TMA 4275 Lifetime analysis
Exercise 8 - solution

Problem 1

a) The MINITAB output:

```
Censoring Information  Count
Uncensored value         18
Estimation Method: Maximum Likelihood
Distribution:   Weibull
Parameter Estimates
                 Standard    95.0% Normal CI
Parameter  Estimate    Error    Lower    Upper
Shape       1.48118  0.266873  1.04050  2.10850
Scale       1105.92   186.088  795.241  1537.99
Log-Likelihood = -140.300
```

The probability plot can be seen in the Figure 1. It can be seen from this figure, that all points are within the confidence intervals and close to the straight line. Therefore it can be concluded that Weibull model is a good fit to the data.
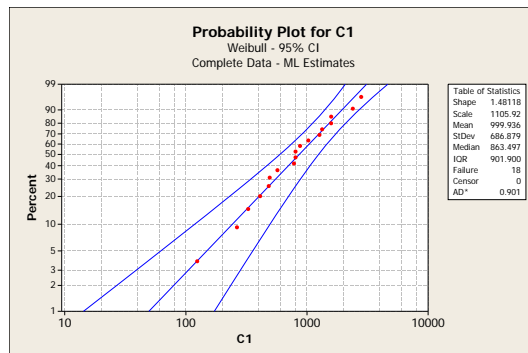

Figure 1: Probability plot for the Weibull fit

b) The MINITAB output:

```
Censoring Information  Count
Uncensored value         18
Estimation Method: Maximum Likelihood
Distribution:   Weibull
Relationship with accelerating variable(s):   Linear
Regression Table
                 Standard                  95.0% Normal CI
Predictor    Coef     Error      Z      P     Lower     Upper
Intercept  5.61015  0.306865  18.28  0.000  5.00870   6.21159
```

```
C2          0.649879  0.143448   4.53  0.000   0.368725  0.931033
Shape       2.21171   0.429650                 1.51138   3.23656
Log-Likelihood = -134.047
```

From the MINITAB output and from the plots in the Figure 2 it can be seen that there
is clear effect of stress factor to the lifetimes (the estimated coefficients are significantly
different from 0 since the p-values are neglible and the points in the probability plot are
clearly separated along three different lines with the same slope). This cannot be seen
from the general Weibull model.

Probability plot is the classical probability plot as a function of the stress variable and the
relation plot is the estimated percentile plot as a function of the stress variable. See the
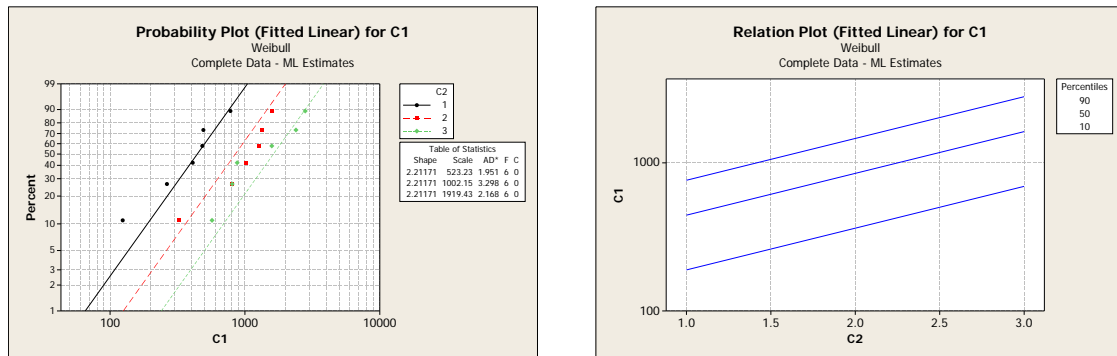lecture slides for more detailed explanation of the plots.

Figure 2: Probability plot and relation plot for ALT

c) The loglikelihood for the pure Weibull model is lower than the loglikelihood for the ALT
model. Using the standard approach by computing twice the difference between the log-
likelihood gives $2(-134.047-(-140.300)) = 12.56$ which is χ_1^2 distributed. Comparing this
value with the quantiles of the χ_1^2 distribution leads to the rejection of the null hypothesis
that there is no effect of the stress factor. Therefore we can conclude that the ALT model
fits better than the pure Weibull model.