TMA4275 Lifetime Analysis (Spring 2013)
Exercise 7

Problem 1 – The two-parameter exponential distribution

The two-parameter exponential distribution has density

$$f(t;\theta,\gamma) = \frac{1}{\theta}\exp\left\{-\frac{t-\gamma}{\theta}\right\} \text{ for } t \geq \gamma$$

Assume we have a censored random sample (y_i, δ_i), $i = 1, \ldots, n$ from this distribution.

a) Find the log-likelihood function $l(\theta, \gamma)$ for these data.

b) Let $(\hat{\theta}, \hat{\gamma})$ be the maximum likelihood estimators of (θ, γ). Verify that

$$\hat{\gamma} \leq t_1$$

where t_1 is the smallest observed time among $y_1, \ldots, y_n$.

Then find explicit expressions for $(\hat{\theta}, \hat{\gamma})$. Show in particular that we always have $\hat{\gamma} = t_1$

c) In the lectures we have considered a likelihood method for constructing confidence intervals for one of two parameters in a model. The method uses the following:

Let $\hat{\gamma}(\theta)$ be the MLE of γ when θ is given. Then

$$W(\theta) = 2(l(\hat{\theta}, \hat{\gamma}) - l(\theta, \hat{\gamma}(\theta)))$$

is approximately χ^2_1 when θ is the true parameter.

Explain how this can be used to construct a confidence interval for θ. Do the calculations of the interval as far as you get.

d) Use MINITAB to estimate the parameters when the Pike cancer data (see page 91 of Slides 3 from lectures) are assumed to follow a two-parameter exponential distribution.

Problem 2 – Censoring and truncation

$n = 10$ units with exponentially distributed life times and MTTF$= \theta$ are put on test. At time $c = 10$ the test is ended (type I censoring), and $r = 4$ units have failed by that time. The observed lifetimes are

$$0.9,\ 2.8,\ 5.9,\ 7.4$$

a) Write down the likelihood function and compute the MLE for θ. Which are the assumptions behind this approach?

b) Assume now that one at the end of the experiment ($c = 10$) does not know how many units were put on test, but only knows that the experiment has gone for 10 time units, with $r = 4$ failures at the times given.

How can you write down a likelihood for this case? (Hint: This is right truncation, see page 95 of Slides 4 from lectures).

Which are the assumptions behind this likelihood?

c) Maximize the likelihood in (b) to find the MLE under the conditions given there.

Problem 3 – Estimation and testing in the gamma distribution

Assume that the life time T is gamma distributed $(2, \lambda)$ (see Ch. 2.11 in the book). We have n independent observations $t_1, \ldots, t_n$ of T (no censoring).

a) Find the MLE $\hat{\lambda}$ for λ. What are the properties of this estimator?

b) What is the estimate for λ when $n = 10$, $\sum t_i = 180$ (months)? Also find an estimate for the variance of $\hat{\lambda}$.

c) Perform a test based on likelihood

$$H_0: \ \lambda = 0.25 \text{ versus } H_1: \ \lambda \neq 0.25$$

What is the conclusion if the significance level is 5%?

d) Make a confidence interval for λ using the loglikelihood.