

# TMA4275 Lifetime Analysis (Spring 2019)

## Exercise 5

### Problem 1 – The two-parameter exponential distribution

The two-parameter exponential distribution has density

$$f(t; \theta, \gamma) = \frac{1}{\theta} \exp \left\{ -\frac{t - \gamma}{\theta} \right\} \text{ for } t \geq \gamma$$

where  $\theta > 0$ ,  $\gamma \geq 0$ .

Assume that we have a right censored sample  $(y_i, \delta_i)$ ,  $i = 1, \dots, n$  from this distribution. Assume that all censorings take place at or after time  $\gamma$ .

- a) Find the log-likelihood function  $l(\theta, \gamma)$  for these data.
- b) Let  $(\hat{\theta}, \hat{\gamma})$  be the maximum likelihood estimators of  $(\theta, \gamma)$ . Why do we have

$$\hat{\gamma} \leq y_{(1)}$$

where  $y_{(1)}$  is the smallest time among  $y_1, \dots, y_n$ ?

Then find explicit expressions for  $(\hat{\theta}, \hat{\gamma})$ . Show in particular that we always have  $\hat{\gamma} = y_{(1)}$

- c) In the lectures we have considered a likelihood method for constructing confidence intervals for one of two parameters in a model. The method uses the following:

Let  $\hat{\gamma}(\theta)$  be the MLE of  $\gamma$  when  $\theta$  is given. Then

$$W(\theta) = 2(\ell(\hat{\theta}, \hat{\gamma}) - \ell(\theta, \hat{\gamma}(\theta)))$$

is approximately  $\chi_1^2$  when  $\theta$  is the true parameter. (Note that  $\tilde{\ell}(\theta) = \ell(\theta, \hat{\gamma}(\theta))$  is the so-called profile log likelihood of  $\theta$ .)

Explain how this can be used to construct a confidence interval for  $\theta$ . Do the calculations of the interval as far as you get.

- d) Use MINITAB to estimate the parameters when the Pike cancer data (see page 25 of Slides 10 from lectures) are assumed to follow a two-parameter exponential distribution.
- e) Reconsider the assumption in the beginning of the Problem, that all censorings take place at or after time  $\gamma$ . Can you think of cases where censorings also before time  $\gamma$  are possible? In such cases, how should the analysis in b) be modified?

## Problem 2 – Censoring and truncation

$n = 10$  units with exponentially distributed life times and  $\text{MTTF} = \theta$  are put on test. At time  $c = 10$  the test is ended (type I censoring), and  $r = 4$  units have failed by that time. The observed lifetimes are

0.9, 2.8, 5.9, 7.4

- a) Write down the likelihood function and compute the MLE for  $\theta$ . Which are the assumptions behind this approach?
- b) Assume now that at the end of the experiment ( $c = 10$ ) one does not know how many units were put on test, but only knows that the experiment has gone for 10 time units, with  $r = 4$  failures at the times given.  
How can you write down a likelihood for this case? (Hint: This is right truncation, see page 3 of Slides 11 from lectures).  
Which are the assumptions behind this likelihood?
- c) Maximize the likelihood in (b) to find the MLE under the conditions given there.

## Problem 3 – Weibull regression

Do Problem 1 b,c in Exam 2013V.