TMA 4275 Lifetime Analysis 2020
Homework 3

Problem 1

Let the observed life times (no censorings) for 12 identical components be:

10.2, 89.6, 54.0, 96.0, 23.3, 30.4, 41.2, 0.8, 73.2, 3.6, 28.0, 31.6

a) Analyse these data using Minitab:

Note that Minitab uses decimal comma instead of point (i.e., 10,2 instead of 10.2)

First use Stat > Basic Statistics to obtain summary statistics and standard plots for the data.

Then use Stat > Reliability/Survival > Distribution Analysis (Right Censoring) to do some parametric (exponential, Weibull, etc.) and non-parametric analyses (Kaplan-Meier, i.e. empirical survival function) of the data.

b) **(Optional)** Analyse the data using R.

(See also R-code given in the lecture plan.)

```
library(survival)
Time = c(10.2,...)
Cens = c(1,1,...)
Data = Surv(Time, Cens)
# or you can use  Data=Surv(Time) since there are here no censorings

#SUMMARY STATISTICS
summary(Data)
sd(Data)

#KAPLAN-MEIER ESTIMATOR
result.km = survfit(Data ~ 1, conf.type="plain")
summary(result.km)
plot(result.km)

#PARAMETRIC ESTIMATION
result.weib = survreg(Data ~ 1, dist="weibull")
summary(result.weib)
```

Instead of Weibull you may write, for example, `"exponential"`, `"lognormal"`, `"loglogistic"`, and some other distributions.

Note that `survreg` parametrizes the Weibull distribution based on its log-location-scale representation μ (Intercept) and σ (scale), while in our parametrization (α, θ) of the Weibull we have $\mu = \ln \theta$ and $\sigma = 1/\alpha$.

Problem 2

a) Load into Minitab the Repair Times data from the course webpage "Data sets". You may either upload the Minitab worksheet (.MTW) which will automatically start Minitab, or copy the .txt file into the Minitab worksheet after you have opened Minitab. (Try both ways!)

These data are 90 repair times for a certain system and can be treated as a complete (i.e. non-censored) dataset of lifetimes.

Plot the *empirical distribution function* using Graph > Empirical CDF. What is the difference between this plot and the *empirical survival function* that you would get by using the Kaplan-Meier method with no censored observations?

It is argued that a log-normal distribution fits the data well (and fits repair data in general). Check this for the present data by plots and analyses using Minitab. Try also other distributions. What is the conclusion?

b) **(Optional)** Use R with the same data and plots etc. The empirical distribution can be plotted by the command `plot(ecdf(Time))`

Problem 3

a) Let T be exponentially distributed with failure rate λ, and let V be exponentially distributed with failure rate μ. Assume that T and V are stochastically independent. Let $Z = \min(T, V)$.

Show that Z is exponentially distributed with failure rate $\lambda + \mu$.

b) Consider a serial system with two components, A and B, with survival times T and V as above. What is the survival function $R(t)$ of this system? (Recall that a series system works as long as both components work).

Assume that the system fails. What is the probability that component A is the failure cause?

c) Consider now instead a parallel system with the two components A and B with lifetime distributions as above. What is now the survival function $R(t)$ of the system? (Recall that a parallel system works as long as at least one of the components works).

d) Suppose now that component A is used alone until it fails, and then component B is activated. The system is assumed to fail when B fails. Express the lifetime of this system as a function of T and V.

What is the MTTF of this system?

What is now the survival function $R(t)$ of the system?