TMA4275 LIFETIME ANALYSIS

Slides 1: Introduction

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After finishing this course you should ....

- know the most common concepts and distributions from lifetime modeling
- be able to use graphical methods for description and comparison of lifetime data
- be able to use statistical methods for statistical inference (estimation, confidence interval, hypothesis testing) of lifetime data
- be able to analyze lifetime data by using computer software (MINITAB)

Reliability engineering:

- Time to failure of a component or a system
- Number of cycles to failure (fatigue testing)
- Times between successive failures of a machine

Medical research:

- Time to death of a patient after start of certain treatment
- Time from entrance to discharge from a hospital
- Times between successive epileptic seizures for patient

Common technical definition of reliability:

The probability that a system or a component will perform its intended task, under given operational conditions, for a specified time period.

Lifetime (survival time) in medical research:

Time to occurrence of some event of interest for individuals in some population. The event may or may not be "death", and is often referred to as "failure".

WHY COLLECT AND ANALYZE LIFETIME/SURVIVAL/RELIABILITY DATA?

Reliability engineering:

- Assess reliability of a system/component/product
- Compare two or more products with respect to reliability
- Predict product reliability in the design phase
- Predict warranty claims for a product in the market

Medical research:

- Compare different treatments with respect to survival or recurrence
- Predict the outcome of an intervention or the life expectancy after the invterention
- Identify risk factors for diseases and assess their magnitude

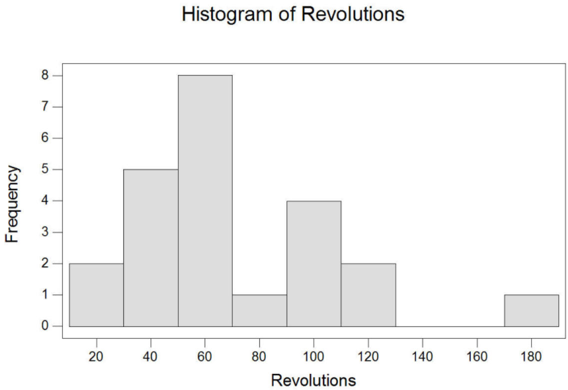
SPECIAL ASPECTS OF LIFETIME ANALYSIS IN STATISTICS

- Definition of starting time and failure time are difficult
- Definition of time scale (operation time, calendar time, number of cycles)
- Censored data (how can we use data from individuals or units for which the event of interest has not occurred within the observation period?)
- Effect of covariates (demographic, medical, environmental)
- What if an individual or unit dies or fails of another cause than the one we would like to study? ("competing risks")
- Recurrent events – what if the system can fail several times; how to analyze recurring stages of a disease?

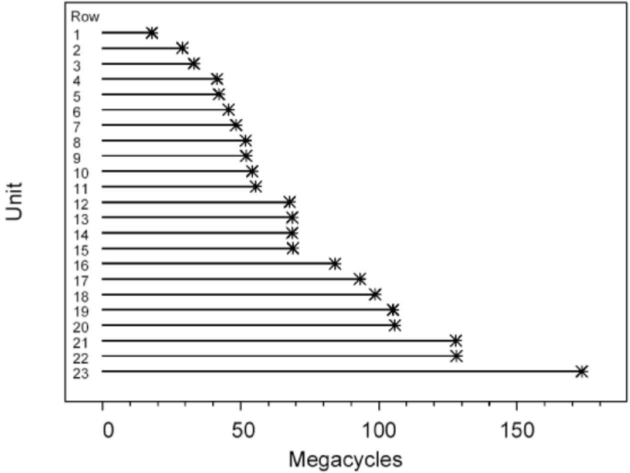
Data: Millions of revolutions to fatigue failure for 23 units

Question: How can we fit a parametric lifetime distribution to these data?

17,88	28,92	33,00	41,52	42,12	45,60	48,40	51,84
51,96	54,12	55,56	67,80	68,64	68,64	68,88	84,12
93,12	98,64	105,12	105,84	127,92	128,04	173,40	



BALL BEARING FAILURE DATA (EVENT PLOT)



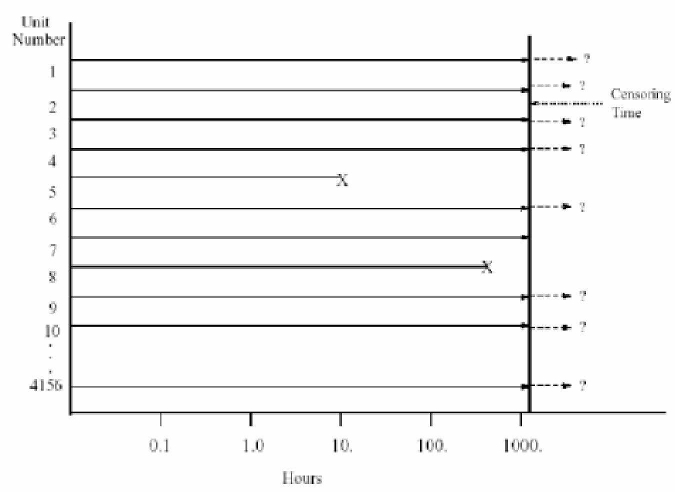
- Integrated circuit failure times in hours
  - n = 4156 ICs tested for 1,370 hours at 80° C and 80% relative humidity
  - There were 28 failures
  - When the test ended at 1,370 hours, 4128 units were still running

.10	.10	.15	.60	.80	.80
1.20	2.5	3.0	4.0	4.0	6.0
10.0	10.0	12.5	20.	20.	43.
43.	48.	48.	54.	74.	84.
94.	168.	263.	593.		

Questions of interest:

- How to estimate the distribution of the failure time when there are censored observations?
- Probability of failure before 100 hours?
- Failure rate by 100 hours?
- Proportion failed after 10^5 hours?

IC DATA (EVENT PLOT)



- *Multiple myeloma* is a malignant disease characterised by the accumulation of abnormal plasma cells, a type of white blood cell, in the bone marrow.
- Data (next slide) from Medical Center of the University of West Virginia, USA.
- **Aim:** To examine the association between certain explanatory variables or covariates and the survival time of patients in months from diagnosis until death from multiple myeloma).

Table 1.3 *Survival times of patients in a study on multiple myeloma.*

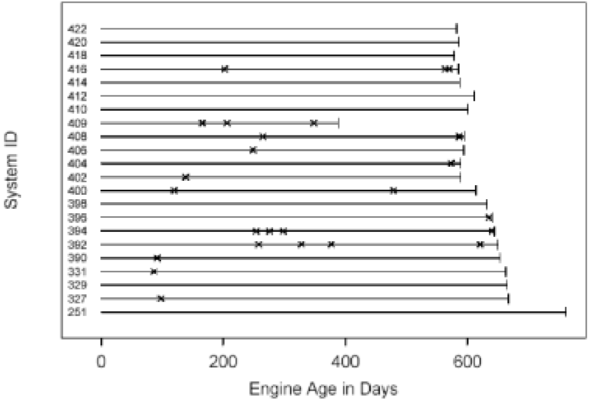
Patient number	Survival time	Status	Age	Sex	*Bun*	*Ca*	*Hb*	*Pcells*	*Protein*
1	13	1	66	1	25	10	14.6	18	1
2	52	0	66	1	13	11	12.0	100	0
3	6	1	53	2	15	13	11.4	33	1
4	40	1	69	1	10	10	10.2	30	1
5	10	1	65	1	20	10	13.2	66	0
6	7	0	57	2	12	8	9.9	45	0
7	66	1	52	1	21	10	12.8	11	1
8	10	0	60	1	41	9	14.0	70	1
9	10	1	70	1	37	12	7.5	47	0
10	14	1	70	1	40	11	10.6	27	0
11	16	1	68	1	39	10	11.2	41	0
12	4	1	50	2	172	9	10.1	46	1
13	65	1	59	1	28	9	6.6	66	0
14	5	1	60	1	13	10	9.7	25	0
15	11	0	66	2	25	9	8.8	23	0
16	10	1	51	2	12	9	9.6	80	0
17	15	0	55	1	14	9	13.0	8	0
18	5	1	67	2	26	8	10.4	49	0
19	76	0	60	1	12	12	14.0	9	0
20	56	0	66	1	18	11	12.5	90	0

Problem 2

A clinical trial to evaluate the efficacy of chemotherapy for a specific cancer was conducted. After reaching a state of remission (disappearance of cancer) through treatment, the patients who entered the study were randomized into two groups. The first group received maintenance chemotherapy, the second (or control) group did not. For a preliminary analysis during the course of the trial the data were as follows:
Length of complete remission (in weeks).

Maintenance group: $9, 13, 13^+, 23, 24^+, 34, 45^+, 55, 161^+$
Control group: $5, 13, 13, 16^+, 20, 21, 43, 45$
$+$ indicates censored observation.

Valve Seat Replacement Times Event Plot
(Nelson and Doganaksoy 1989)

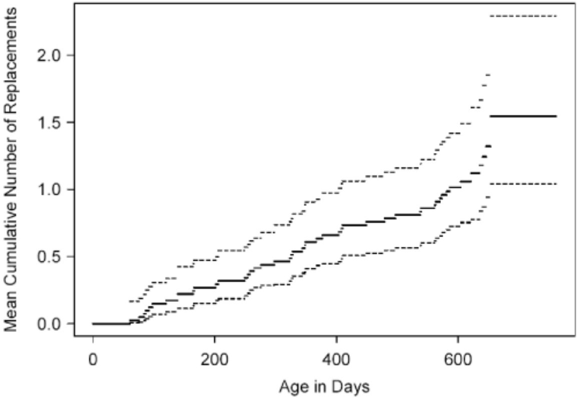


Data on previous slide are collected from valve seats from a fleet of 41 diesel engines. Each engine has 16 valves. (Time unit is days of operation).

Questions of interest:

- Does the replacement rate increase with age?
- How many replacement valves will be needed in the future?
- Can valve life in these systems be modeled as a renewal process?

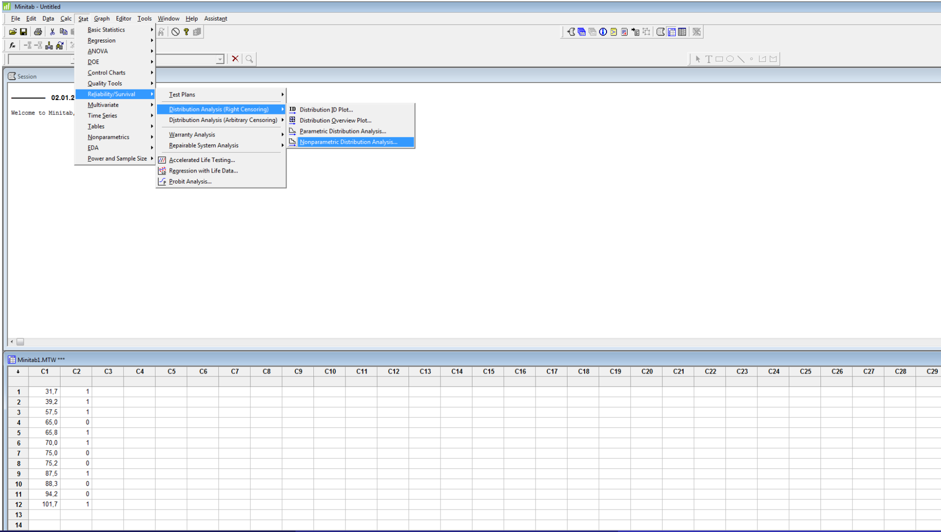
- Middle curve is cumulative estimated number of replacements for one engine, as a function of age.
- Lower and upper curves are 95% confidence limits.



MINITAB EXAMPLE
- THE DATA WORKSHEET

C1: Lifetimes

C2: Censoring indicators (0 = censored)



MINITAB EXAMPLE
- ESTIMATION OF SURVIVAL FUNCTION