

For en regulær prosess vil dermed

$$\frac{\text{Forv. ant. feil i } (t,t+h]}{h} = \frac{1 \cdot P(\text{ant. feil i } (t,t+h] = 1) + 2 \cdot P(\text{ant. feil i } (t,t+h]) + \dots}{h}$$

$$\frac{\sum_{k=0}^{\infty} k P(k \text{ feil i } (t,t+h))}{h} = \frac{0 \dots}{h} + \frac{1 \cdot P(1 \text{ feil i } (t,t+h))}{h}$$

$$+ 2 \cdot \frac{P(2 \text{ feil i } (t,t+h))}{h} + \dots$$

$$= \frac{P(1 \text{ feil i } (t,t+h))}{h} + \underbrace{\frac{2o(h)}{h}}_{\to 0} + \underbrace{\dots}_{\to 0}$$

$$\Rightarrow$$

dvs ROCOF $w(t) = \lim_{h \to 0} \frac{P(\text{feil i } (t,t+h))}{h}$

for en REGULÆR PROSESS.

dvs (ved regulær)

$$P(\text{hendelse} \in (t, t+h)) \approx w(t)\,h$$

Dette er ganske analogt med hasardraten for en levetid T: . . .

$$P(t < T \le t+h \mid T > t) \approx z(t)\,h$$

(~~Men~~ som i boka også kalles FOM = force of mortality. Men må ikke forveksles! Bruker for kompensator begge ordene til «dør».

~~Parametrisk~~

Modeller:

~~1. Renewal process~~ $T_1, T_2, \ldots$ er u.i.f.

2. HPP (Homogen Poisson Process, spesielt er denne renewal med eksp)

~~2. Renewal Process~~

3. NHPP (Nonhomogenous Poisson Process)

~~1. HPP - kjent fra grunnkurs. Knyttes til eksponensialfordelingen.~~ Kommer tilbake til

~~Def~~

1. Antar at $T_1, T_2, \ldots$ er uavh og identisk fordelte med fordeling $f(t)$, $z(t)$, $Z(t)$, $R(t)$ etc. Estimerer da ~~noen~~ fordelingene som før

   Boka -Ch. 7.3- ~~inneholder~~ mye om sannsynlighetsteori, feks om hvordan $N(t)$ og $W(t)$ oppfører seg

2. HPP er kjent fra grunnkurs + Stok-pros.

3. Viktig fordeling, som vi skal se på i detalj fremover.

$\boxed{\text{Kap 7.4 NHPP.}}$

Def

1. $N(0)=0$
2. $\{N(t), t \geq 0\}$ har uavh. inkrementer, dvs

$$0 \qquad t_1 \qquad t_2 \qquad \cdots \qquad t_{k-1} \qquad t_k$$

$N(t_1), N(t_2)-N(t_1), \ldots, N(t_k)-N(t_{k-1})$, er stok uavh.

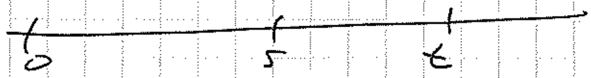
In other words: For two disjoint intervals $(s_1,t_1]$, $(s_2,t_2]$ are $N(s_1,t_1)$, $N(s_2,t_2)$ etc stoch. independent and Poisson distributed

3. Regulær: $P(N(t+h)-N(t) \geq 2) = o(h)$
4. $P(N(t+h)-N(t)=1) = w(t)h + o(h)$

$\boxed{\text{dvs ikke [illegible] stasjonære inkrementer}}$

$w(t)$ kalles intensitetsfunksjonen

Hovedresultat:



$$N(s,t) \equiv N(t) - N(s) \sim \text{Poisson}\left(\int_s^t w(u)\,du\right)$$
$$= \text{Poisson}(W(t) - W(s))$$

Spesialtilfellet HPP: $w(t) = \lambda$ = konstant

$$\Rightarrow N(t) - N(s) \sim \text{Poisson}(\lambda(t-s))$$

Dermed $N(t) \sim \text{Poisson}\left(\int_0^t w(u)\,du\right)$

$$\stackrel{=}{_{def}} W(t)$$

dvs $E(N(t)) = W(t)$ dvs

Men det betyr at $w(t) = W'(t)$

er ROCOF.

Fordel med NHPP framfor HPP og RP er at den kan brukes til å modellere en trend i feilintensiteten. Dette fordi som vi husker, $w(t) \cdot h \approx P(\text{feil i } (t, t+h))$

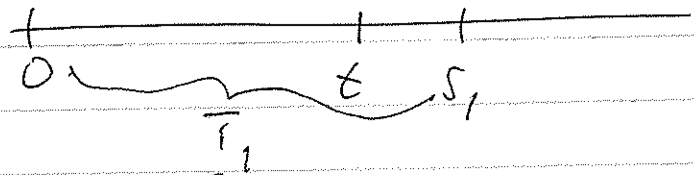
$w(t)$ voksende : Økende feilfrekvens ("ageing or deteriorating system")

$w(t) \downarrow$: Avtagende " ("improving system")

F.eks. software pålitelighet

Noen egenskaper ved NHPP:

Tid til første feil: Vanlig HPP(λ): Vet at dette er eksp(λ).



La T_1 = tid til første feil.

$$\underbrace{P(T_1 > t) = P(N(0,t)}_{\text{ekvivalent!}} = 0)$$

Poisson($\int_0^t w(t)$)

$$= \frac{W(t)^0}{0!} e^{-W(t)} = e^{-W(t)} \qquad (7.109)$$

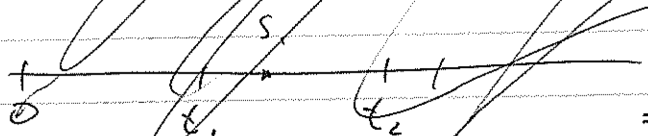
dvs $f_{T_1}(t) = -R_{T_1}'(t) = \underline{\underline{w(t) e^{-W(t)}}}$

(som kan være hvilken som helst fordeling.

Simultanfordeling for S_1 og S_2:

$$P(S_1 > t_1 \cap S_2 > t_2) = P(S_1 > t_2, S_2 > t_2) + P(t_1 < S_1 \le t_2, S_2 > t_2)$$

$=$



$$= P(N(0,t_2) = 0) + P(N(0,t_1) = 0 \cap N(t_1,t_2) = 1)$$

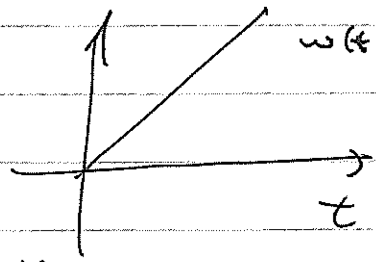
$$= e^{-W(t_2)} + e^{-W(t_1)}(W(t_2) - W(t_1))e^{-(W(t_2) - W(t_1))}$$

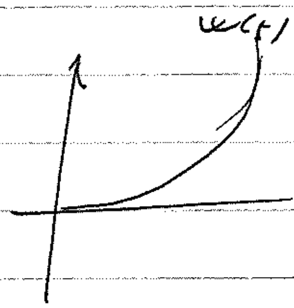
Example:

Suppose $T_1, T_2, \dots$ is an NHPP with

$w(t) = 2t$

$W(t) = t^2$



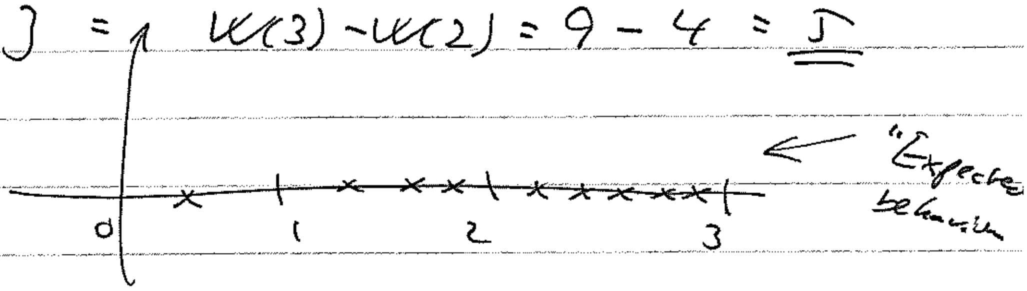


What is the expected # failures in $[0,1]$?

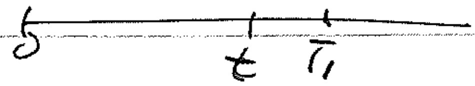
$$E[N(0,1)] = W(1) - W(0) = \underline{1}$$

$$E[N(1,2)] = W(2) - W(1) = 2^2 - 1^2 = \underline{\underline{3}}$$

$$E[N(2,3)] = W(3) - W(2) = 9 - 4 = \underline{\underline{5}}$$



Time to first failure:



$$R_{T_1}(t) = P(T_1 > t) = P(N(0,t) = 0) = \frac{W(t)^0}{0!} e^{-W(t)}$$

$$= e^{-t^2}$$

$$\Rightarrow f_{T_1}(t) = -R_{T_1}'(t) = 2te^{-t^2} \quad (= w(t)e^{-W(t)}$$

which is a Weibull-distribution