TMA4275 LIFETIME ANALYSIS

Slides 13: Accelerated lifetime models

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Suppose we want to find the distribution ($R(t)$, MTTF, etc.) for the lifetime of a product.

Problem: MTTF may be so *large* that one would need to let experiments last several years.

Solution: Increase stress, use a regression model, and then extrapolate to normal conditions.

Suppose you want to investigate the deterioration of an insulation used for electric motors. The motors normally run between 80 and 100 degrees Celsius.

To save time and money, you decide to use *accelerated life testing.*

First you gather failure times for the insulation at abnormally high temperatures: 110, 130, 150, and 170 degrees Celsius, to speed up the deterioration. With failure time information at these temperatures, you can then extrapolate to 80 and 100 degrees.

It is known that an *Arrhenius* relationship exists between temperature and failure time.

This is an example which is included in MINITAB.

THE ARRHENIUS MODEL

We need a connection between T = *lifetime*; s = *stress* (temperature).

Some standard relations are known to be useful in accelerated testing:

$$\ln T = \beta_0 + \beta_1(\text{function of stress}) + \sigma U$$

i.e. $\ln T = \beta_0 + \beta_1 g(s) + \sigma U$ for some function $g(\cdot)$ of the stress.

The model used in the example is the Arrhenius model:

$$\ln T = \beta_0 + \beta_1 \cdot \frac{11604.83}{s + 273.16} + \frac{1}{\alpha} W$$

where W is Gumbel distributed and s = temperature in $^\circ\mathrm{C}$,
so $s + 273.16$ = temp in $^\circ\mathrm{K}$ (absolute temperature).

This is the same as computing a transformed covariate

$$x = g(s) = \frac{11604.83}{s + 273.16}$$

1. **Inverse temperature:**

$$\ln T = \beta_0 + \beta_1 \cdot \frac{1}{\text{temperature in } {}^\circ\mathrm{C} + 273.16} + \sigma U$$

i.e. $g(s) = \frac{1}{s+273.16}$

2. **Ln (power)**:

$$\ln T = \beta_0 + \beta_1 \ln(\text{accelerating variable}) + \sigma U$$

i.e. $g(s) = \ln s$

3. **Linear:**

$$\ln T = \beta_0 + \beta_1 \cdot \text{accelerating variable} + \sigma U$$

i.e. $g(s) = s$

We could have done all this with ordinary lifetime regression in MINITAB, but the ALT module has in addition some intersting *plots*.

MINITAB INSULATION EXAMPLE

1. Open the worksheet INSULATE.MTW (from where you installed MINITAB)
2. Choose Stat $>$ Reliability/Survival $>$ Accelerated Life Testing.
3. In Variables/Start variables, enter FailureT. In Accelerating variable, enter Temp.
4. From Relationship, choose Arrhenius.
5. Click Censor. In Use censoring columns, enter Censor, then click OK.
6. Click Graphs. In Enter design value to include on plot, enter 80. Click OK.
7. Click Estimate. In Enter new predictor values, enter Design, then click OK in each dialog box.

Insulate.MTW ***

↓	C1	C2	C3	C4	C5-T	C6	C7	C8	C9	C10	C11
	Temp	ArrTemp	Plant	FailureT	Censor	Design	NewTemp	ArrNewT	NewPlant		
1	170	26,1865	1	343	F	80	80	32,8600	1		
2	170	26,1865	1	869	F	100	80	32,8600	2		
3	170	26,1865	1	244	C		100	31,0988	1		
4	170	26,1865	1	716	F		100	31,0988	2		
5	170	26,1865	1	531	F						
6	170	26,1865	1	738	F						
7	170	26,1865	1	461	F						
8	170	26,1865	1	221	F						
9	170	26,1865	1	665	F						
10	170	26,1865	1	384	C						
11	170	26,1865	2	394	C						
12	170	26,1865	2	369	F						
13	170	26,1865	2	366	F						
14	170	26,1865	2	507	F						
15	170	26,1865	2	461	F						
16	170	26,1865	2	431	F						
17	170	26,1865	2	479	F						
18	170	26,1865	2	106	F						
19	170	26,1865	2	545	F						
20	170	26,1865	2	536	F						
21	150	27,4242	1	2134	C						
22	150	27,4242	1	2746	F						
23	150	27,4242	1	2859	F						
24	150	27,4242	1	1826	C						

MINITAB Help

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Example of Accelerated Life Testing

main topic interpreting results session command see also

Suppose you want to investigate the deterioration of an insulation used for electric motors. The motors normally run between 80 and 100° C. To save time and money, you decide to use accelerated life testing.

First you gather failure times for the insulation at abnormally high temperatures – 110, 130, 150, and 170° C – to speed up the deterioration. With failure time information at these temperatures, you can then extrapolate to 80 and 100° C. It is known that an Arrhenius relationship exists between temperature and failure time.

To see how well the model fits, you will draw a probability plot based on the standardized residuals.

1. Open the worksheet INSULATE.MTW.
2. Choose **Stat > Reliability/Survival > Accelerated Life Testing**.
3. In **Variables/Start variables**, enter **FailureT**. In **Accelerating variable**, enter **Temp**.
4. From **Relationship**, choose **Arrhenius**.
5. Click **Censor**. In **Use censoring columns**, enter **Censor**, then click **OK**.
6. Click **Graphs**. In **Enter design value to include on plot**, enter **80**. Click **OK**.
7. Click **Estimate**. In **Enter new predictor values**, enter **Design**, then click **OK** in each dialog box.

Session window output

Regression with Life Data: FailureT versus Temp

```
Response Variable:  FailureT

Censoring Information              Count
Uncensored value                      66
Right censored value                  14
Censoring value:  Censor = C

Estimation Method:  Maximum Likelihood
Distribution:  Weibull
Transformation on accelerating variable:  Arrhenius
```

```
Regression Table
                          Standard                          95.0% Normal CI
Predictor        Coef        Error        Z      P        Lower        Upper
Intercept    -15.1874       0.9862   -15.40  0.000     -17.1203     -13.2546
Temp          0.83072      0.03504    23.71  0.000      0.76204      0.89940
Shape          2.8246       0.2570                       2.3633       3.3760

Log-Likelihood = -564.693

Anderson-Darling (adjusted) Goodness-of-Fit

At each accelerating level
Level      Fitted Model
110                  *
130                  *
150                  *
170                  *

Table of Percentiles
                                      Standard        95.0% Normal CI
 Percent        Temp  Percentile         Error        Lower        Upper
      50     80.0000    159584.5      27446.85     113918.2     223557.0
      50    100.0000    36948.57      4216.511     29543.36     46209.94
```

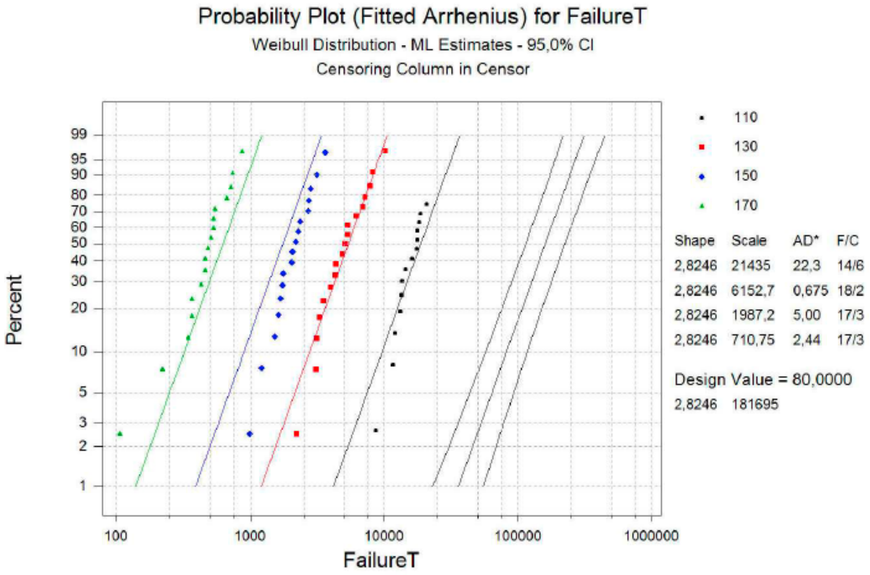
Recall the Arrhenius model:

$$\ln T = \beta_0 + \beta_1 \cdot \frac{11604.83}{s + 273.16} + \frac{1}{\alpha} W$$

where W is Gumbel and s = temperature in °C.

Estimated model:

$$\ln T = -15.874 + 0.83072 \cdot \frac{11604.83}{s + 273.16} + \frac{1}{2.8246} W$$


Probability Plot (Fitted Arrhenius) for FailureT
Weibull Distribution - ML Estimates - 95,0% CI
Censoring Column in Censor
Percent
99
95
90
80
70
60
50
40
30
20
10
5
3
2
1
100
1000
10000
100000
1000000
FailureT
110
130
150
170
Shape Scale AD* F/C
2,8246 21435 22,3 14/6
2,8246 6152,7 0,675 18/2
2,8246 1987,2 5,00 17/3
2,8246 710,75 2,44 17/3
Design Value = 80,0000
2,8246 181695

Normal temperature is 80-100C.

Experiment temperatures: 110, 130, 150, 170. Needs to extrapolate to 80-100, using Arrhenuis model.

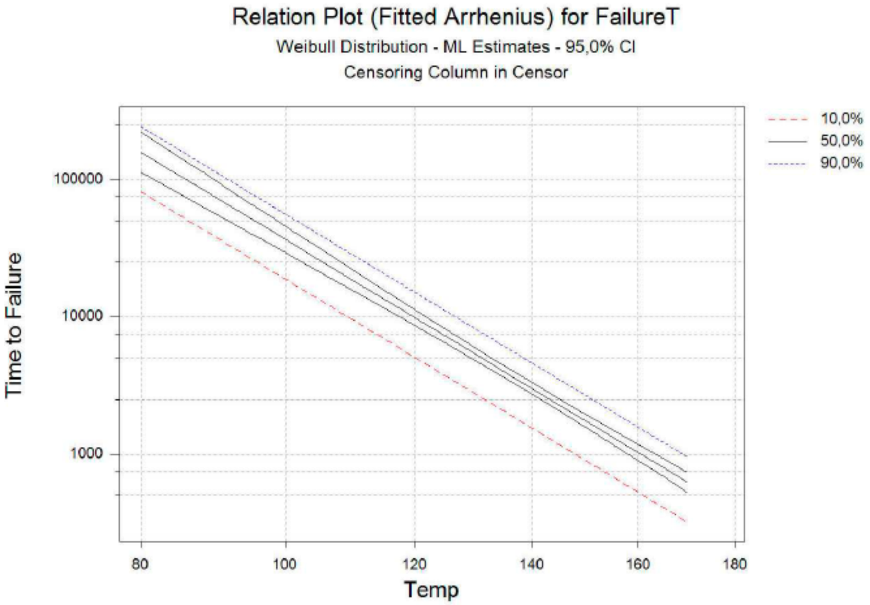
Recall probability plot for Weibull:

$\ln(-\ln R(t)) = \alpha \ln t - \alpha \ln \theta$

So:

- Slope α is the same for all lines
- Scale $\theta = \exp\{\beta_0 + \beta_1 \cdot \frac{11604.83}{s+273.16}\}$ depends on temperature s.

RELATION PLOT



Plot $\hat{t}_p(s)$ as function of s.

Recall general formula:

$$\ln \hat{t}_p(\mathbf{x}) = \beta_0 + \boldsymbol{\beta}'\mathbf{x} + \sigma\Psi^{-1}(p)$$

where for Weibull/Gumbel we have $\Psi^{-1}(p) = \ln(-\ln(1-p))$.

In example:

$$\ln \hat{t}_p(s) = -15.1874 + 0.83072 \cdot \frac{11.60483}{s + 273.16} + \frac{1}{2.8246} \cdot \ln(-\ln(1-p))$$

Figure shows median, $p = 0.50$, together with 95% confidence curves; and in addition the curves for $p = 0.10$ and $p = 0.90$.