Censored and Truncated Data

- An observation is right censored at y:
  Unit is in our data, we know $T > y$.
  Contribution to L: $P(T > y) = R(y)$.

- An observation is left censored at y:
  Unit is in our data, we know $T < y$.
  Contribution to L: $P(T < y) = F(y)$.

- An observation is right truncated at y:
  Unit is in our data only if $T \leq y$. We do not know about the units with $T > y$.
  Contribution to L of observed failure at t:
  $\Delta^{-1} P(t \leq T \leq t + \Delta | T \leq y) \approx f(t)/F(y)$.

- An observation is left truncated at y:
  Unit is in our data only if $T \geq y$. We do not know about the units with $T < y$.
  Contribution to L of observed failure at t:
  $\Delta^{-1} P(t \leq T \leq t + \Delta | T \geq y) \approx f(t)/R(y)$.

Examples of left truncation:

- Ultrasonic inspection of material. Signal amplitude only trusted when above limit τ. Condition for being in the data set is $T > \tau$.

- Life data with pretest screening. Electronic component is burn-in tested for 1000 hours. Only the ones that passed this test are observed later. The number of components failing at burn-in is unknown. Condition for being in the data set is $T > 1000$.

Example of right truncation:

- Casting for automobile engine mounts. Pore size distribution below 10 microns only are recorded (other units are immediately discarded). Condition for being in the data set is $T < 10$ microns.

- Study group of individuals with AIDS diagnosis before July 1, 1986, and known date of HIV-infection (due to blood-transfusion). Let T_i = time from HIV-infection to AIDS diagnosis for ith individual. Then condition for being in the data set is that $T_i \leq v_i$ where v_i is time from HIV-infection of the ith individual to July 1, 1986. (Kalbfleisch and Lawless, 1989)

COMPUTER PROGRAM EXECUTION TIME vs SYSTEM LOAD

Data: 17 observations of (T,x)

- Time to complete a computationally intensive task.
- Information from the Unix uptime command
- Predictions needed for scheduling subsequent steps in a multi-step computational process.

Seconds (T)	Load (x)	Seconds (T)	Load (x)
123	2,74	110	,60
704	5,47	213	2,10
184	2,13	284	3,10
113	1,00	317	5,86
94	,32	142	1,18
76	,31	127	,57
78	,51	96	1,10
98	,29	111	1,89
240	,96		

Covariates (explanatory variables) for failure times

Useful covariates explain/predict why some units fail quickly and some units survive a long time:

- Continuous variables like stress, temperature, voltage, and pressure.
- Discrete variables like number of hardening treatments or number of simultaneous users of a system.
- Categorical variables like manufacturer, design, and location.

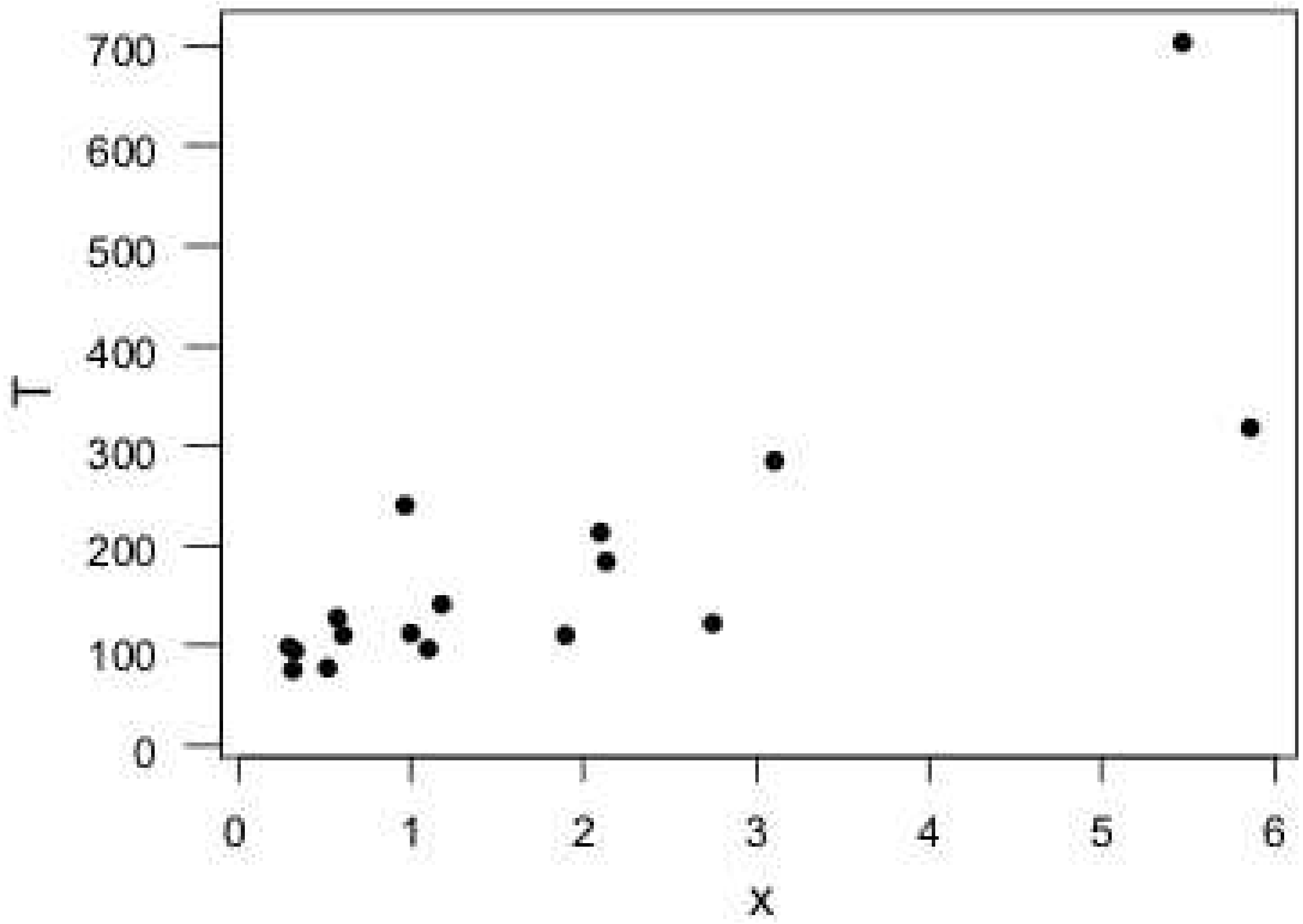
Regression model relates failure time distribution to covariates $x = (x_1, \ldots, x_k)$:

$$P(T \leq t) = F(t) = F(t; x)$$

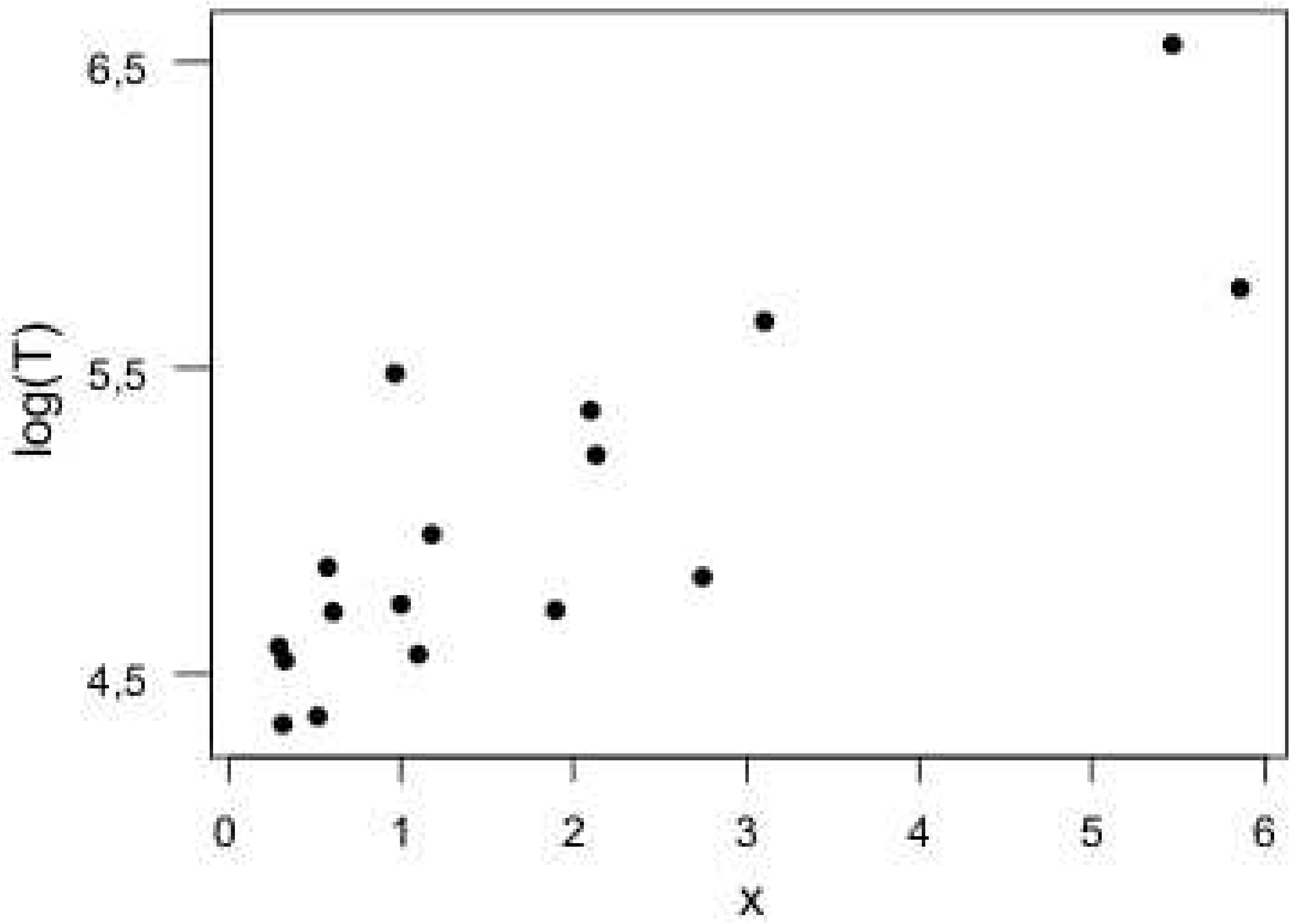
Why regression models?

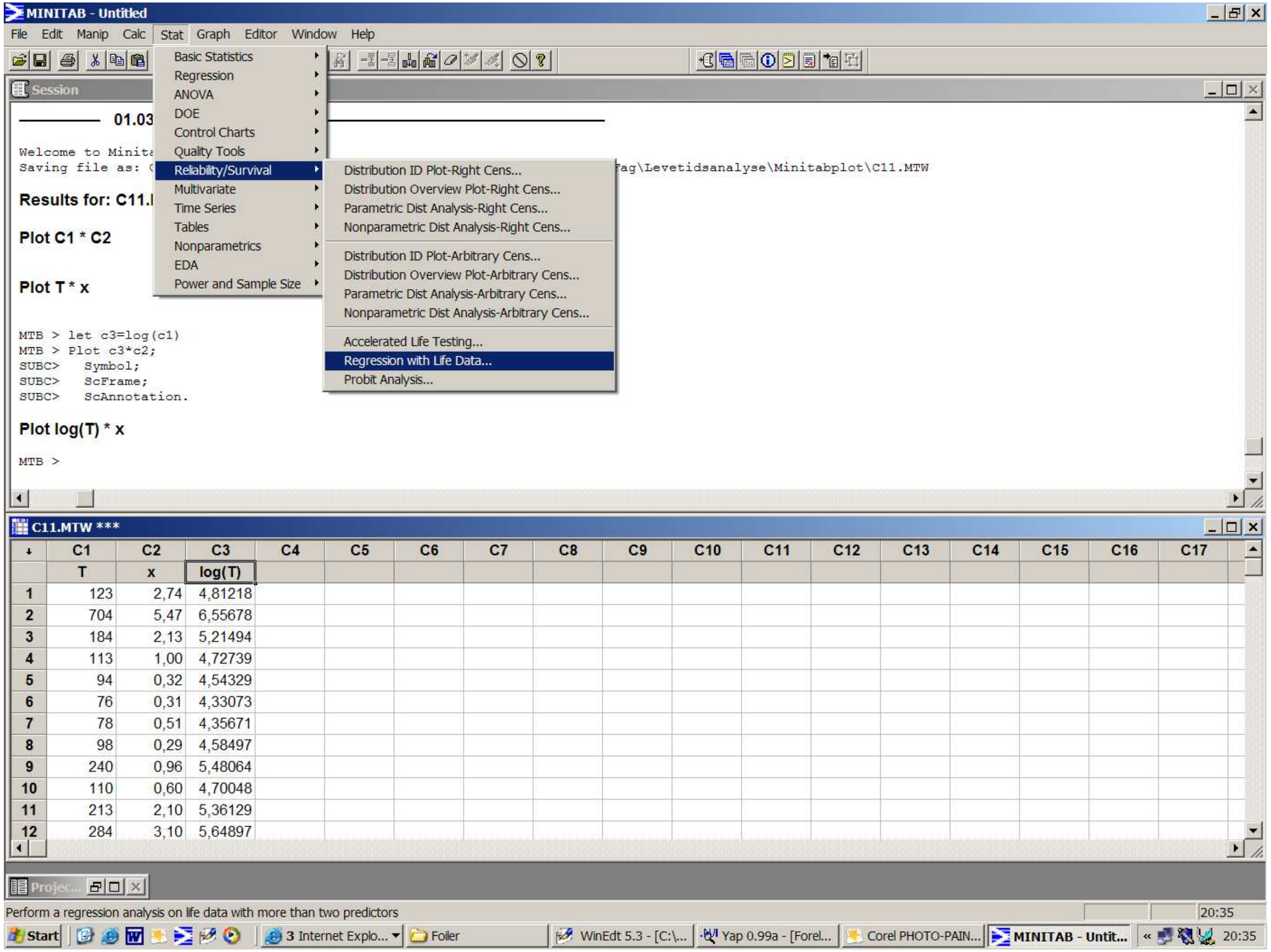
- Want to find factors which explain the reliability of an item
- Want to exclude factors which do not influence the reliability
- Obtain new knowledge about failure mechanisms
- Make better predictions for reliability of an item

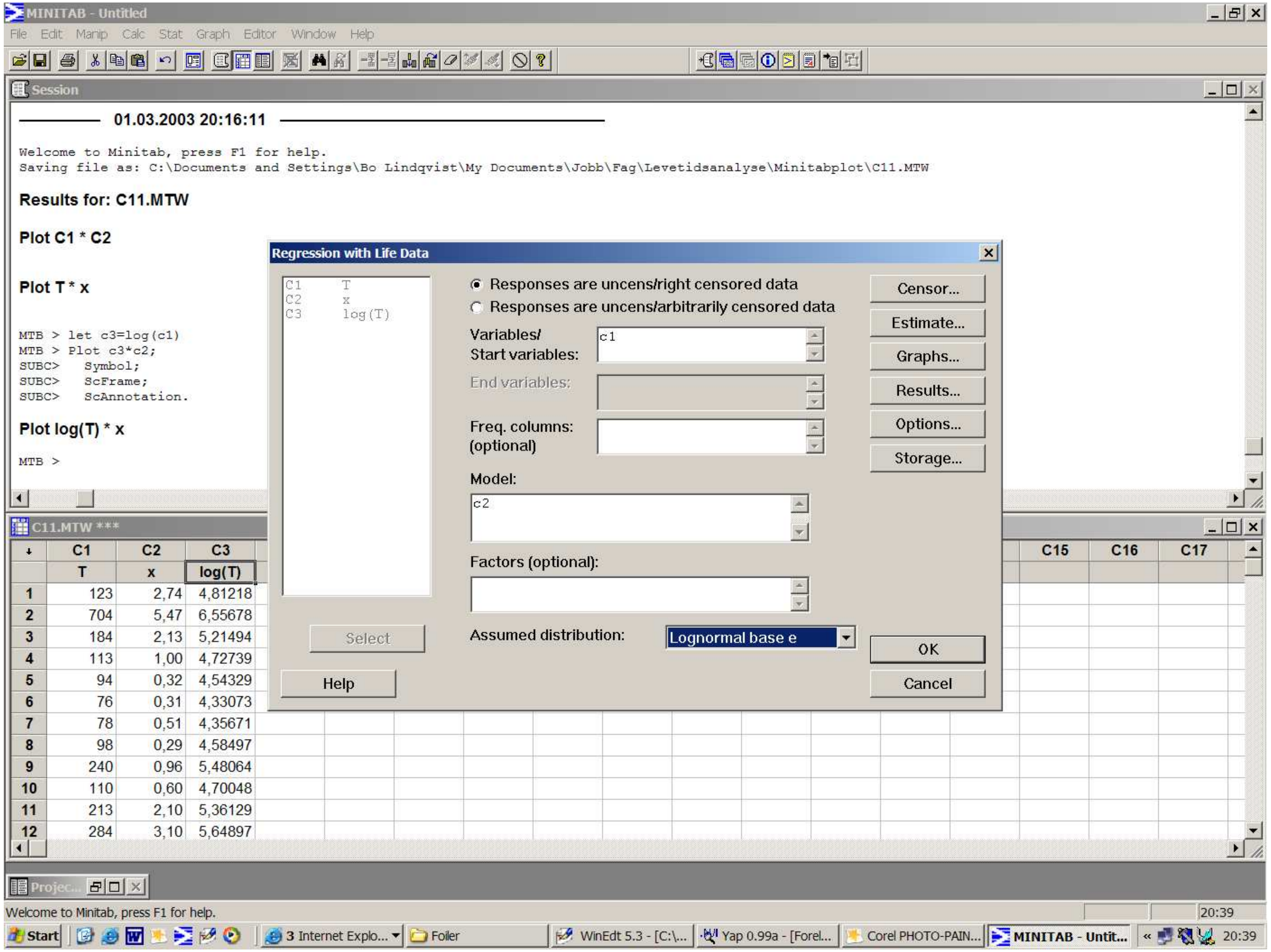
Computer data



Computer data




MINITAB - Untitled
File Edit Manip Calc Stat Graph Editor Window Help
Basic Statistics
Regression
ANOVA
DOE
Control Charts
Quality Tools
Reliability/Survival
Multivariate
Time Series
Tables
Nonparametrics
EDA
Power and Sample Size
Distribution ID Plot-Right Cens...
Distribution Overview Plot-Right Cens...
Parametric Dist Analysis-Right Cens...
Nonparametric Dist Analysis-Right Cens...
Distribution ID Plot-Arbitrary Cens...
Distribution Overview Plot-Arbitrary Cens...
Parametric Dist Analysis-Arbitrary Cens...
Nonparametric Dist Analysis-Arbitrary Cens...
Accelerated Life Testing...
Regression with Life Data...
Probit Analysis...
Session
Results for: C11.
Plot C1 * C2
Plot T * x
MTB > let c3=log(c1)
MTB > Plot c3*c2;
SUBC> Symbol;
SUBC> ScFrame;
SUBC> ScAnnotation.
Plot log(T) * x
MTB >
C11.MTW ***
C1 T
C2 x
C3 log(T)
1 123 2,74 4,81218
2 704 5,47 6,55678
3 184 2,13 5,21494
4 113 1,00 4,72739
5 94 0,32 4,54329
6 76 0,31 4,33073
7 78 0,51 4,35671
8 98 0,29 4,58497
9 240 0,96 5,48064
10 110 0,60 4,70048
11 213 2,10 5,36129
12 284 3,10 5,64897
Perform a regression analysis on life data with more than two predictors


MINITAB - Untitled
File Edit Manip Calc Stat Graph Editor Window Help
Session
01.03.2003 20:16:11
Welcome to Minitab, press F1 for help.
Saving file as: C:\Documents and Settings\Bo Lindqvist\My Documents\Jobb\Fag\Levetidsanalyse\Minitabplot\C11.MTW
Results for: C11.MTW
Plot C1 * C2
Plot T * x
MTB > let c3=log(c1)
MTB > Plot c3*c2;
SUBC> Symbol;
SUBC> ScFrame;
SUBC> ScAnnotation.
Plot log(T) * x
MTB >
C11.MTW ***
C1 T
C2 x
C3 log(T)
1 123 2,74 4,81218
2 704 5,47 6,55678
3 184 2,13 5,21494
4 113 1,00 4,72739
5 94 0,32 4,54329
6 76 0,31 4,33073
7 78 0,51 4,35671
8 98 0,29 4,58497
9 240 0,96 5,48064
10 110 0,60 4,70048
11 213 2,10 5,36129
12 284 3,10 5,64897
C15 C16 C17
Regression with Life Data
Responses are uncens/right censored data
Responses are uncens/arbitrarily censored data
Variables/ Start variables:
c1
End variables:
Freq. columns: (optional)
Model:
c2
Factors (optional):
Assumed distribution:
Lognormal base e
Select
Help
Censor...
Estimate...
Graphs...
Results...
Options...
Storage...
OK
Cancel
Welcome to Minitab, press F1 for help.
20:39

Regression with Life Data: T versus x

```
Response Variable:  T

Censoring Information                Count
Uncensored value                        17

Estimation Method:  Maximum Likelihood
Distribution:   Lognormal base e
```

Regression Table

		Standard			95,0% Normal CI	
Predictor	Coef	Error	Z	P	Lower	Upper
Intercept	4,4936	0,1112	40,39	0,000	4,2756	4,7116
x	0,29075	0,04595	6,33	0,000	0,20069	0,38080
Scale	0,31247	0,05359			0,22327	0,43730

```
Log-Likelihood = -89,498

Anderson-Darling (adjusted) Goodness-of-Fit

Standardized Residuals = 0,8356; Cox-Snell Residuals = 0,8170
```

Regression with Life Data: C1 versus C2

Response Variable: C1

Censoring Information	Count
Uncensored value	17

Estimation Method: Maximum Likelihood
Distribution: Weibull

Regression Table

Predictor	Coef	Standard Error	Z	P	95,0% Normal CI Lower	Upper
Intercept	4,6182	0,1219	37,88	0,000	4,3792	4,8572
C2	0,31118	0,04939	6,30	0,000	0,21437	0,40799
Shape	3,0604	0,5245			2,1873	4,2820

Log-Likelihood = -91,504

Anderson-Darling (adjusted) Goodness-of-Fit

Likelihood for Lognormal Distribution Simple Regression Model with Right Censored Data

The likelihood for n independent observations has the form

$$
\begin{aligned}
L(\beta_0, \beta_1, \sigma) &= \prod_{i=1}^{n} L_i(\beta_0, \beta_1, \sigma; \text{data}_i) \\
&= \prod_{i=1}^{n} \left\{ \frac{1}{\sigma t_i} \phi_{\text{nor}} \left[\frac{\log(t_i) - \mu_i}{\sigma} \right] \right\}^{\delta_i} \left\{ 1 - \Phi_{\text{nor}} \left[\frac{\log(t_i) - \mu_i}{\sigma} \right] \right\}^{1-\delta_i}
\end{aligned}
$$

where $\text{data}_i = (x_i, t_i, \delta_i)$, $\mu_i = \beta_0 + \beta_1 x_i$,

$$
\delta_i = \begin{cases} 1 & \text{exact observation} \\ 0 & \text{right censored observation} \end{cases}
$$

$\phi_{\text{nor}}(z)$ is the standardized normal pdf and $\Phi_{\text{nor}}(z)$ is the corresponding normal cdf.

The parameters are $\boldsymbol{\theta} = (\beta_0, \beta_1, \sigma)$.

Estimated Parameter Variance-Covariance Matrix

Local (observed information) estimate

$$\hat{\Sigma}_{\hat{\boldsymbol{\theta}}} = \begin{bmatrix} \widehat{\text{Var}}(\hat{\beta}_0) & \widehat{\text{Cov}}(\hat{\beta}_0, \hat{\beta}_1) & \widehat{\text{Cov}}(\hat{\beta}_0, \hat{\sigma}) \\ \widehat{\text{Cov}}(\hat{\beta}_1, \hat{\beta}_0) & \widehat{\text{Var}}(\hat{\beta}_1) & \widehat{\text{Cov}}(\hat{\beta}_1, \hat{\sigma}) \\ \widehat{\text{Cov}}(\hat{\sigma}, \hat{\beta}_0) & \widehat{\text{Cov}}(\hat{\sigma}, \hat{\beta}_1) & \widehat{\text{Var}}(\hat{\sigma}) \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{\partial^2 \mathcal{L}(\beta_0,\beta_1,\sigma)}{\partial \beta_0^2} & -\frac{\partial^2 \mathcal{L}(\beta_0,\beta_1,\sigma)}{\partial \beta_0 \partial \beta_1} & -\frac{\partial^2 \mathcal{L}(\beta_0,\beta_1,\sigma)}{\partial \beta_0 \partial \sigma} \\ -\frac{\partial^2 \mathcal{L}(\beta_0,\beta_1,\sigma)}{\partial \beta_1 \partial \beta_0} & -\frac{\partial^2 \mathcal{L}(\beta_0,\beta_1,\sigma)}{\partial \beta_1^2} & -\frac{\partial^2 \mathcal{L}(\beta_0,\beta_1,\sigma)}{\partial \beta_1 \partial \sigma} \\ -\frac{\partial^2 \mathcal{L}(\beta_0,\beta_1,\sigma)}{\partial \sigma \partial \beta_0} & -\frac{\partial^2 \mathcal{L}(\beta_0,\beta_1,\sigma)}{\partial \sigma \partial \beta_1} & -\frac{\partial^2 \mathcal{L}(\beta_0,\beta_1,\sigma)}{\partial \sigma^2} \end{bmatrix}^{-1}$$

Partial derivatives are evaluated at $\hat{\beta}_0, \hat{\beta}_1, \hat{\sigma}$.

Standard Errors and Confidence Intervals for Parameters

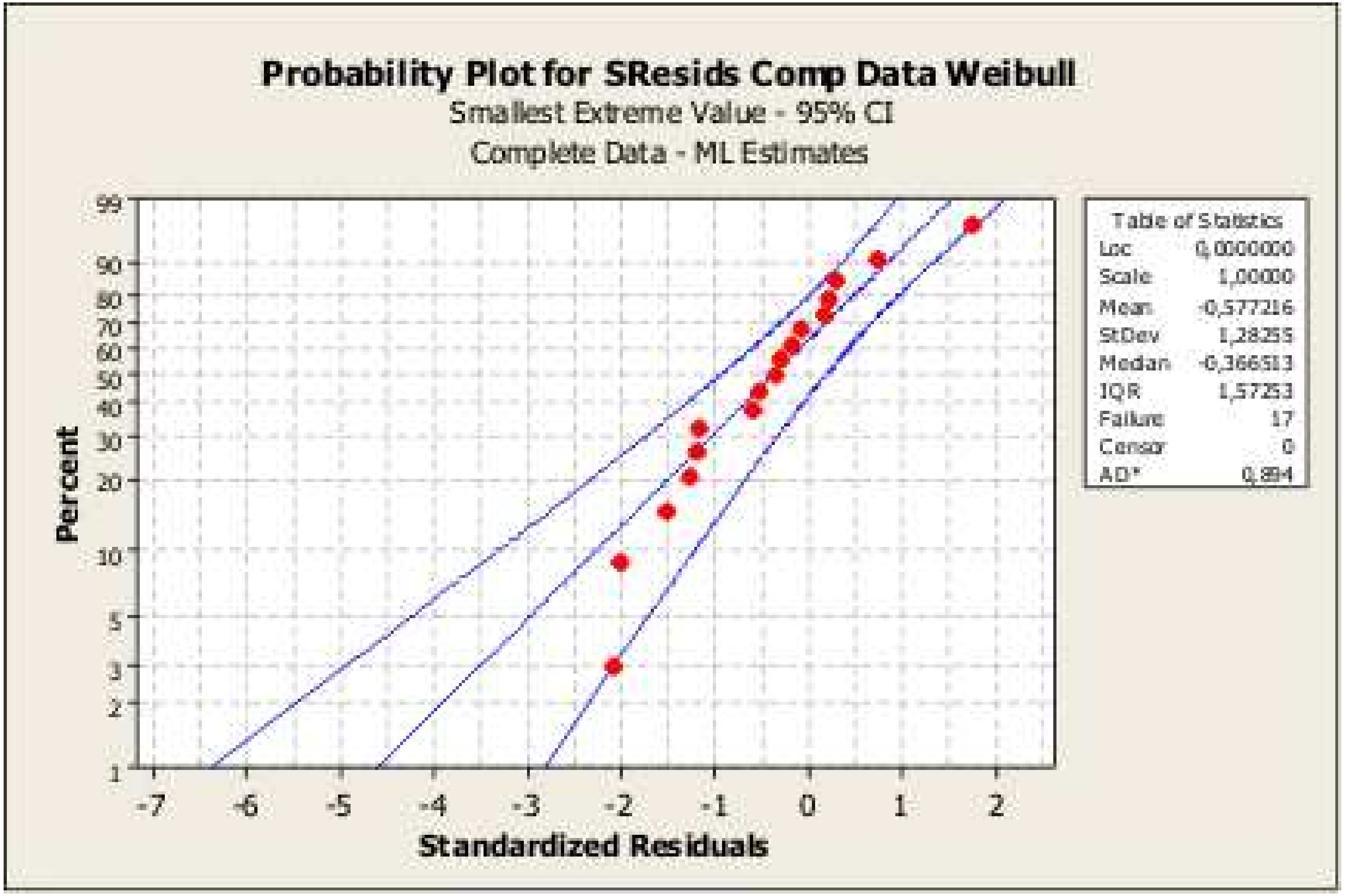
- Lognormal ML estimates for the computer time experiment were $\widehat{\boldsymbol{\theta}} = (\widehat{\beta}_0, \widehat{\beta}_1, \widehat{\sigma}) = (4.49, .290, .312)$ and an estimate of the variance-covariance matrix for $\widehat{\boldsymbol{\theta}}$ is

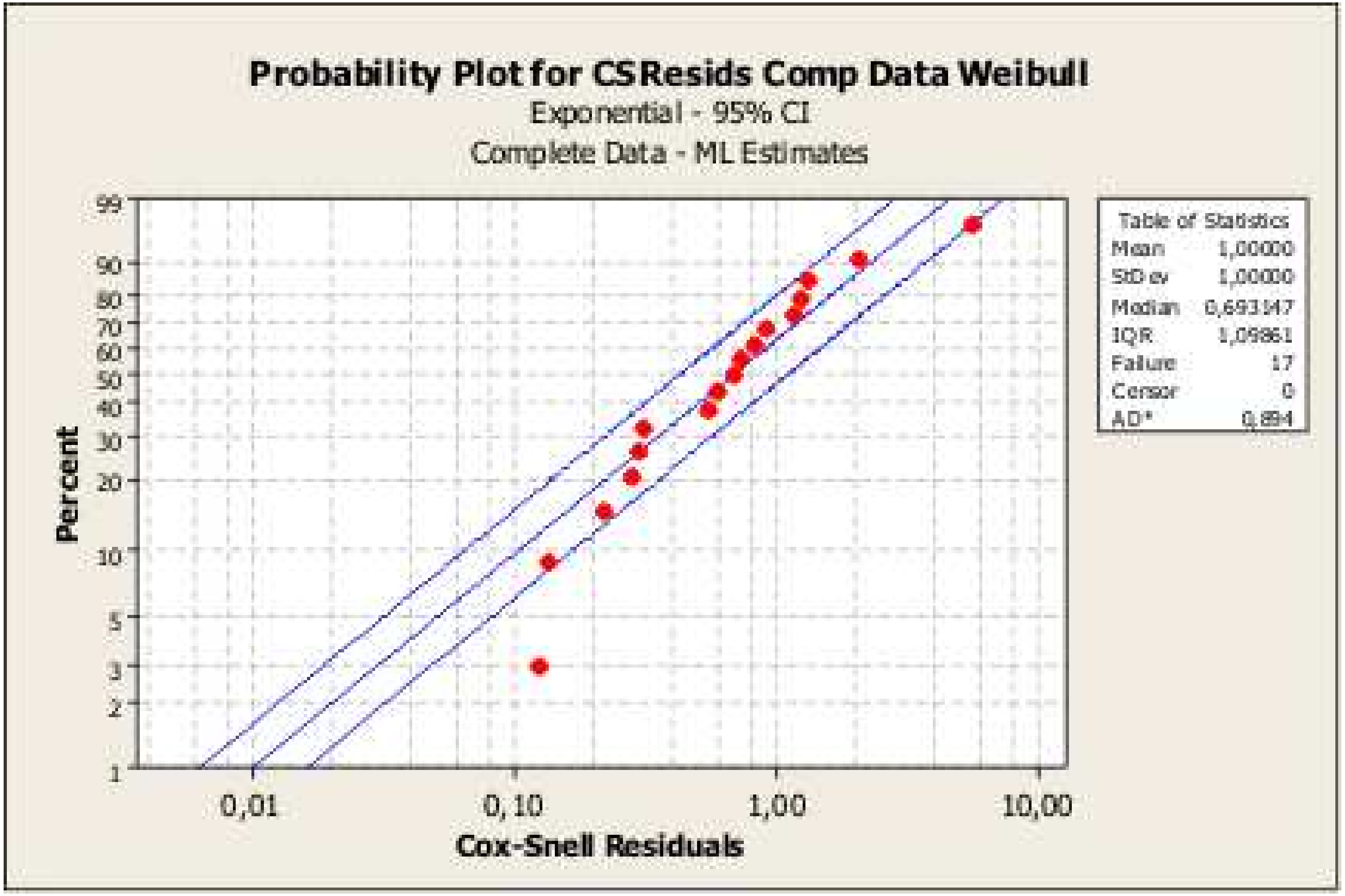
$$\widehat{\Sigma}_{\widehat{\boldsymbol{\theta}}} = \begin{bmatrix} .012 & -.0037 & 0 \\ -.0037 & .0021 & 0 \\ 0 & 0 & .0029 \end{bmatrix}.$$

- Normal-approximation confidence interval for the computer execution time regression slope is

$$[\underset{\sim}{\beta_1}, \quad \widetilde{\beta}_1] = \widehat{\beta}_1 \pm z_{(.975)} \widehat{\text{se}}_{\widehat{\beta}_1} = .290 \pm 1.96(.046) = [.20, \quad .38]$$

where $\widehat{\text{se}}_{\widehat{\beta}_1} = \sqrt{.0021} = .046$.


Probability Plot for SResids Comp Data Weibull
Smallest Extreme Value - 95% CI
Complete Data - ML Estimates
Percent
Standardized Residuals
Table of Statistics
Loc 0,0000000
Scale 1,00000
Mean -0,577216
StDev 1,28255
Median -0,366513
IQR 1,57253
Failure 17
Censor 0
AD* 0,894



Probability Plot for CSResids Comp Data Weibull
Exponential - 95% CI
Complete Data - ML Estimates
Percent
Cox-Snell Residuals
0,01
0,10
1,00
10,00
Table of Statistics
Mean 1,00000
StDev 1,00000
Median 0,693147
IQR 1,09861
Failure 17
Censor 0
AD* 0,894

Ordinary residuals

$$y_i - x'_i\hat{\beta}$$

where

y_i is the ith response value

x'_i is the vector of predictor values associated with the ith response value

$\hat{\beta}$ represents the estimated regression coefficients

Standardized residuals

$$\frac{y_i - x'_i\hat{\beta}}{\hat{\sigma}}$$

where $\hat{\sigma}$ is the estimated scale parameter.

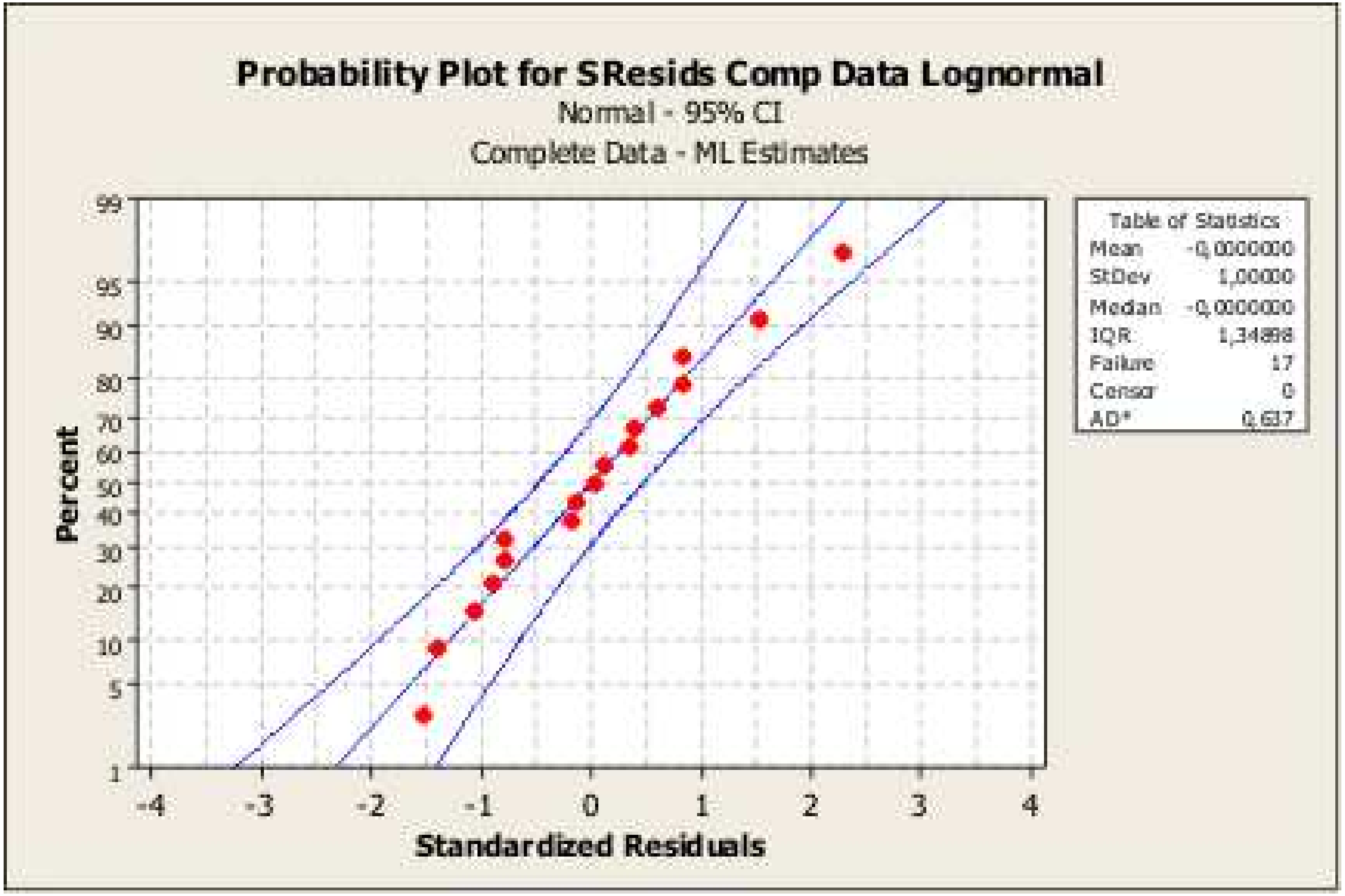
Cox-Snell residuals

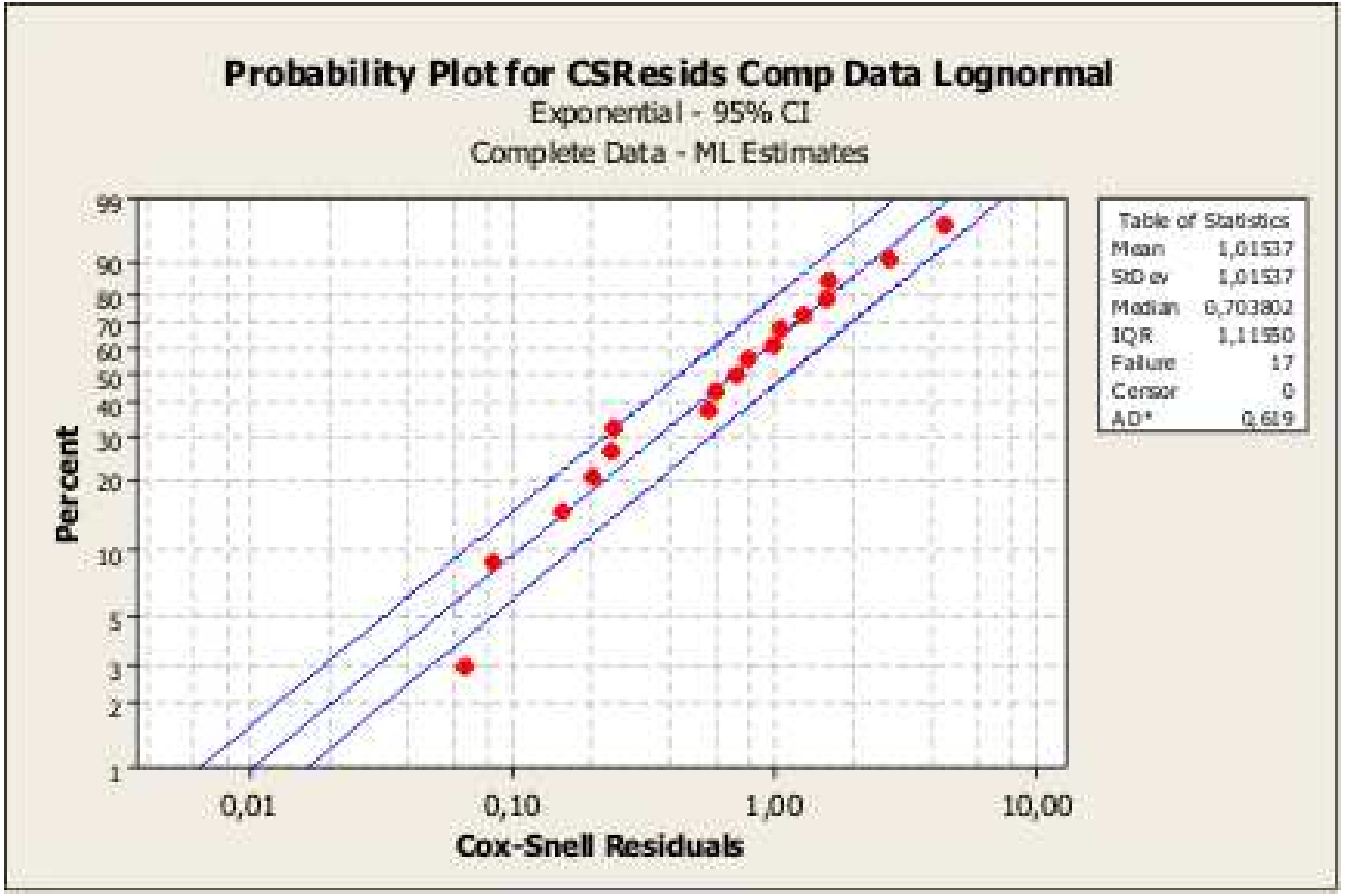
$$-\ln(\hat{R}(y_i))$$

where

$\hat{R}(y_i)$ is the estimated survival (reliability) probability for the response value y_i

$\ln(x)$ is the natural log of x


Probability Plot for SResids Comp Data Lognormal
Normal - 95% CI
Complete Data - ML Estimates
Percent
Standardized Residuals
99
95
90
80
70
60
50
40
30
20
10
5
1
-4
-3
-2
-1
0
1
2
3
4
Table of Statistics
Mean -0,0000000
StDev 1,00000
Median -0,0000000
IQR 1,34898
Failure 17
Censor 0
AD* 0,637


Probability Plot for CSResids Comp Data Lognormal
Exponential - 95% CI
Complete Data - ML Estimates
Percent
Cox-Snell Residuals
99
90
80
70
60
50
40
30
20
10
5
3
2
1
0,01
0,10
1,00
10,00
Table of Statistics
Mean 1,01537
StDev 1,01537
Median 0,703802
IQR 1,11550
Failure 17
Censor 0
AD* 0,619

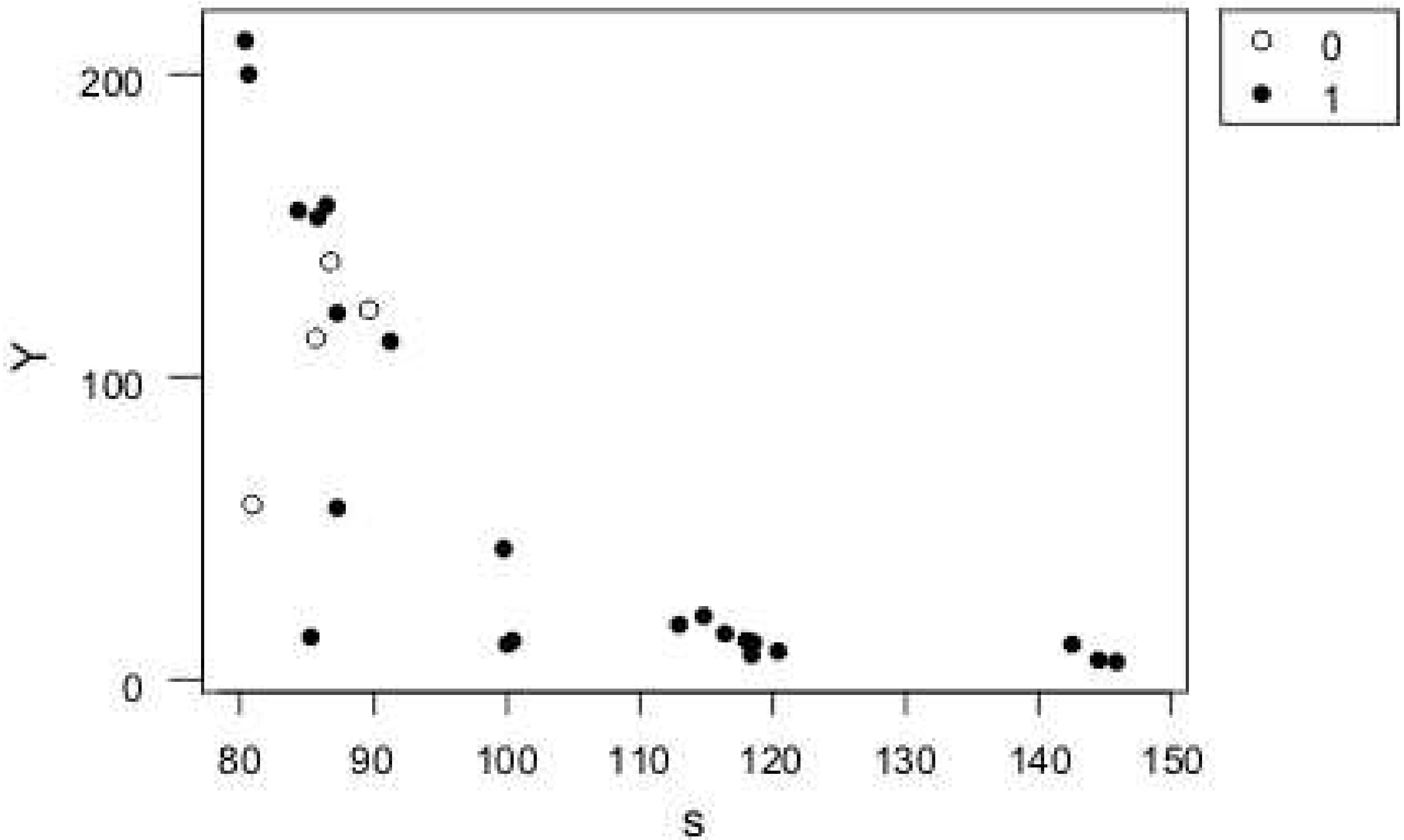
Row	Pseudo-stress	k-Cycles	Status (1=failed, 0=censored)
i	s	Y	C
1	80,3	211,629	1
2	80,6	200,027	1
3	80,8	57,923	0
4	84,3	155,000	1
5	85,2	13,949	1
6	85,6	112,968	0
7	85,8	152,680	1
8	86,4	156,725	1
9	86,7	138,114	0
10	87,2	56,723	1
11	87,3	121,075	1
12	89,7	122,372	0
13	91,3	112,002	1
14	99,8	43,331	1
15	100,1	12,076	1
16	100,5	13,181	1
17	113,0	18,067	1
18	114,8	21,300	1
19	116,4	15,616	1
20	118,0	13,030	1
21	118,4	8,489	1
22	118,6	12,434	1
23	120,4	9,750	1
24	142,5	11,865	1
25	144,5	6,705	1
26	145,9	5,733	1

DATA DESCRIPTION:
Low-Cycle Fatigue Life of Nickel-Base Superalloy Specimens (in units of thousands of cycles to failure).

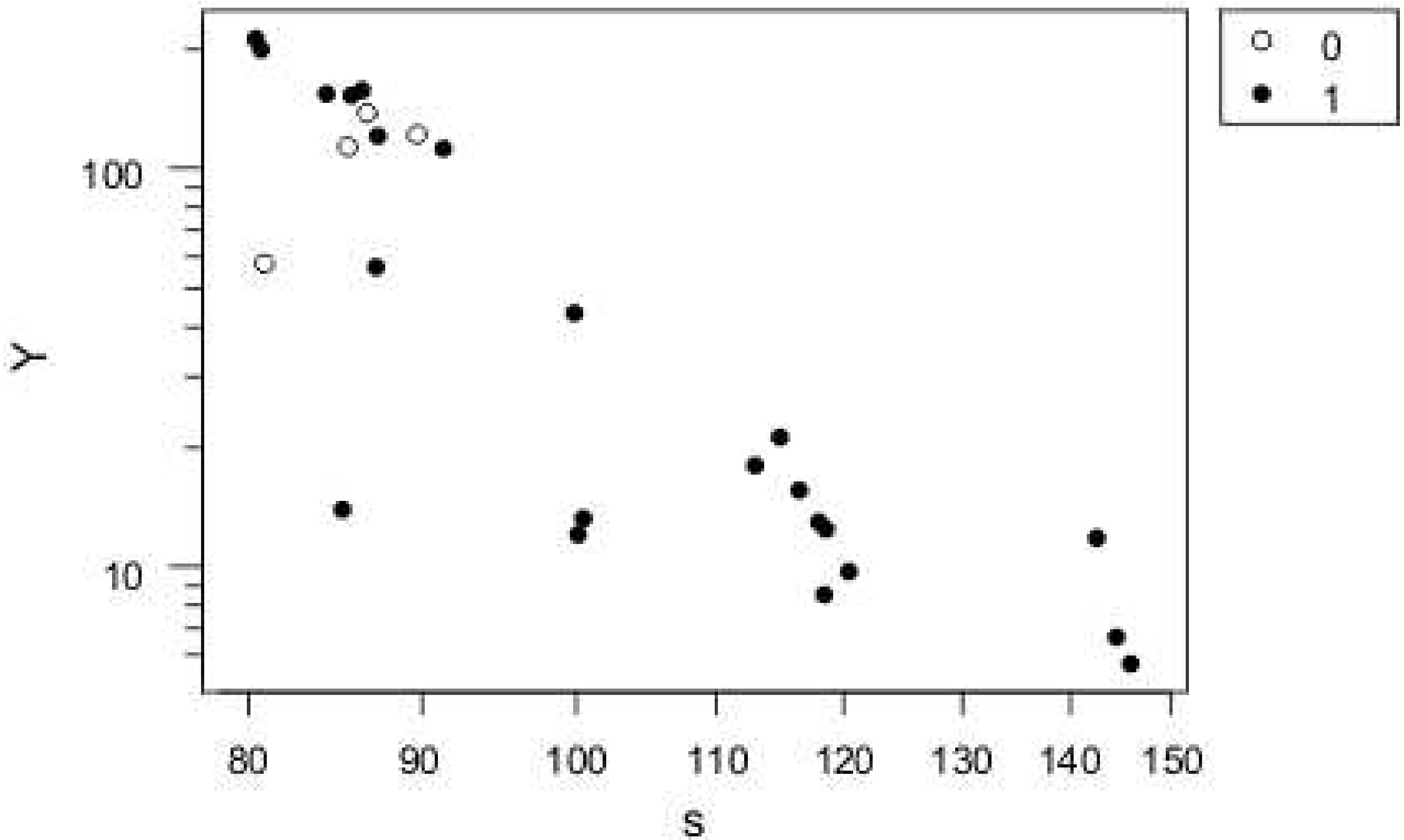
Data from Nelson (1990):

SUPER ALLOY DATA

Plot of Y vs s



Plot of log(Y) vs log(s)



Regression with Life Data: Y versus x

Response Variable: Y

Censoring Information	Count
Uncensored value	22
Right censored value	4

Censoring value: C = 0

Estimation Method: Maximum Likelihood
Distribution: Weibull

Regression Table

		Standard			95,0% Normal CI	
Predictor	Coef	Error	Z	P	Lower	Upper
Intercept	31,432	2,008	15,65	0,000	27,496	35,368
x	-5,9600	0,4329	-13,77	0,000	-6,8085	-5,1116
Shape	2,2105	0,3894			1,5651	3,1221

Log-Likelihood = -97,155

Anderson-Darling (adjusted) Goodness-of-Fit

Standardized Residuals = 1,0768; Cox-Snell Residuals = 1,0768

Regression with Life Data: Y versus x

Response Variable: Y

Censoring Information	Count
Uncensored value	22
Right censored value	4

Censoring value: C = 0

Estimation Method: Maximum Likelihood
Distribution: Weibull

Regression Table

Predictor	Coef	Standard Error	Z	P	95,0% Normal CI Lower	Upper
Intercept	217,61	62,13	3,50	0,000	95,83	339,39
x	-85,52	26,55	-3,22	0,001	-137,55	-33,49
x*x	8,483	2,831	3,00	0,003	2,934	14,032
Shape	2,6685	0,4777			1,8789	3,7900

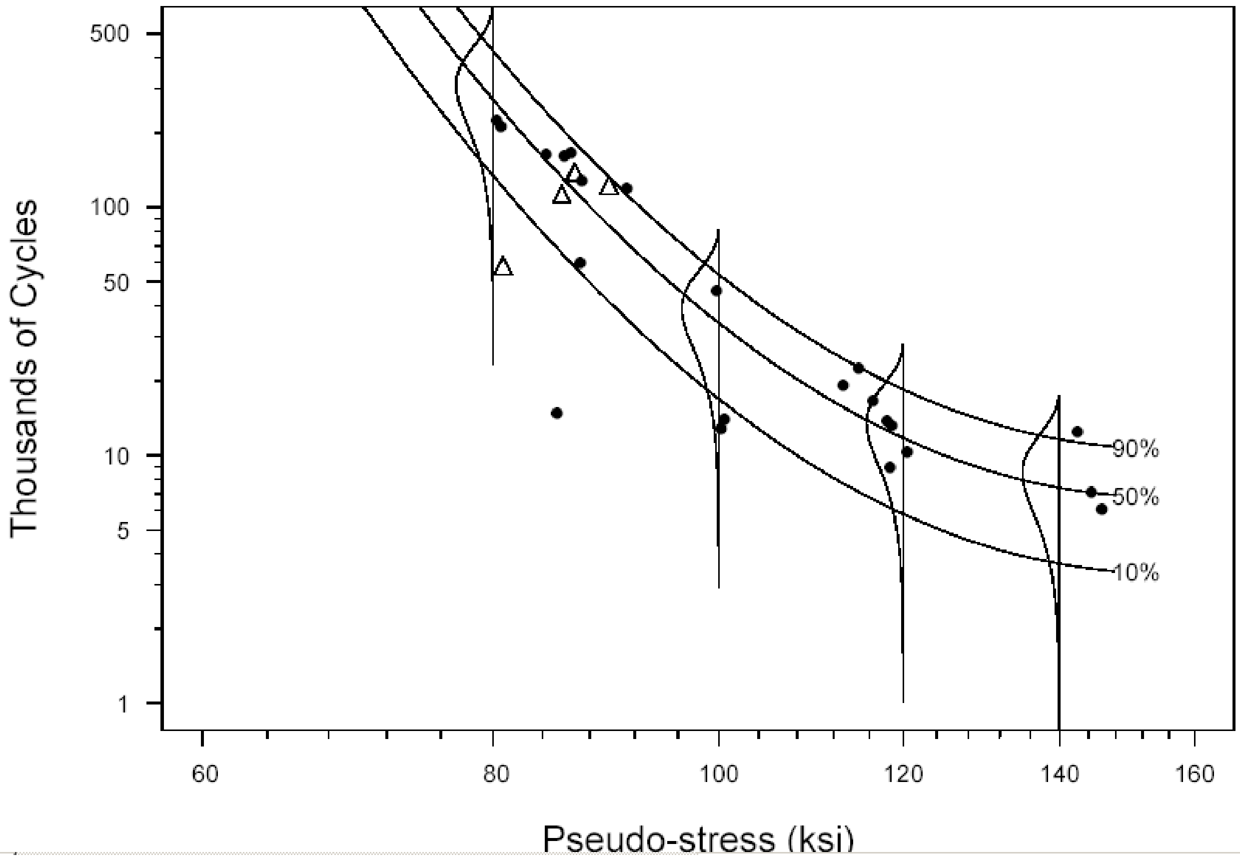
Log-Likelihood = -93,382

Anderson-Darling (adjusted) Goodness-of-Fit

Standardized Residuals = 0,9283; Cox-Snell Residuals = 0,9283

Log-Quadratic Weibull Regression Model with Constant ($\beta = 1/\sigma$) Fit to the Fatigue Data

$$\log[\widehat{t}_p(x)] = \widehat{\mu}(x) + \Phi^{-1}_{\text{sev}}(p)\widehat{\sigma}, \ x = \log(\textbf{pseudo-stress})$$



Regression with Life Data: Y versus x

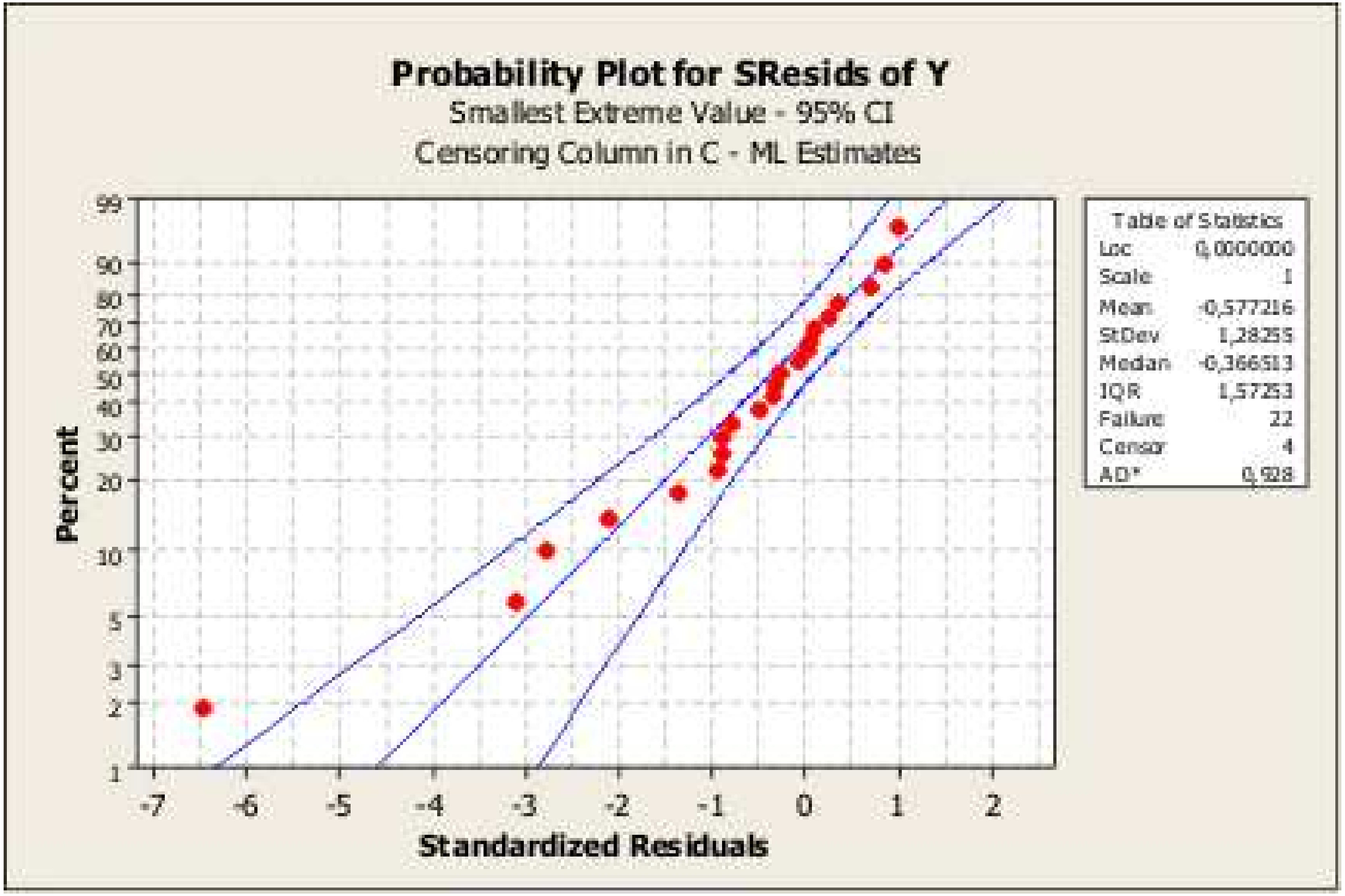
Response Variable: Y

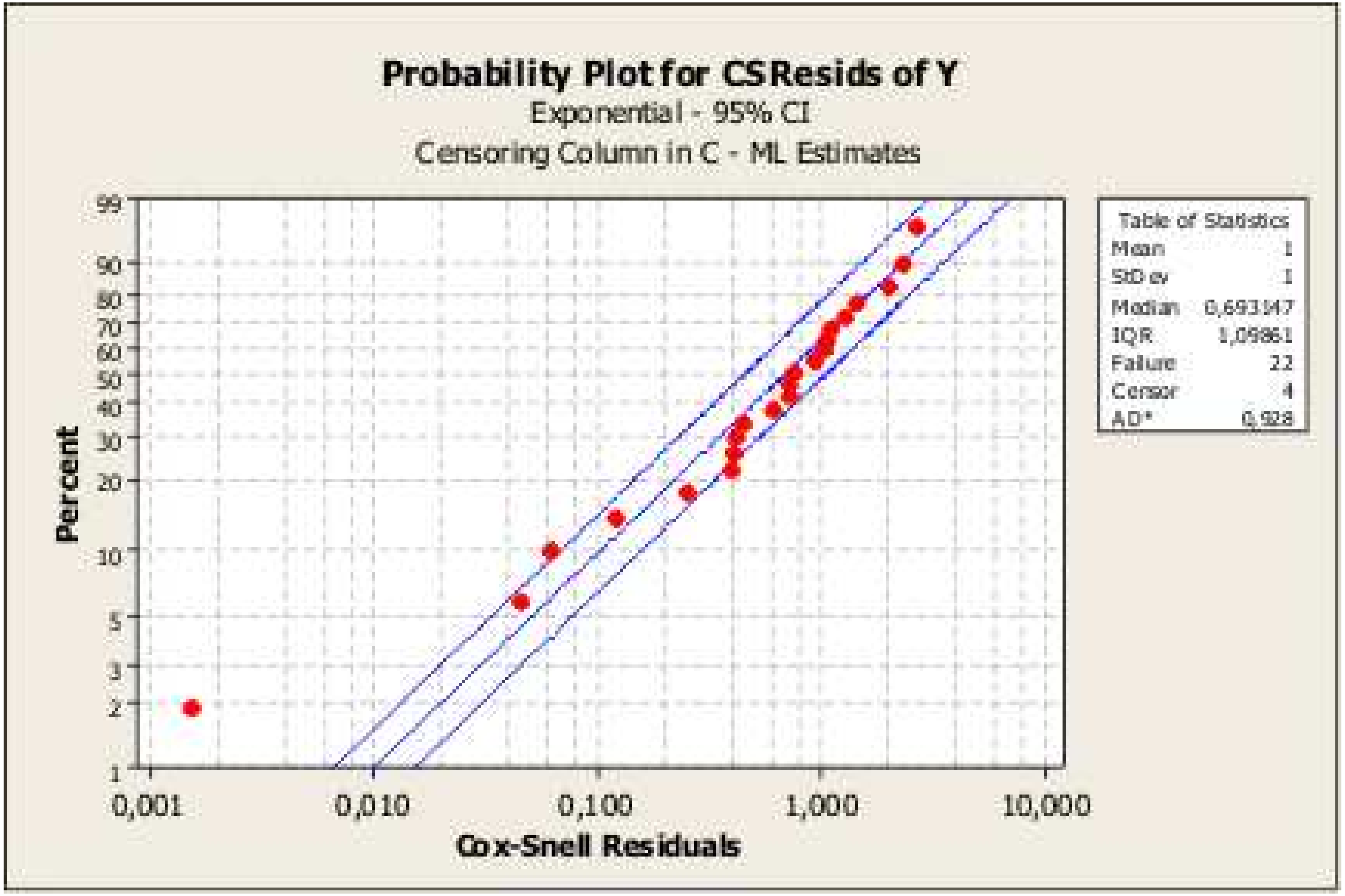
Table of Percentiles

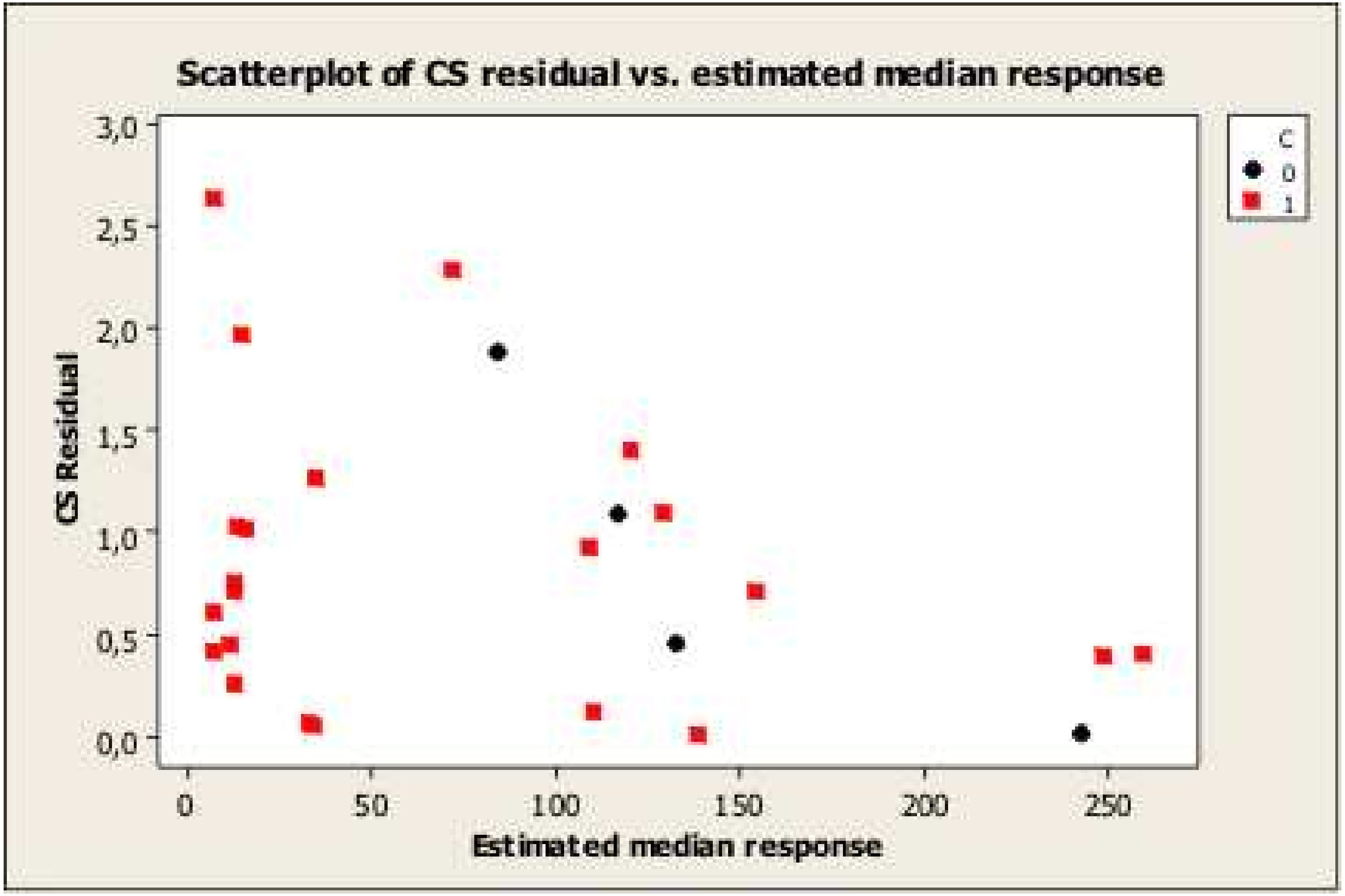
Percent	s	x	Percentile	Standard Error	95,0% Normal CI Lower	95,0% Normal CI Upper
10	80	4,3820	133,3747	34,0579	80,8565	220,0048
10	100	4,6052	16,7928	3,4263	11,2577	25,0494
10	120	4,7875	5,7830	1,2364	3,8034	8,7929
10	140	4,9416	3,6458	0,8760	2,2766	5,8386
50	80	4,3820	270,1879	56,0580	179,9121	405,7621
50	100	4,6052	34,0186	4,3027	26,5494	43,5891
50	120	4,7875	11,7151	1,5950	8,9713	15,2980
50	140	4,9416	7,3856	1,2828	5,2547	10,3807
90	80	4,3820	423,6933	90,4646	278,8097	643,8659
90	100	4,6052	53,3461	6,8162	41,5281	68,5272
90	120	4,7875	18,3709	2,4567	14,1351	23,8760
90	140	4,9416	11,5817	1,9813	8,2824	16,1952

ESTIMERT KOVARIANSMATRISE FOR $(\widehat{\beta}_0, \widehat{\beta}_1, \widehat{\beta}_2, \widehat{\sigma})$

3860,37	-1649,17	175,82	-0,80
-1649,17	704,70	-75,15	0,33
175,82	-75,15	8,02	-0,03
-0,80	0,33	-0,03	0,23


Probability Plot for SResids of Y
Smallest Extreme Value - 95% CI
Censoring Column in C - ML Estimates
Percent
Standardized Residuals
Table of Statistics
Loc 0,0000000
Scale 1
Mean -0,577216
StDev 1,28255
Median -0,366513
IQR 1,57253
Failure 22
Censor 4
AD* 0,928


Probability Plot for CSResids of Y
Exponential - 95% CI
Censoring Column in C - ML Estimates
Percent
Cox-Snell Residuals
0,001
0,010
0,100
1,000
10,000
99
90
80
70
60
50
40
30
20
10
5
3
2
1
Table of Statistics
Mean 1
StDev 1
Median 0,693147
IQR 1,09861
Failure 22
Censor 4
AD* 0,928


Scatterplot of CS residual vs. estimated median response
CS Residual
3,0
2,5
2,0
1,5
1,0
0,5
0,0
0
50
100
150
200
250
Estimated median response
C
0
1

SIMPLE EXAMPLE COX-REGRESSION

j	Y_j	x_j	δ_j
1	5	12	0
2	10	10	1
3	40	3	0
4	80	5	0
5	120	3	1
6	400	4	1
7	600	1	0

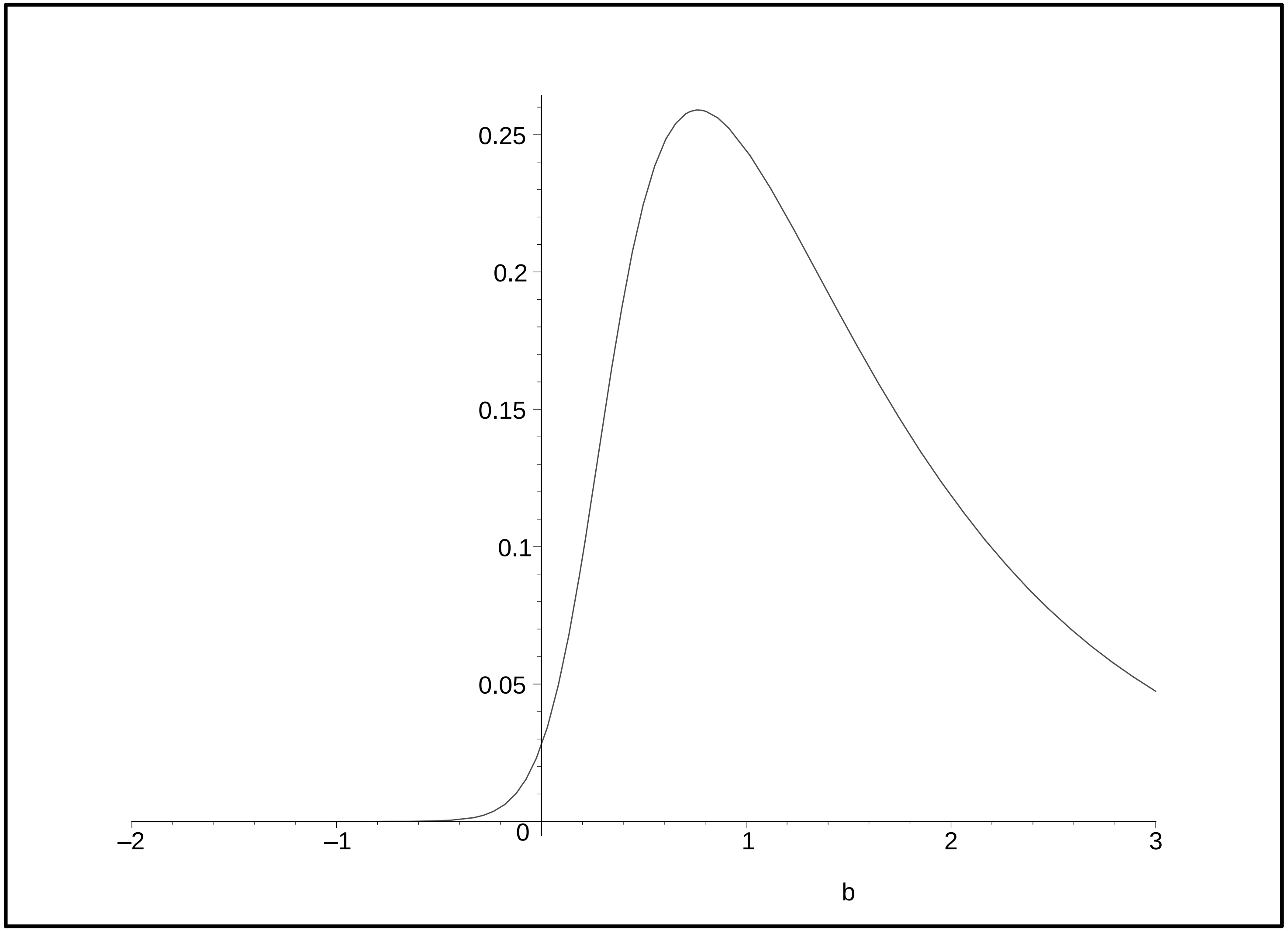
Model:

- $z(t|x) = z_0(t)\,\mathrm{exp}\{\beta x\}$

Partial likelihood:

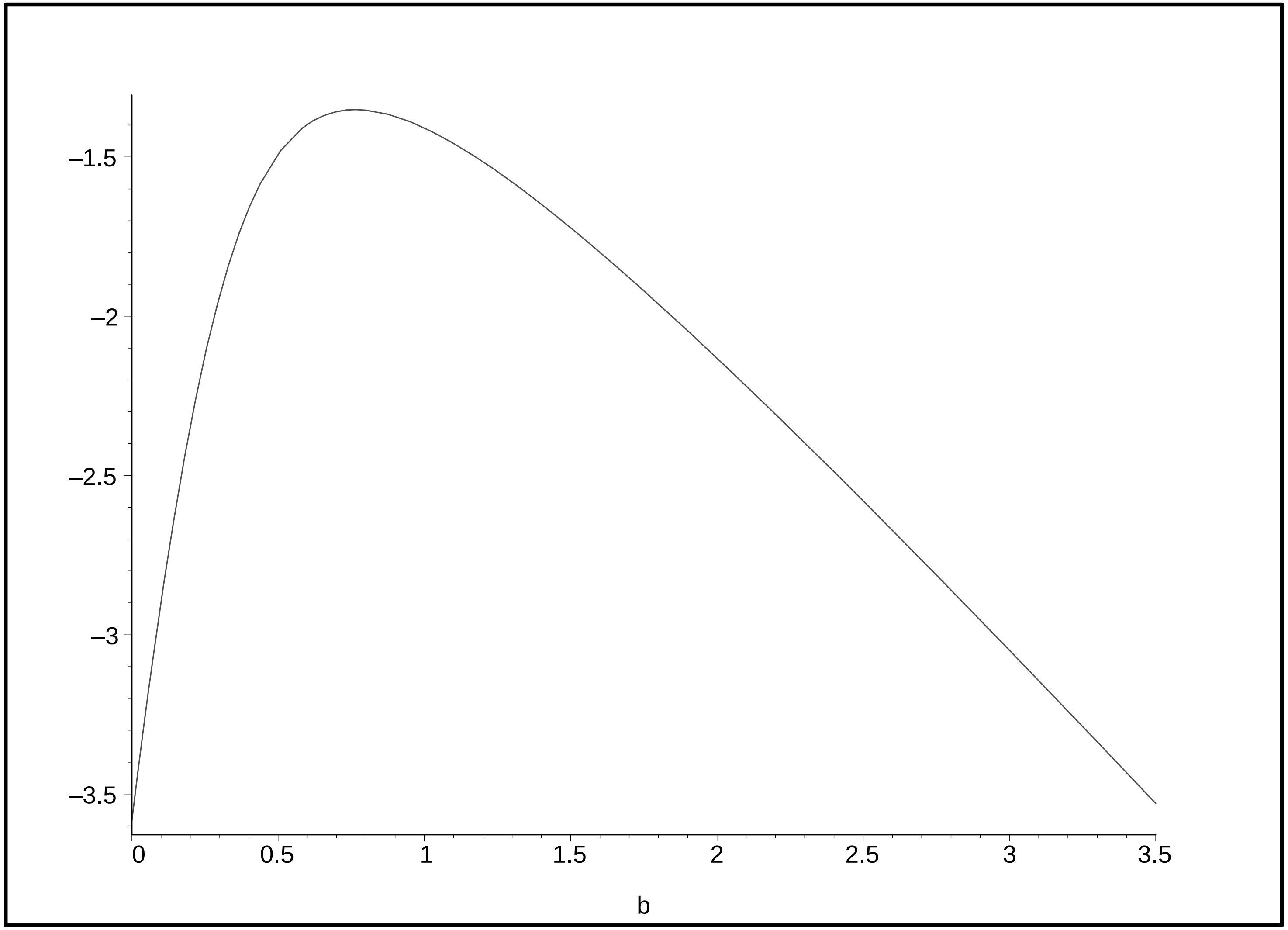
$$L(\beta) = \frac{e^{10\beta}}{e^{10\beta}+e^{3\beta}+e^{5\beta}+e^{3\beta}+e^{4\beta}+e^{\beta}} \cdot \frac{e^{3\beta}}{e^{3\beta}+e^{4\beta}+e^{\beta}} \cdot \frac{e^{4\beta}}{e^{4\beta}+e^{\beta}}$$

Cox' partial likelihood $L(\beta)$ in the example:



Maximum likelihood estimate: $\widehat{\beta} = 0.765$.

Cox' partial log-likelihood $l(\beta)$ in the example:



Maximum likelihood estimate: $\widehat{\beta} = 0.765$.

95% likelihood confidence interval: $(0.1, 3.2)$.

Weibull regression (Cox-example)

```
Estimation Method: Maximum Likelihood

Distribution:   Weibull

Relationship with accelerating variable(s):   Linear


Regression Table

                          Standard                      95,0% Normal CI
Predictor        Coef        Error      Z      P       Lower       Upper
Intercept     7,58636     0,548229  13,84  0,000     6,51185     8,66087
x           -0,468235    0,0842830  -5,56  0,000   -0,633427   -0,303044
Shape         2,05563     0,872169                  0,894943     4,72167

Log-Likelihood = -17,450
```

Worksheet 1 ***

↓	C1	C2	C3	C4	C5	C6	C7	C8
	Y	x	d					
1	5	12	0					
2	10	10	1					
3	40	3	0					
4	80	5	0					
5	120	3	1					
6	400	4	1					
7	600	1	0					

Data from Ansell & Phillips (s. 63)

Table 3.2. Lifetimes (in cycles) of sodium sulphur batteries

Batch 1	164	164	218	230	263	467	538	639	669
	917	1148	1678+	1678+	1678+	1678+			
Batch 2	76	82	210	315	385	412	491	504	522
	646+	678	775	884	1131	1446	1824	1827	2248
	2385	3077							

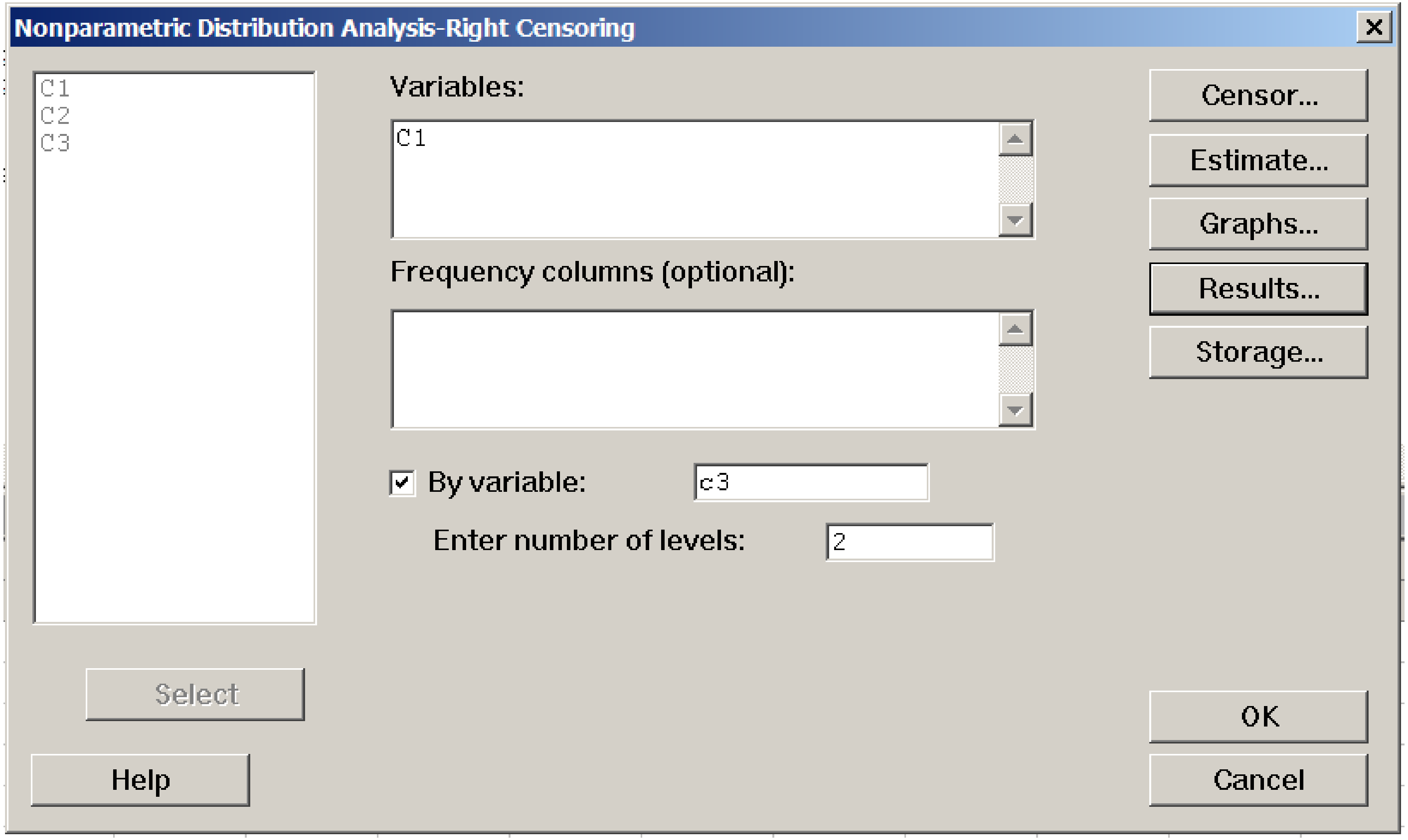
Note: Lifetimes with + are right censored observations, not failures.

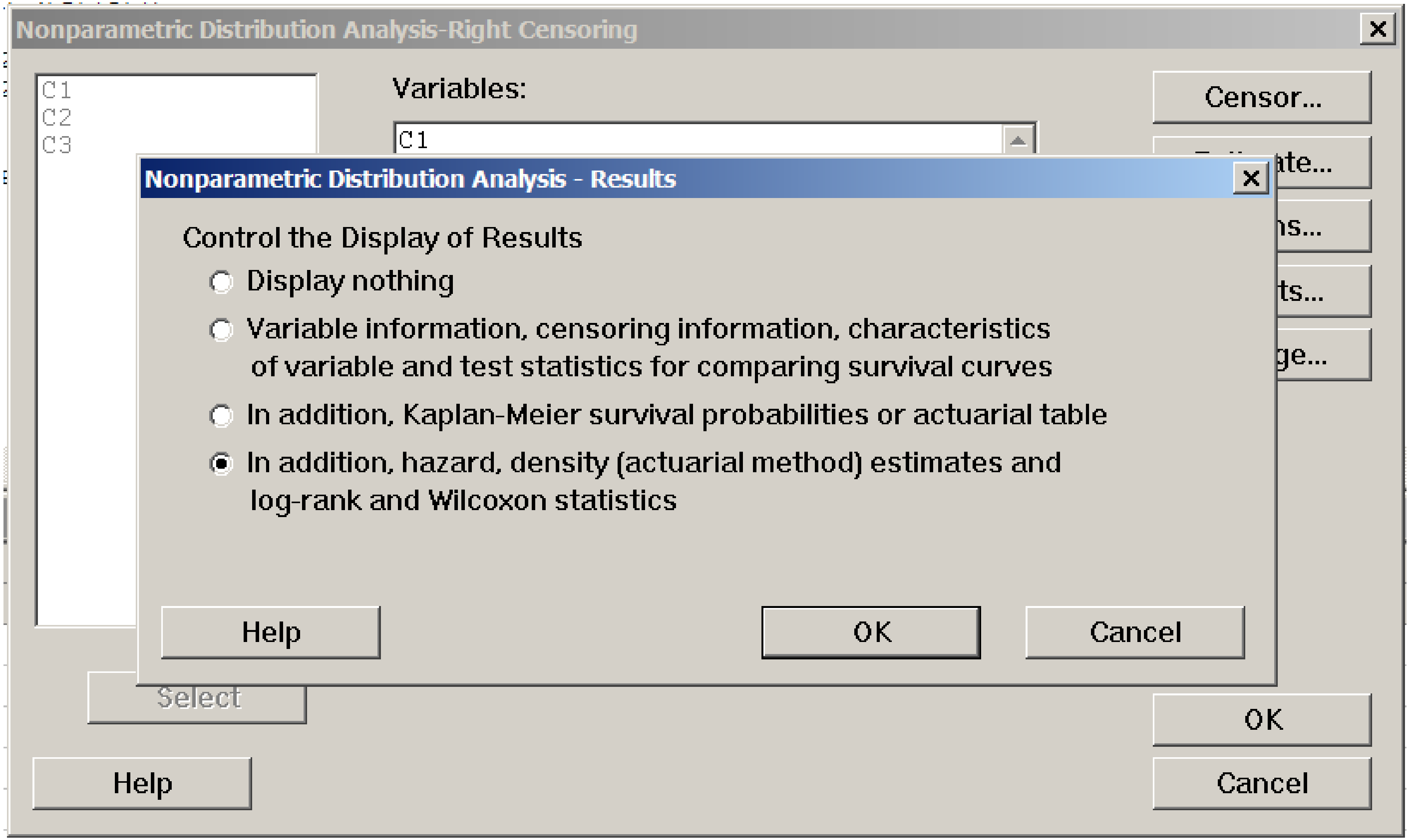
C1=Obs. times, C2=censoring, C3="batch" no.

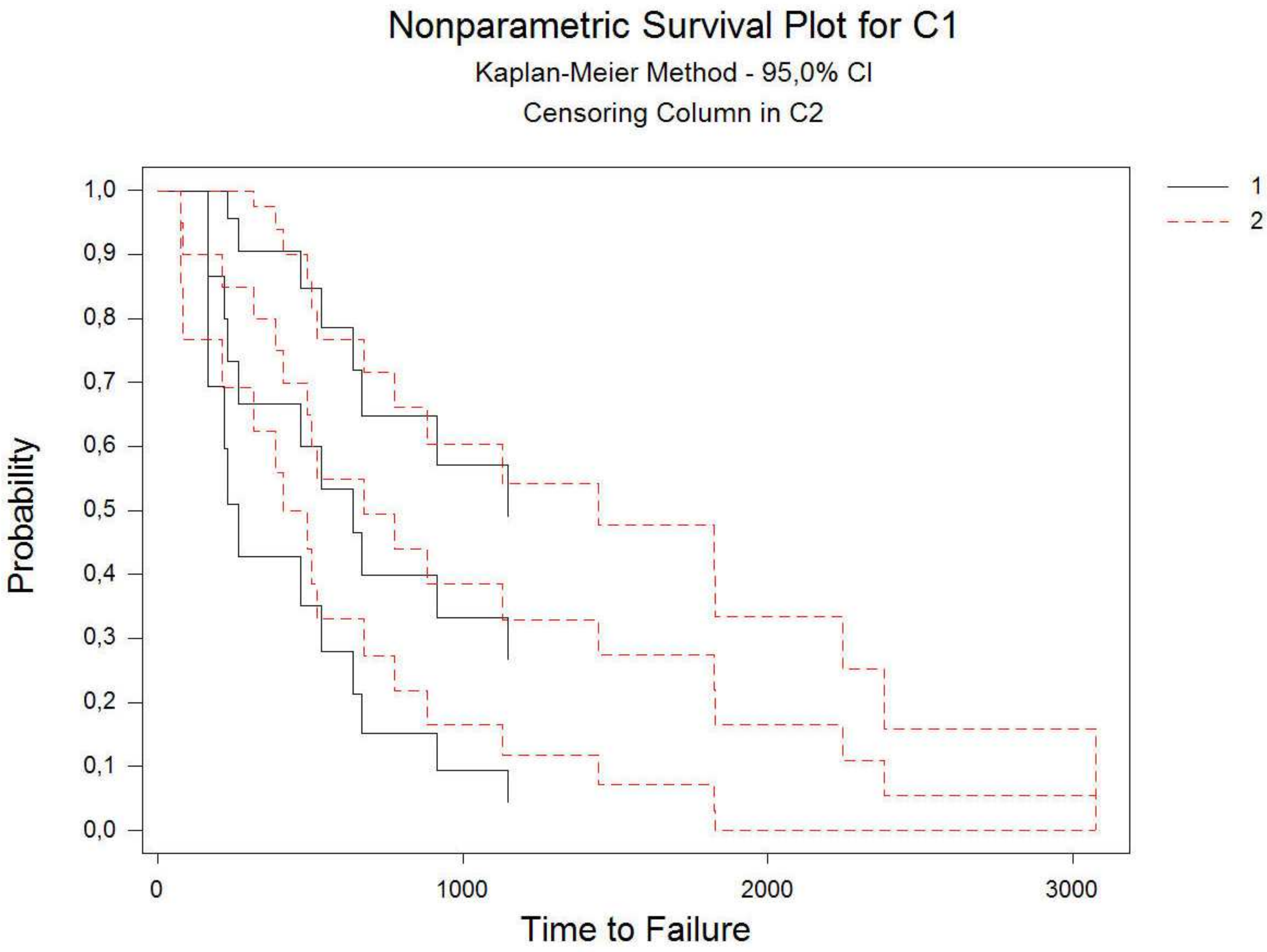
Ansell32.MTW ***

↓	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12
1	164	1	1									
2	164	1	1									
3	218	1	1									
4	230	1	1									
5	263	1	1									
6	467	1	1									
7	538	1	1									
8	639	1	1									
9	669	1	1									
10	917	1	1									
11	1148	1	1									
12	1678	0	1									
13	1678	0	1									
14	1678	0	1									
15	1678	0	1									
16	76	1	2									
17	82	1	2									
18	210	1	2									
19	315	1	2									
20	385	1	2									
21	412	1	2									
22	491	1	2									
23	504	1	2									
24	522	1	2									
25	646	0	2									

MINITAB-analysis of data from Ansell & Phillips:




Nonparametric Distribution Analysis-Right Censoring
C1
C2
C3
Variables:
C1
Censor...
Select
Help
OK
Cancel
Nonparametric Distribution Analysis - Results
Control the Display of Results
Display nothing
Variable information, censoring information, characteristics of variable and test statistics for comparing survival curves
In addition, Kaplan-Meier survival probabilities or actuarial table
In addition, hazard, density (actuarial method) estimates and log-rank and Wilcoxon statistics
Help
OK
Cancel


Nonparametric Survival Plot for C1
Kaplan-Meier Method - 95,0% CI
Censoring Column in C2
Probability
1,0
0,9
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0,0
0
1000
2000
3000
Time to Failure
1
2

MINITAB-result (among other output):

```
Test Statistics Method Chi-Square    DF    P-Value Log-Rank
                         0,04855     1     0,8256
```

This is in accordance with the result of p. 72 (Example 3.5.2) in Ansell & Phillips.

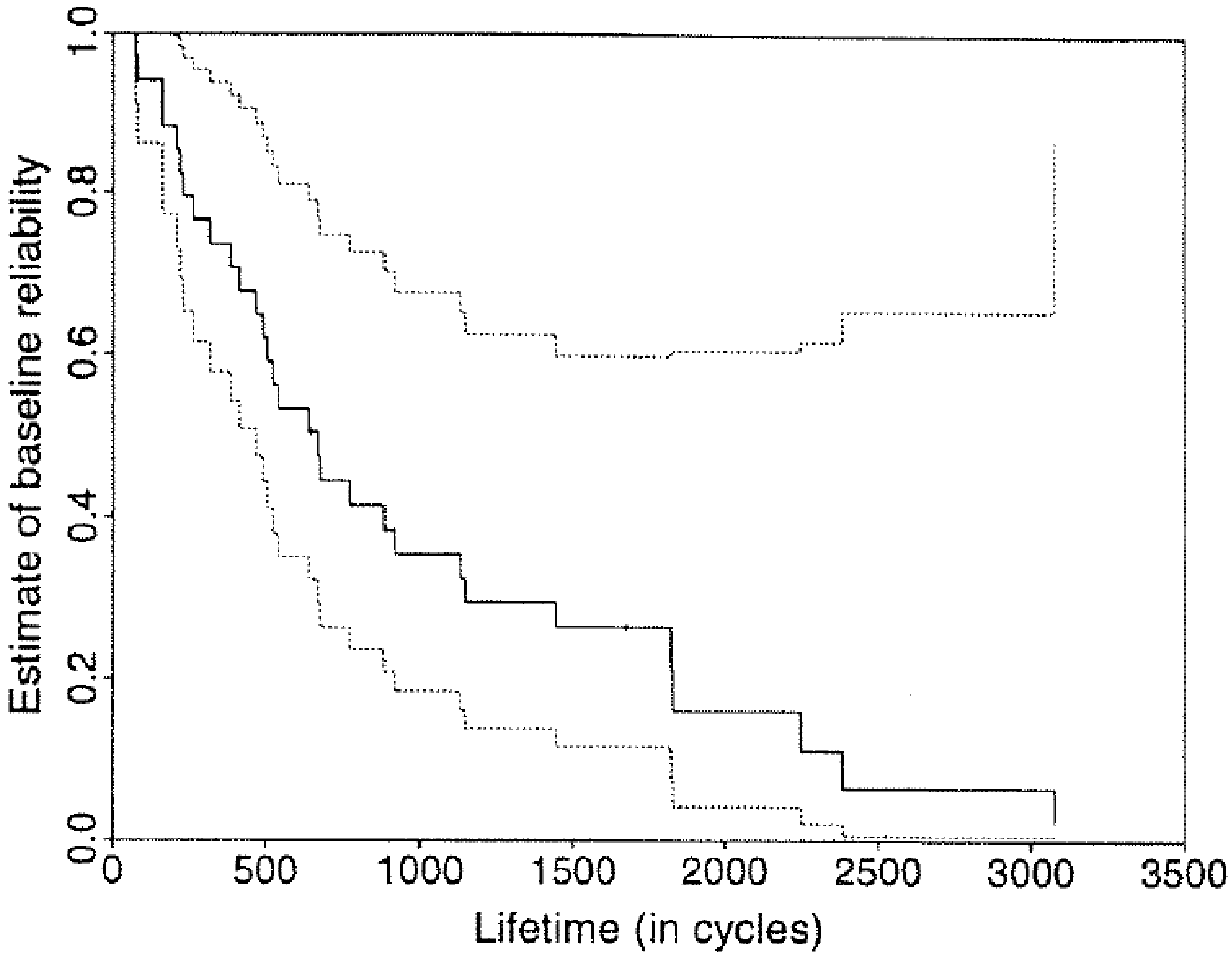


Fig. 3.3. Plot of the baseline reliability function for the proportional hazards model for the sodium sulphur battery data with 95% confidence limits

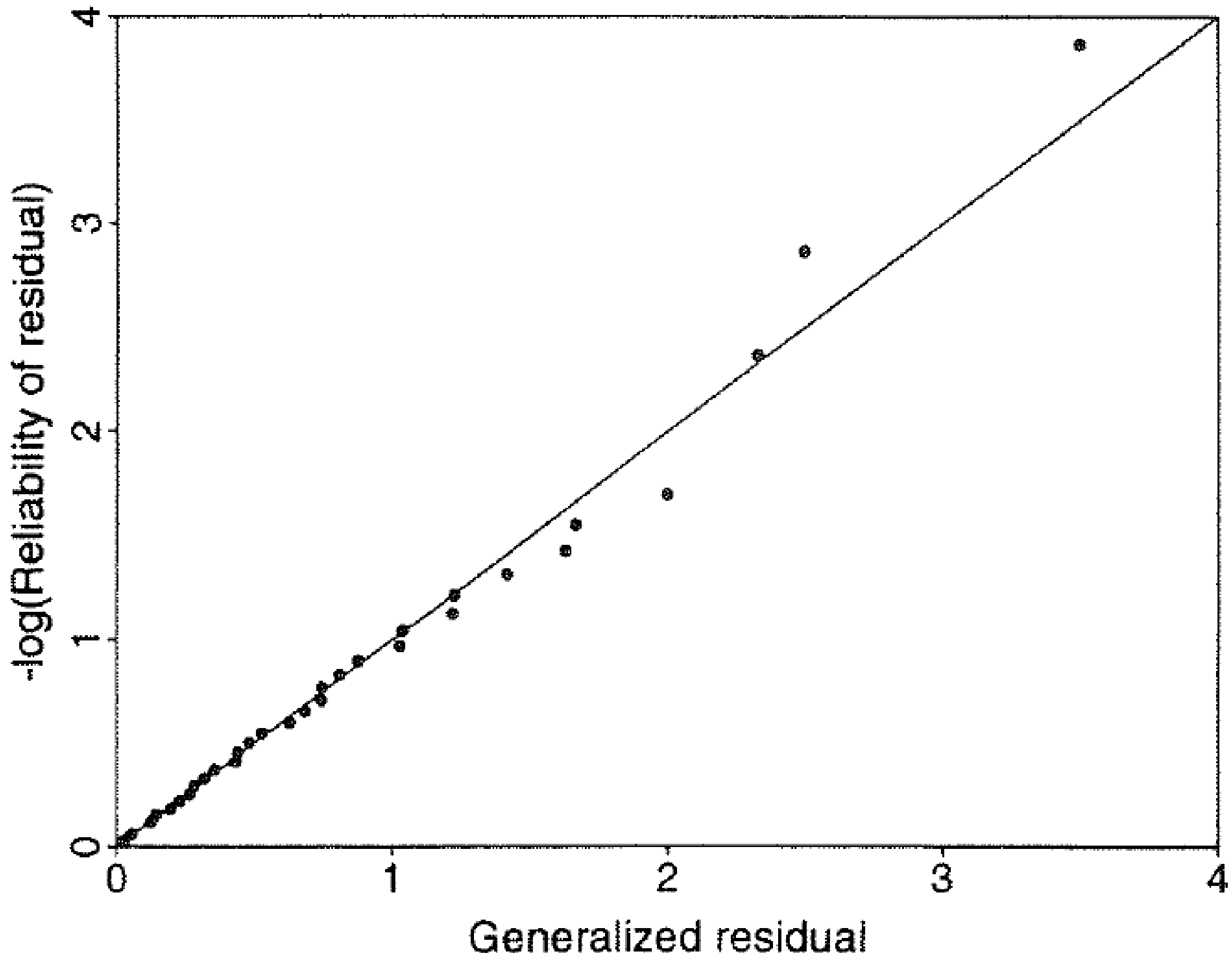


Fig. 3.5. Plot of the generalized residuals of the proportional hazards model for the sodium sulphur battery data

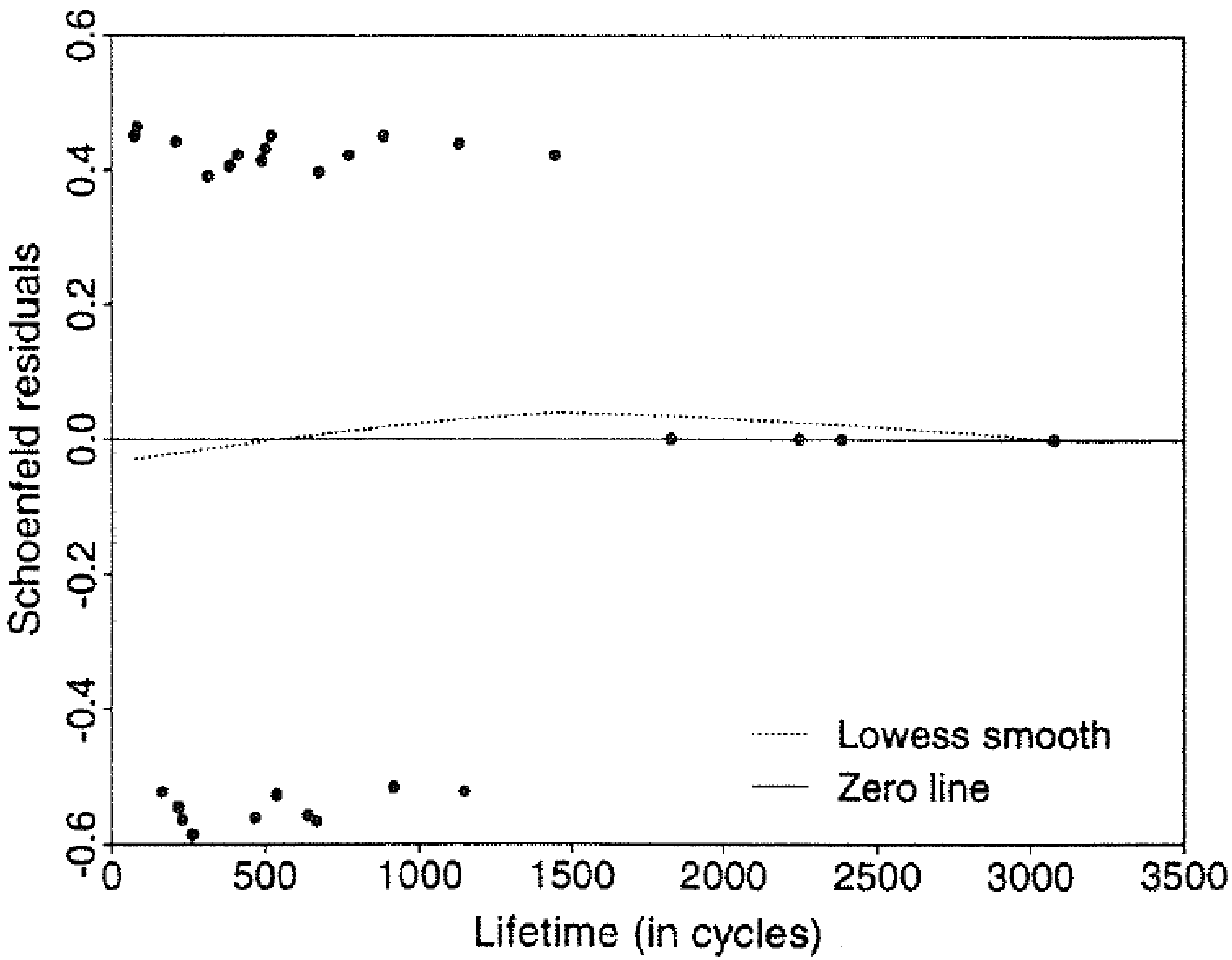
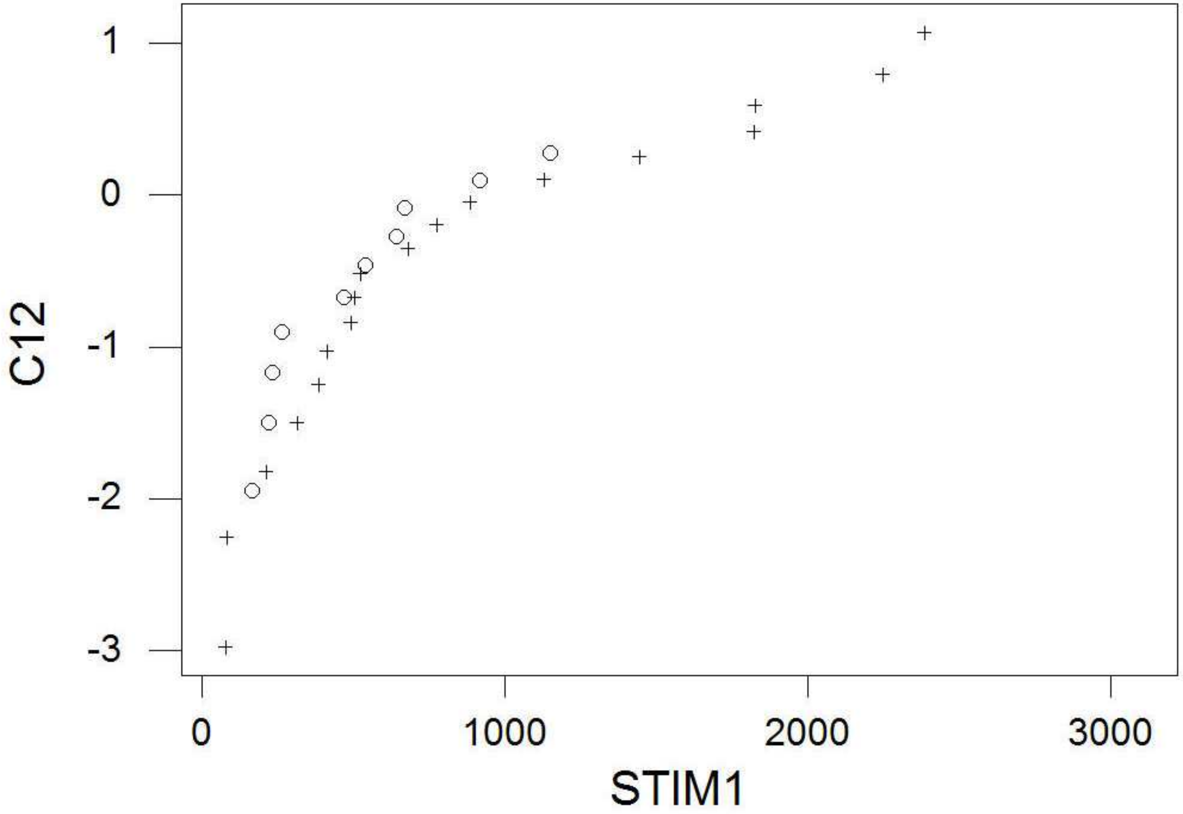


Fig. 3.7. Plot of the Schoenfeld residuals for batch of the proportional hazards model for the sodium sulphur battery data

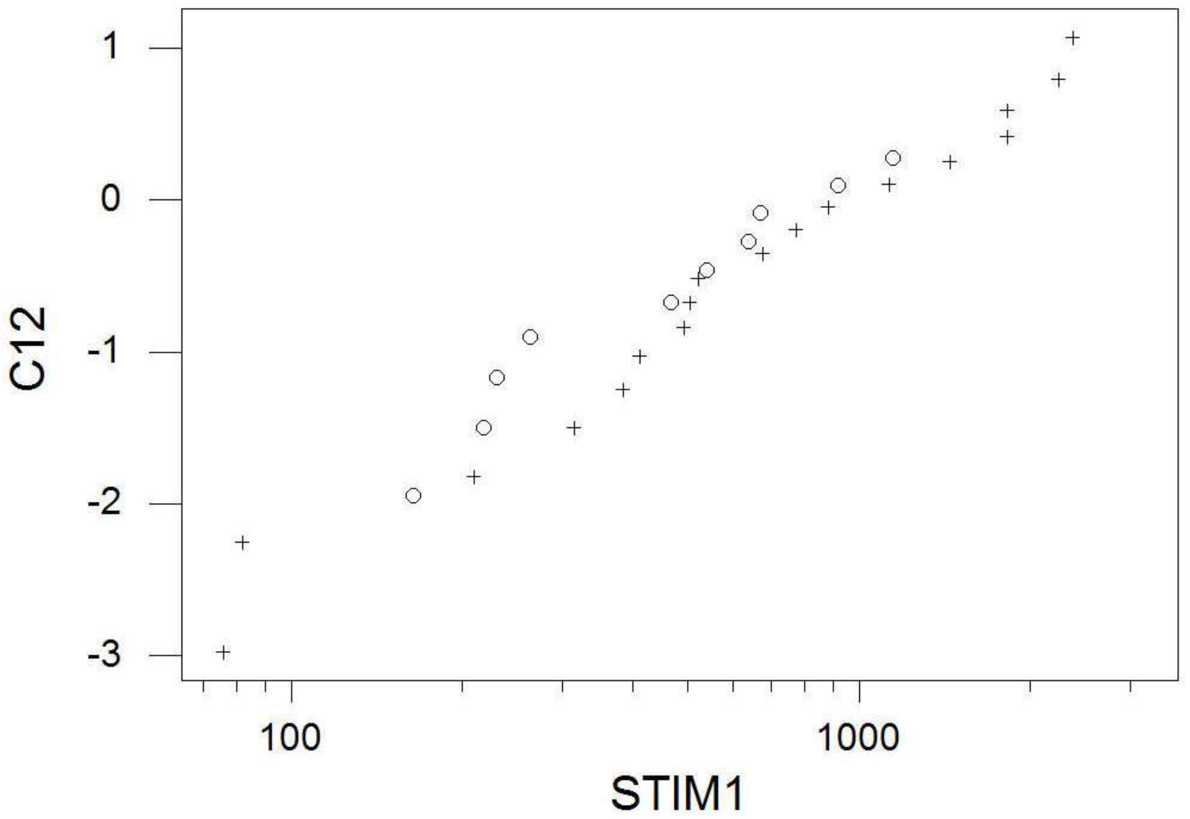
LOG MINUS LOG PLOT vs. "t" FOR A&P DATA (s.63)

Parallel plots indicate Proportional Hazard



LOG MINUS LOG PLOT vs. "log t" FOR A&P DATA (s.63)

Straight lines indicate Weibull distribution

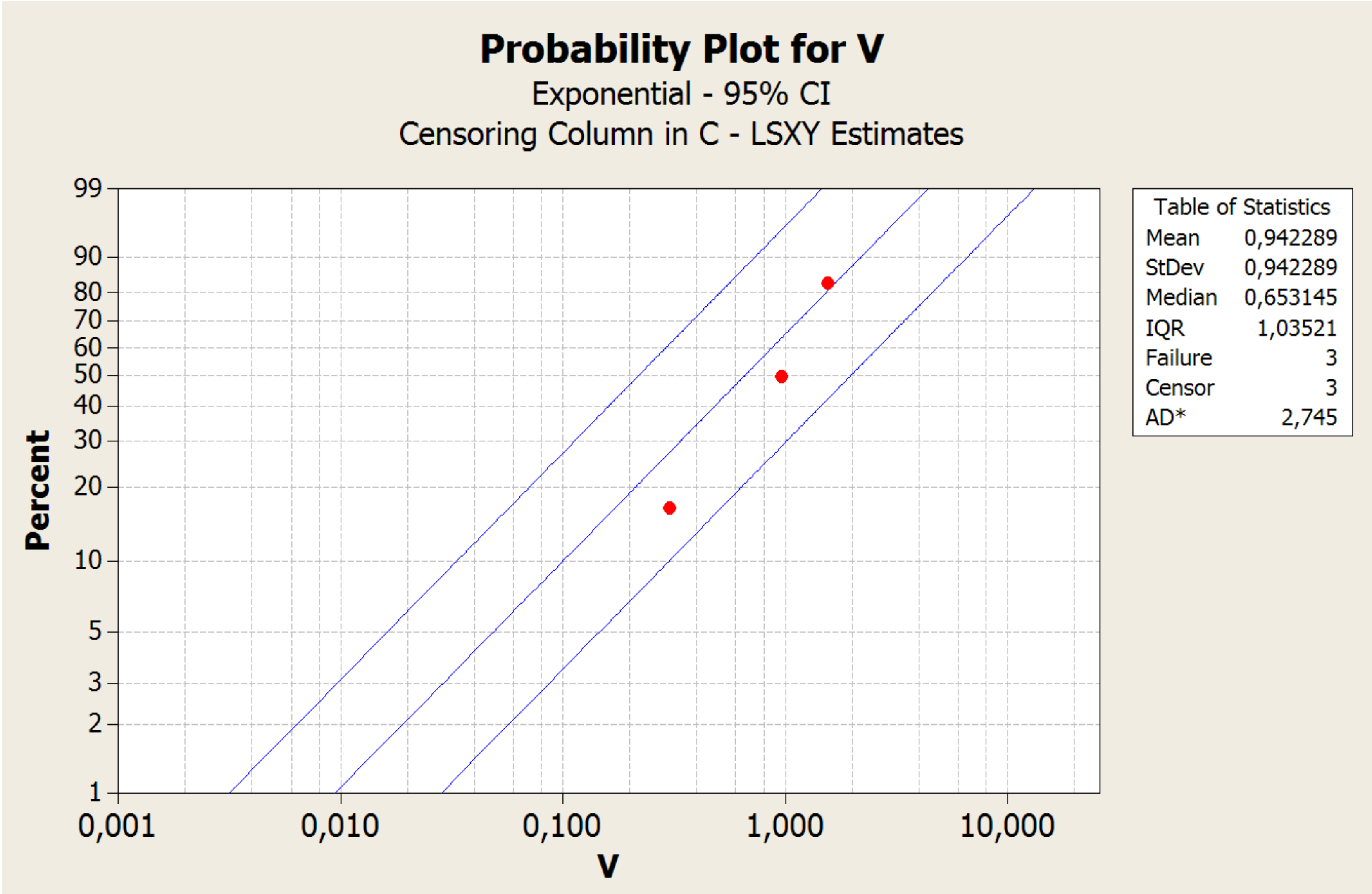


COX-SNELL RESIDUALS SIMPLE COX-EXAMPLE

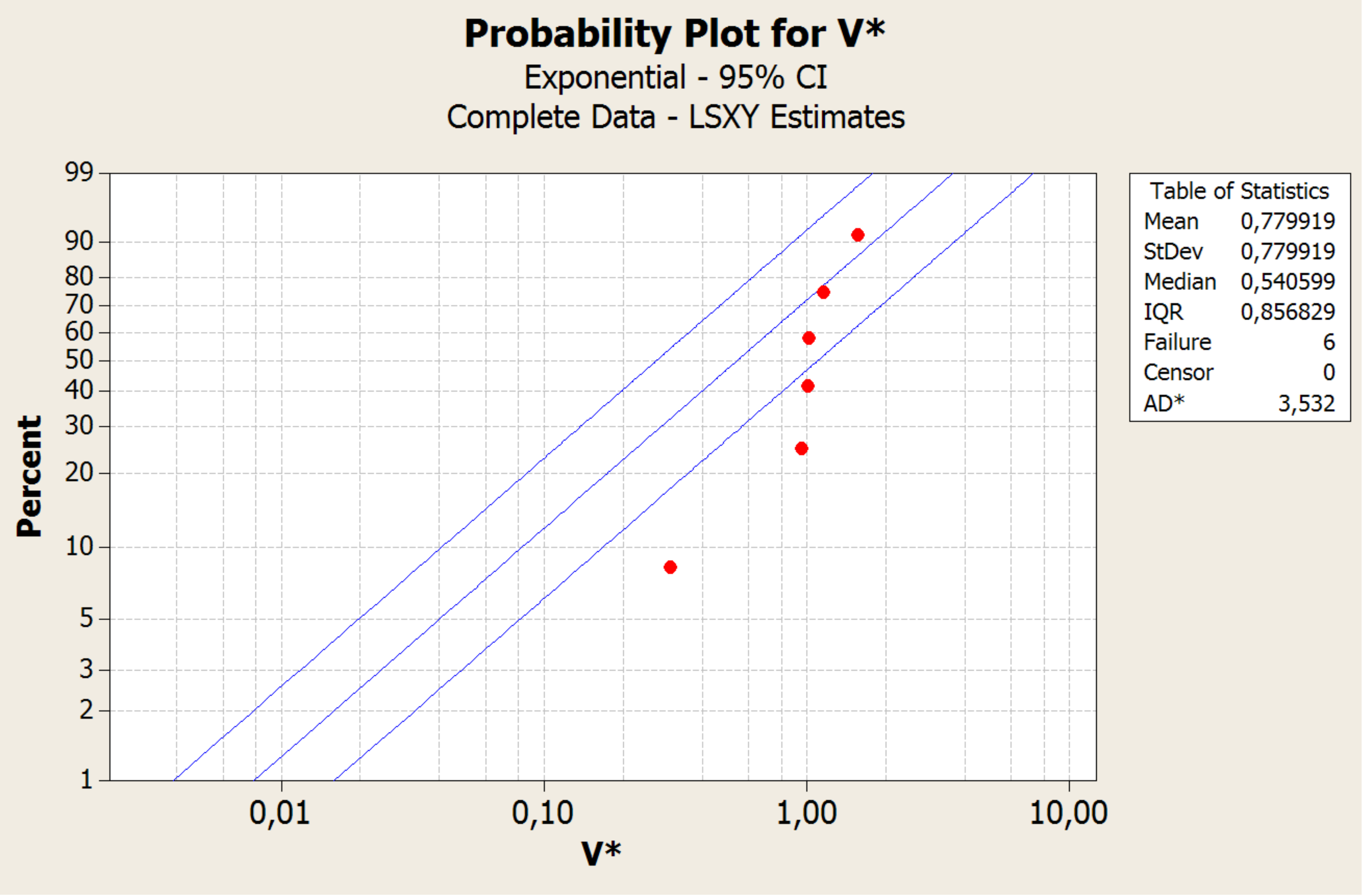
Worksheet 1 ***

↓	C1	C2	C3
	V	C	V*
1	0,9593	1	0,9593
2	0,0045	0	1,0045
3	0,0209	0	1,0209
4	0,3017	1	0,3017
5	1,5567	1	1,5567
6	0,1569	0	1,1569
7			

COX-SNELL RESIDUALS SIMPLE COX-EXAMPLE



MODIFIED COX-SNELL RESIDUALS SIMPLE COX-EXAMPLE



Methods and Formulas – Accelerated Life Testing

Equation

Lifetime regression

The regression model estimates the percentiles of the failure time distribution:

$Y = \beta_0 + \beta_1 X + \sigma\varepsilon$

where:

Y	= either failure time or log(failure time)
β_0	= y-intercept (constant)
β_1	= regression coefficient
X	= predictor values (may be transformed)
σ	= 1/shape (Weibull distribution) or scale (other distributions)
ε	= random error term

Response variable

Depending on the distribution, Y = failure time or log (failure time):

- For the Weibull, exponential, lognormal, and loglogistic distributions, Y = log (failure time)
- For the normal, extreme value, and logistic distributions, Y = failure time

When Y = log (failure time), Minitab takes the antilog to display the percentiles on the original scale.

Error term

The value of the error distribution also depends on the distribution chosen.

- For the normal distribution, the error distribution is the standard normal distribution − normal (0,1). For the lognormal distribution, Minitab takes the log base$_e$ of the data and uses a normal distribution.
- For the logistic distribution, the error distribution is the standard logistic distribution − logistic (0, 1). For the loglogistic distribution, Minitab takes the log of the data and uses a logistic distribution.
- For the extreme value distribution, the error distribution is the standard extreme value distribution − extreme value (0, 1). For the Weibull distribution and the exponential distribution (a type of Weibull distribution), Minitab takes the log of the data and uses the extreme value distribution.

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Models

Linear

$Y = \beta_0 + \beta_1 *$ accelerating variable $+ \sigma\varepsilon$

where:

- Y is the failure time or log failure time
- σ is the reciprocal of the shape parameter (Weibull distribution) or the scale parameter (other distributions)
- ε is the random error term

Arrhenius

$Y = \beta_0 + \beta_1 *$ [11604.83/Degrees Celsius + 273.16)] $+ \sigma\varepsilon$

where:

- Y is the failure time or log failure time
- σ is the reciprocal of the shape parameter (Weibull distribution) or the scale parameter (other distributions)
- ε is the random error term

Inverse temp

$Y = \beta_0 + \beta_1 *$ [1/(Degrees Celsius + 273.16)] $+ \sigma\varepsilon$

where:

- Y is the failure time or log failure time
- σ is the reciprocal of the shape parameter (Weibull distribution) or the scale parameter (other distributions)
- ε is the random error term

Loge (Power)

$Y = \beta_0 + \beta_1 *$ log(accelerating variable) $+ \sigma\varepsilon$

where:

- Y is the failure time or log failure time
- σ is the reciprocal of the shape parameter (Weibull distribution) or the scale parameter (other distributions)
- ε is the random error term

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Insulate.MTW ***

↓	C1	C2	C3	C4	C5-T	C6	C7	C8	C9	C10	C11
	Temp	ArrTemp	Plant	FailureT	Censor	Design	NewTemp	ArrNewT	NewPlant		
1	170	26,1865	1	343	F	80	80	32,8600	1		
2	170	26,1865	1	869	F	100	80	32,8600	2		
3	170	26,1865	1	244	C		100	31,0988	1		
4	170	26,1865	1	716	F		100	31,0988	2		
5	170	26,1865	1	531	F						
6	170	26,1865	1	738	F						
7	170	26,1865	1	461	F						
8	170	26,1865	1	221	F						
9	170	26,1865	1	665	F						
10	170	26,1865	1	384	C						
11	170	26,1865	2	394	C						
12	170	26,1865	2	369	F						
13	170	26,1865	2	366	F						
14	170	26,1865	2	507	F						
15	170	26,1865	2	461	F						
16	170	26,1865	2	431	F						
17	170	26,1865	2	479	F						
18	170	26,1865	2	106	F						
19	170	26,1865	2	545	F						
20	170	26,1865	2	536	F						
21	150	27,4242	1	2134	C						
22	150	27,4242	1	2746	F						
23	150	27,4242	1	2859	F						
24	150	27,4242	1	1826	C						

MINITAB Help

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Example of Accelerated Life Testing

main topic interpreting results session command see also

Suppose you want to investigate the deterioration of an insulation used for electric motors. The motors normally run between 80 and 100° C. To save time and money, you decide to use accelerated life testing.

First you gather failure times for the insulation at abnormally high temperatures – 110, 130, 150, and 170° C – to speed up the deterioration. With failure time information at these temperatures, you can then extrapolate to 80 and 100° C. It is known that an Arrhenius relationship exists between temperature and failure time.

To see how well the model fits, you will draw a probability plot based on the standardized residuals.

1 Open the worksheet INSULATE.MTW.

2 Choose **Stat > Reliability/Survival > Accelerated Life Testing**.

3 In **Variables/Start variables**, enter **FailureT**. In **Accelerating variable**, enter **Temp**.

4 From **Relationship**, choose **Arrhenius**.

5 Click **Censor**. In **Use censoring columns**, enter **Censor**, then click **OK**.

6 Click **Graphs**. In **Enter design value to include on plot**, enter **80**. Click **OK**.

7 Click **Estimate**. In **Enter new predictor values**, enter **Design**, then click **OK** in each dialog box.

Session window output

Regression with Life Data: FailureT versus Temp

```
Response Variable:  FailureT

Censoring Information                Count
Uncensored value                        66
Right censored value                    14
Censoring value:  Censor = C

Estimation Method:  Maximum Likelihood
Distribution:  Weibull
Transformation on accelerating variable:  Arrhenius
```

Regression Table

Predictor	Coef	Standard Error	Z	P	95.0% Normal CI Lower	Upper
Intercept	-15.1874	0.9862	-15.40	0.000	-17.1203	-13.2546
Temp	0.83072	0.03504	23.71	0.000	0.76204	0.89940
Shape	2.8246	0.2570			2.3633	3.3760

Log-Likelihood = -564.693

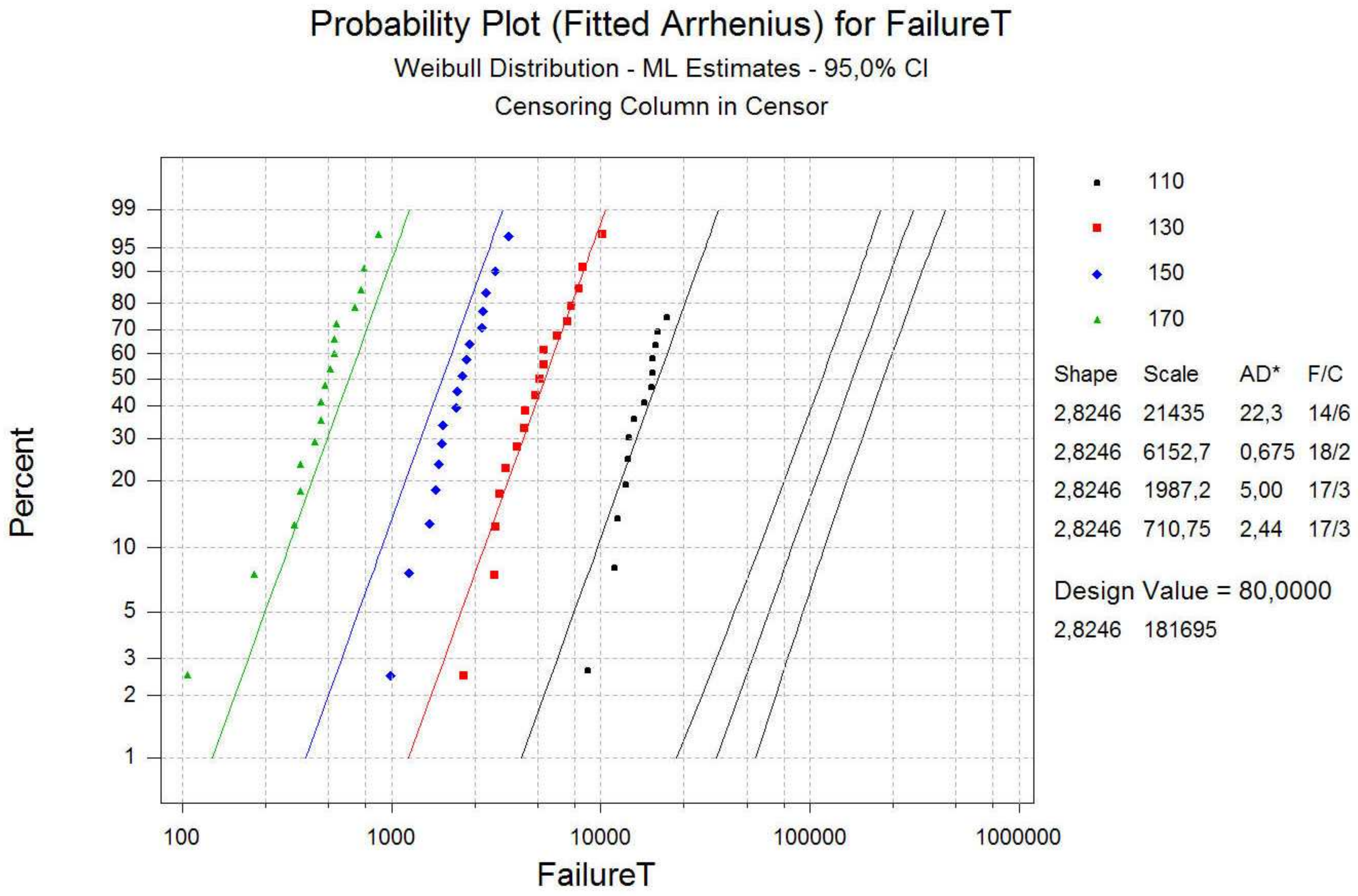
Anderson-Darling (adjusted) Goodness-of-Fit

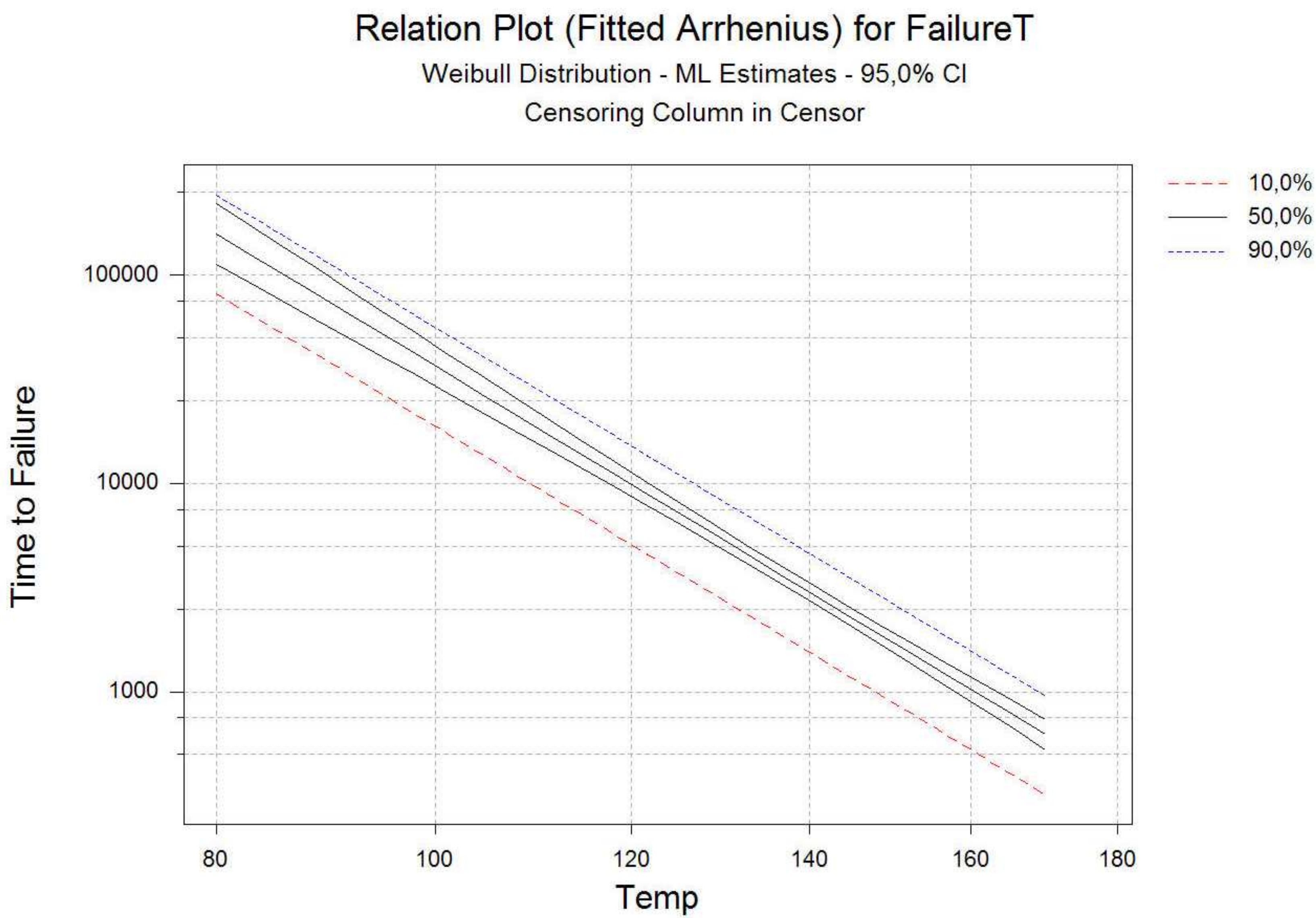
At each accelerating level

Level	Fitted Model
110	*
130	*
150	*
170	*

Table of Percentiles

Percent	Temp	Percentile	Standard Error	95.0% Normal CI Lower	Upper
50	80.0000	159584.5	27446.85	113918.2	223557.0
50	100.0000	36948.57	4216.511	29543.36	46209.94


Probability Plot (Fitted Arrhenius) for FailureT
Weibull Distribution - ML Estimates - 95,0% CI
Censoring Column in Censor
Percent
99
95
90
80
70
60
50
40
30
20
10
5
3
2
1
100
1000
10000
100000
1000000
FailureT
110
130
150
170
Shape Scale AD* F/C
2,8246 21435 22,3 14/6
2,8246 6152,7 0,675 18/2
2,8246 1987,2 5,00 17/3
2,8246 710,75 2,44 17/3
Design Value = 80,0000
2,8246 181695


Relation Plot (Fitted Arrhenius) for FailureT
Weibull Distribution - ML Estimates - 95,0% CI
Censoring Column in Censor
Time to Failure
100000
10000
1000
80
100
120
140
160
180
Temp
10,0%
50,0%
90,0%

ADDING THE FACTOR "PLANT":