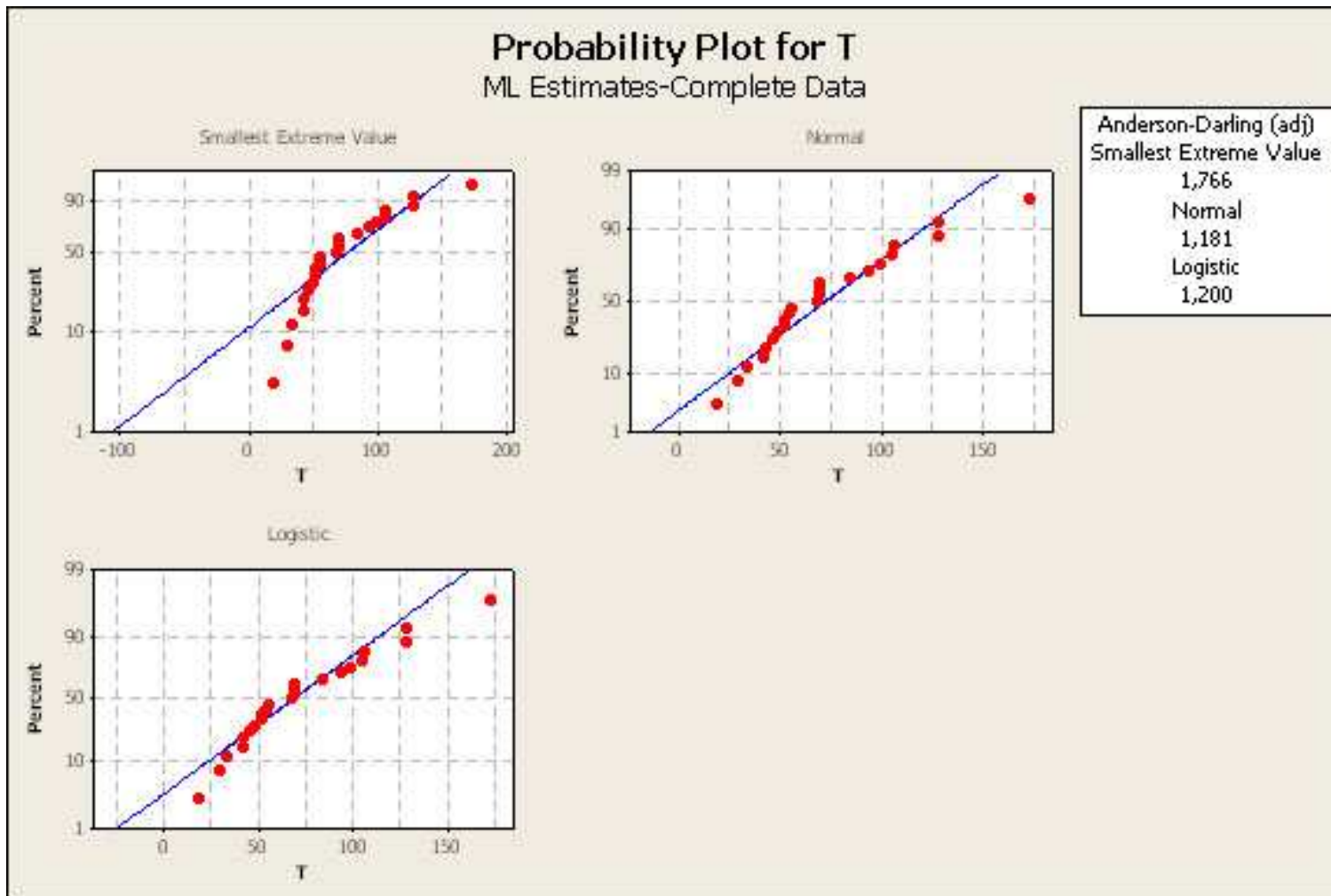
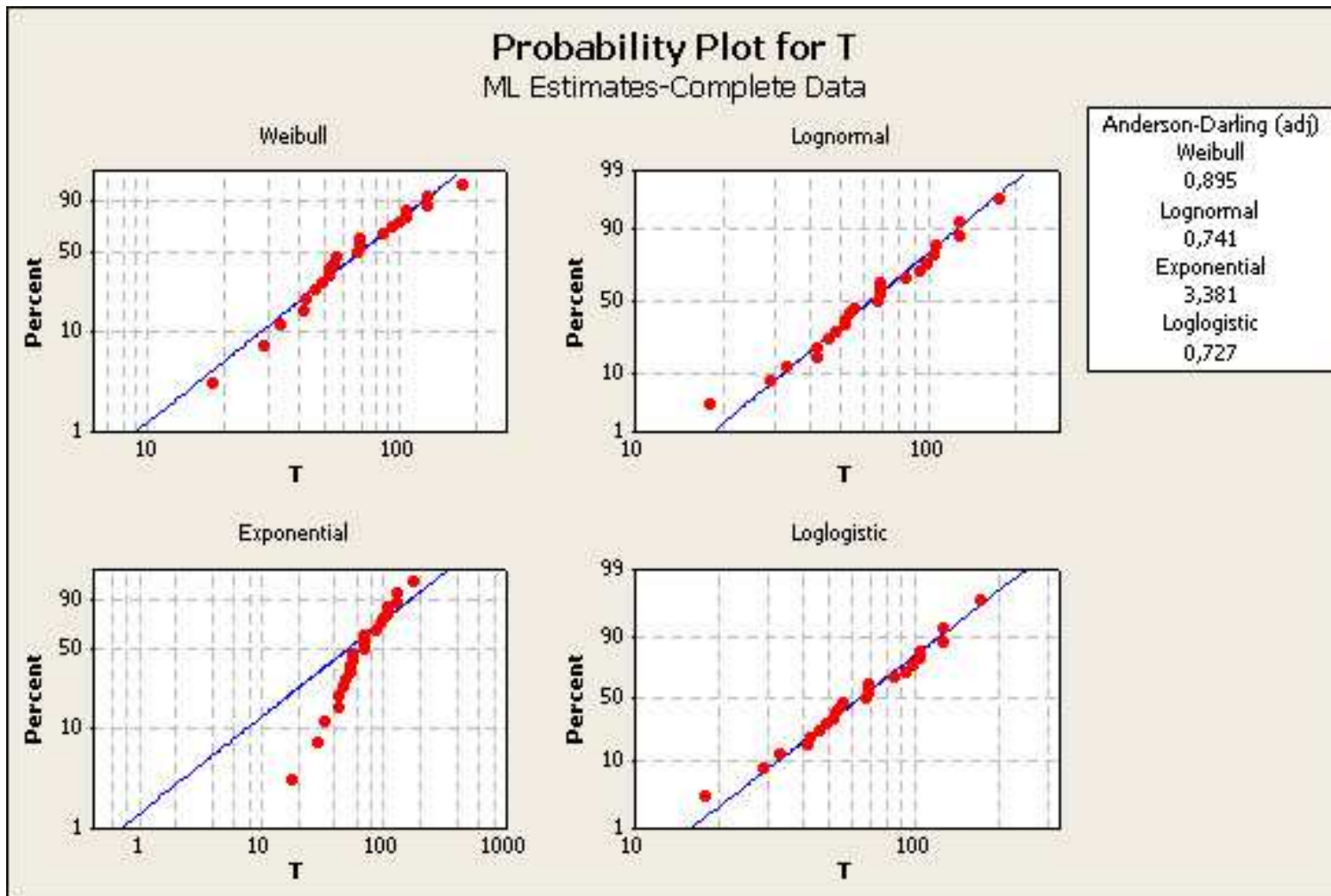


BALL BEARINGS DATA



BALL BEARINGS DATA



BALL BEARING DATA

Distribution ID Plot: T

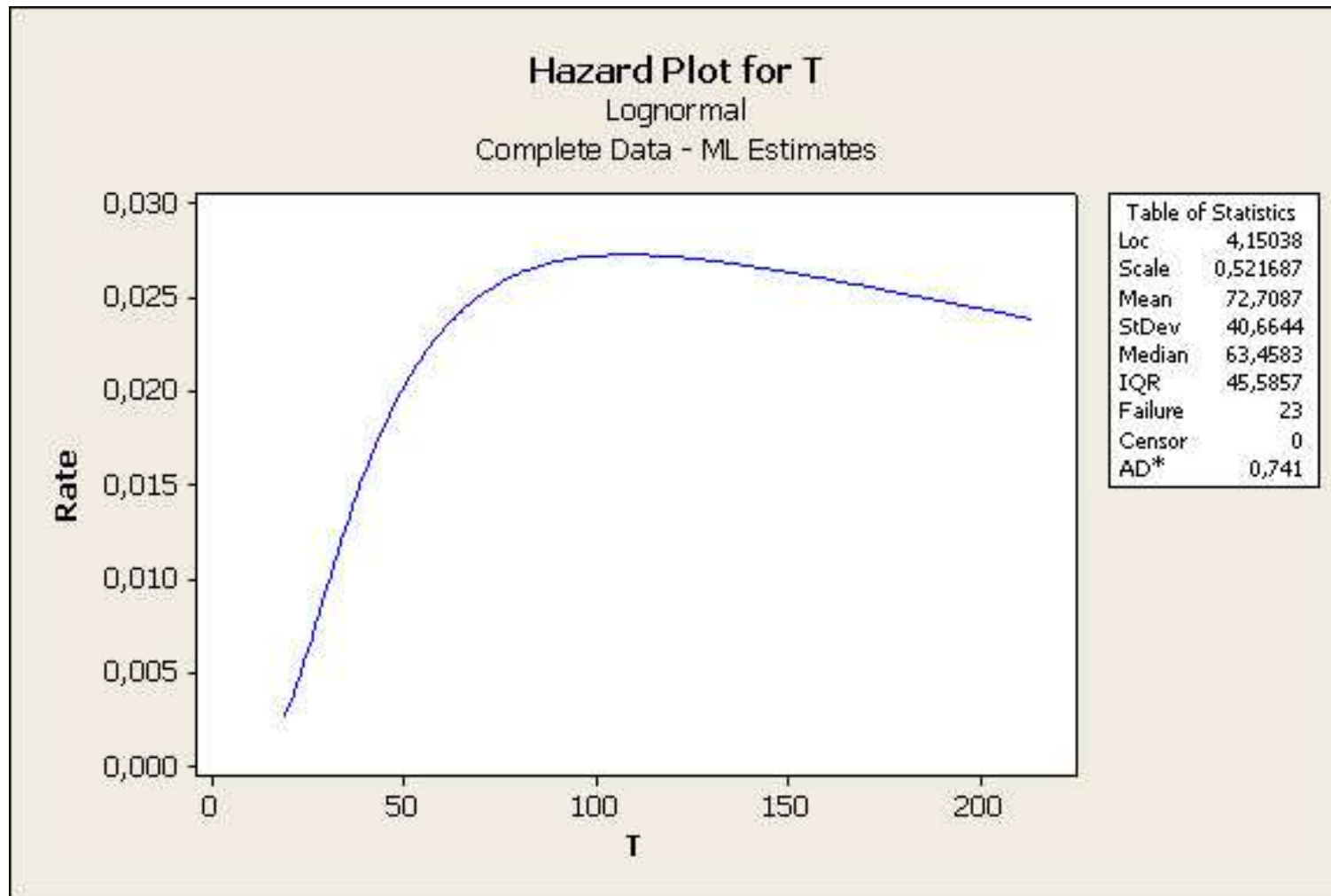
Goodness-of-Fit

| | Anderson-Darling |
|--------------|------------------|
| Distribution | (adj) |
| Exponential | 3,381 |
| Weibull | 0,895 |
| Lognormal | 0,741 |
| Loglogistic | 0,727 |

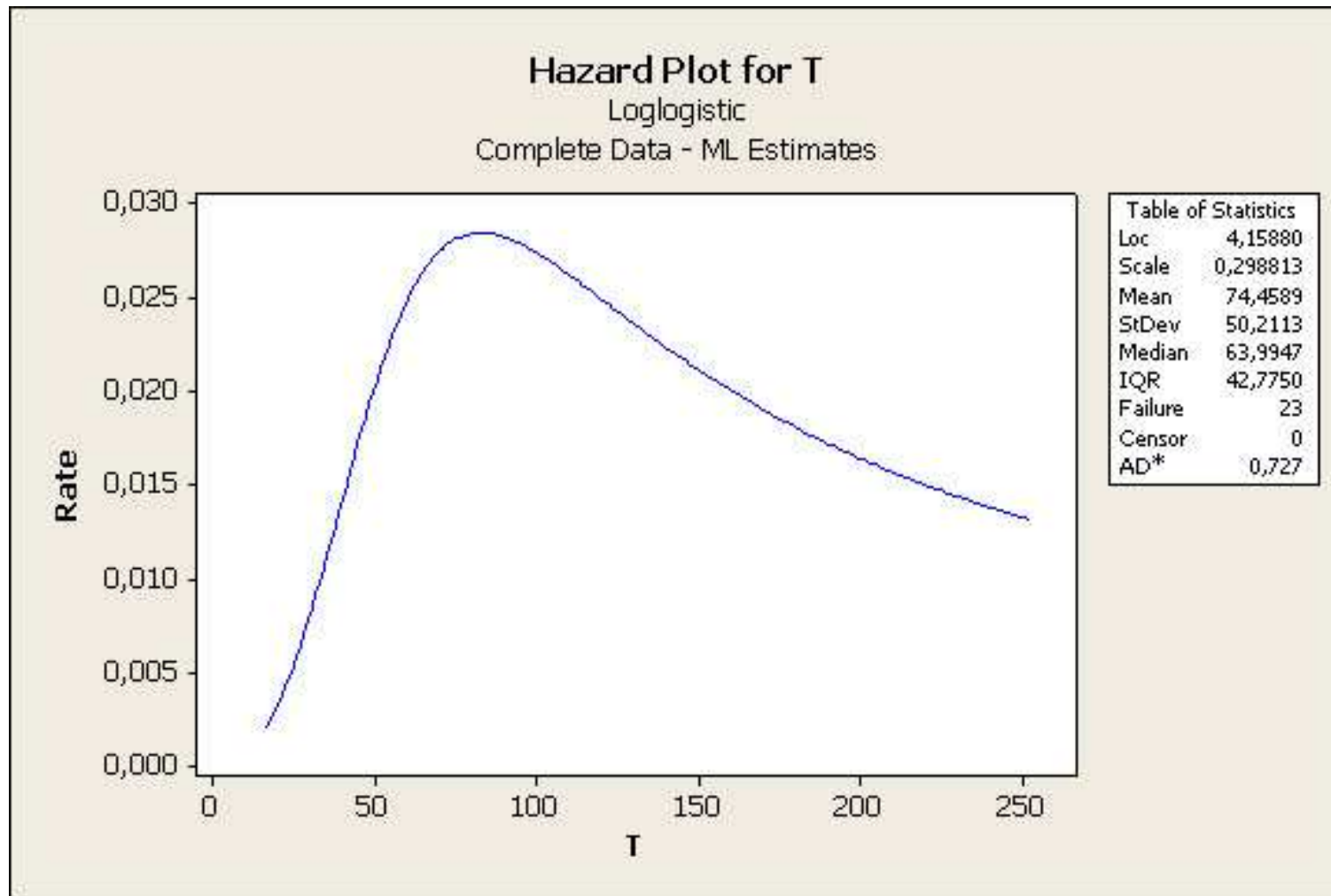
Table of MTF

| | | Standard | 95% Normal CI | |
|--------------|---------|----------|---------------|---------|
| Distribution | Mean | Error | Lower | Upper |
| Exponential | 72,2209 | 15,0591 | 47,9927 | 108,680 |
| Weibull | 72,5154 | 7,5939 | 59,0596 | 89,037 |
| Lognormal | 72,7087 | 8,4302 | 57,9288 | 91,260 |
| Loglogistic | 74,4589 | 9,0701 | 58,6447 | 94,538 |

BALL BEARINGS DATA



BALL BEARINGS DATA



Distribution: Loglogistic

Parameter Estimates

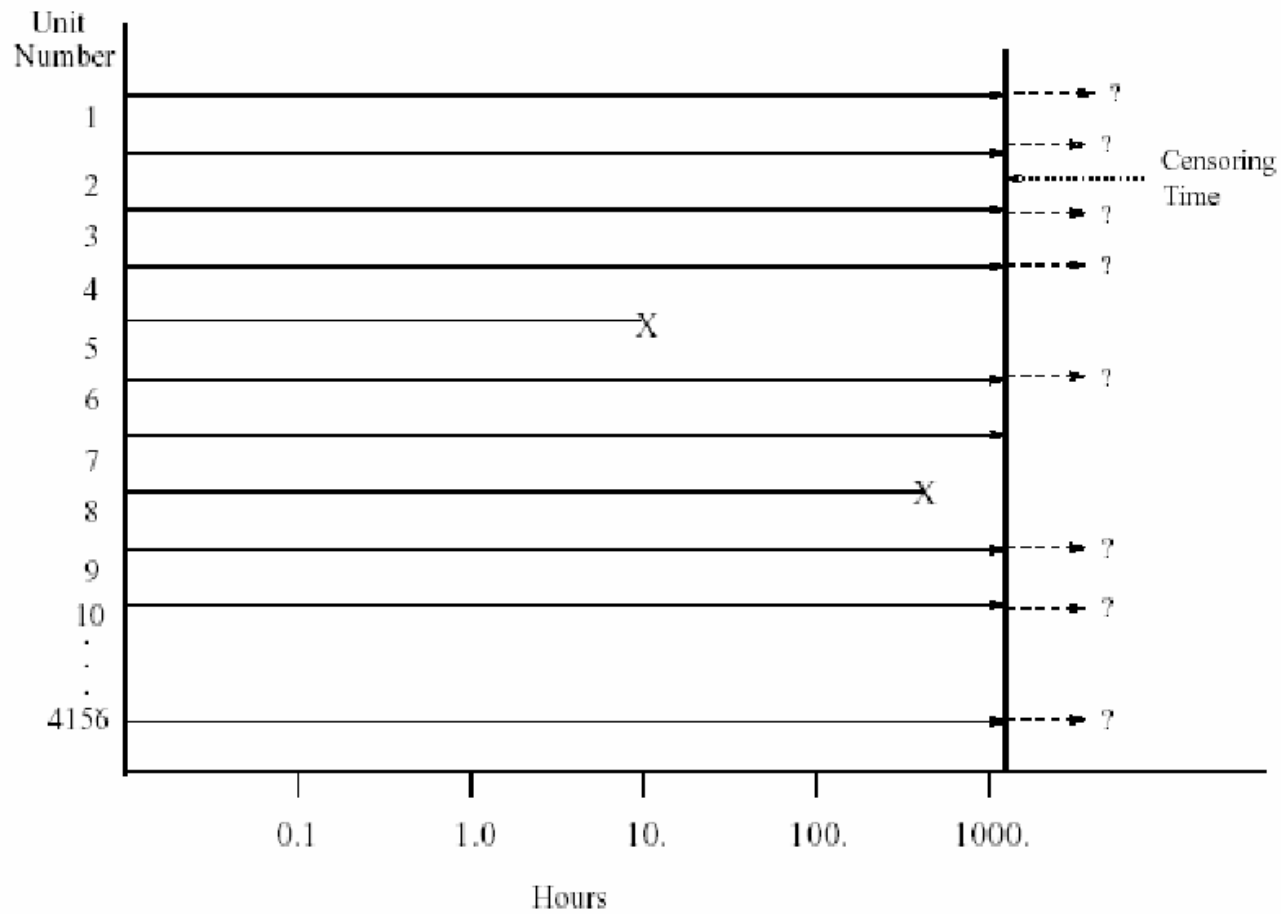
| Parameter | Estimate | Standard Error | 95,0% Normal CI Lower | 95,0% Normal CI Upper |
|-----------|----------|----------------|-----------------------|-----------------------|
| Location | 4,15880 | 0,109047 | 3,94507 | 4,37253 |
| Scale | 0,298813 | 0,0515266 | 0,213118 | 0,418965 |

Characteristics of Distribution

| | Estimate | Standard Error | 95,0% Normal CI Lower | 95,0% Normal CI Upper |
|--------------------------|----------|----------------|-----------------------|-----------------------|
| Mean(MTTF) | 74,4589 | 9,07013 | 58,6447 | 94,5377 |
| Standard Deviation | 50,2113 | 17,0541 | 25,8044 | 97,7034 |
| Median | 63,9947 | 6,97843 | 51,6801 | 79,2438 |
| First Quartile(Q1) | 46,0866 | 5,65417 | 36,2363 | 58,6144 |
| Third Quartile(Q3) | 88,8616 | 10,9338 | 69,8199 | 113,096 |
| Interquartile Range(IQR) | 42,7750 | 8,96475 | 28,3658 | 64,5036 |

EXAMPLE: CENSORING

IC Data Failure Pattern



Example of Censoring: Leukemia Data

Remission times for 42 children with acute leukemia

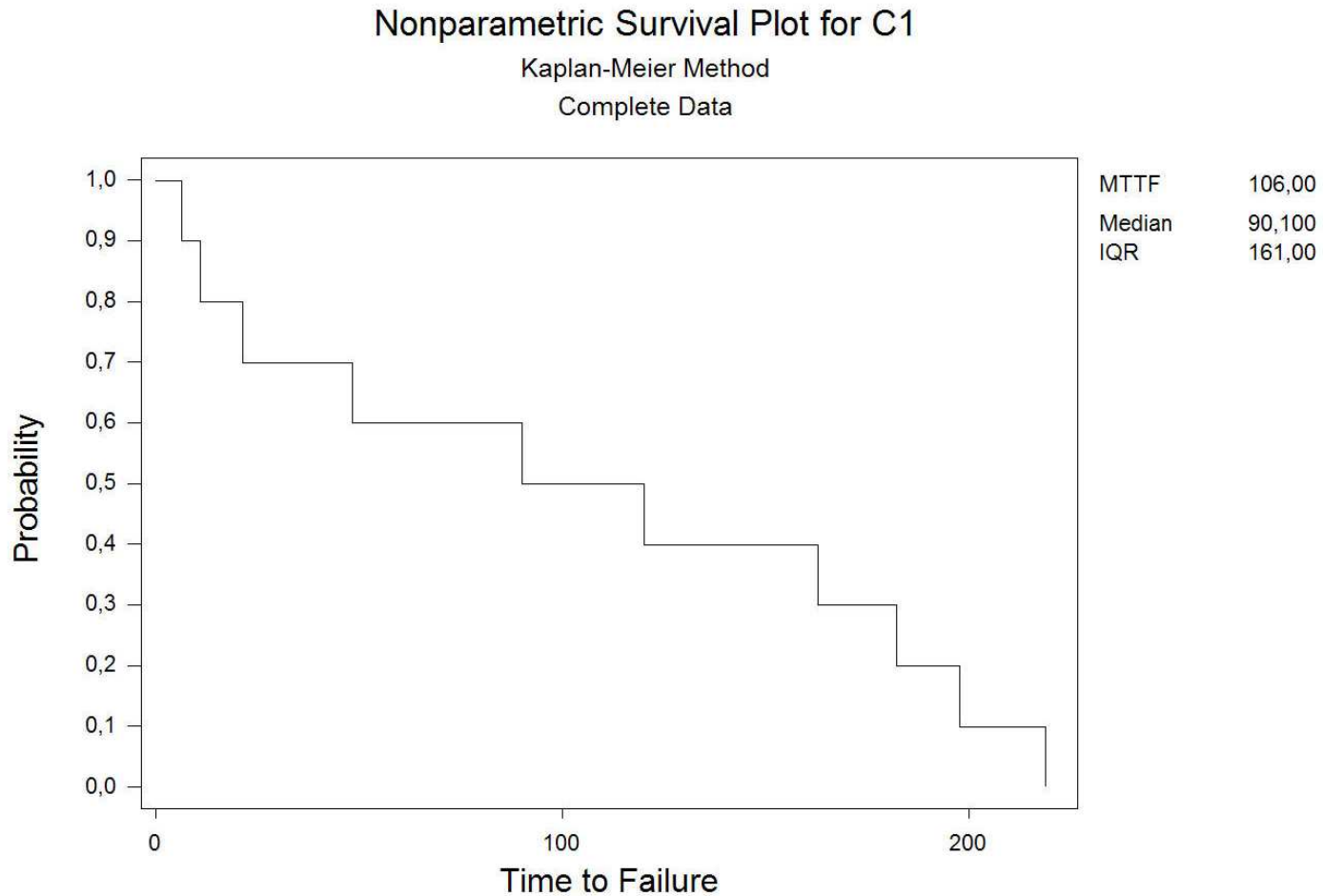
T=remission time (free of symptoms), C=censoring
(0=censored)

| | Placebo | | 6MP treatment | |
|---|---------|---|---------------|---|
| | T | C | T | C |
| 1 | 1 | 1 | 6 | 1 |
| 2 | 1 | 1 | 6 | 1 |
| 3 | 2 | 1 | 6 | 1 |
| 4 | 2 | 1 | 6 | 0 |
| 5 | 3 | 1 | 7 | 1 |
| 6 | 4 | 1 | 9 | 0 |
| 7 | 4 | 1 | 10 | 1 |
| 8 | 5 | 1 | 10 | 0 |
| 9 | 5 | 1 | 11 | 0 |

| | Placebo | | 6MP treatment | |
|----|---------|---|---------------|---|
| | T | C | T | C |
| 10 | 8 | 1 | 13 | 1 |
| 11 | 8 | 1 | 16 | 1 |
| 12 | 8 | 1 | 17 | 0 |
| 13 | 8 | 1 | 19 | 0 |
| 14 | 11 | 1 | 20 | 0 |
| 15 | 11 | 1 | 22 | 1 |
| 16 | 12 | 1 | 23 | 1 |
| 17 | 12 | 1 | 25 | 0 |
| 18 | 15 | 1 | 32 | 0 |
| 19 | 17 | 1 | 32 | 0 |
| 20 | 22 | 1 | 34 | 0 |
| 21 | 23 | 1 | 35 | 0 |

EMPIRICAL SURVIVAL FUNCTION – COMPLETE DATA (Example 11.8)

Data: 6.3, 11.0, 21.5, 48.4, 90.1, 120.2, 163.0, k 182.5, 198.0, 219.0

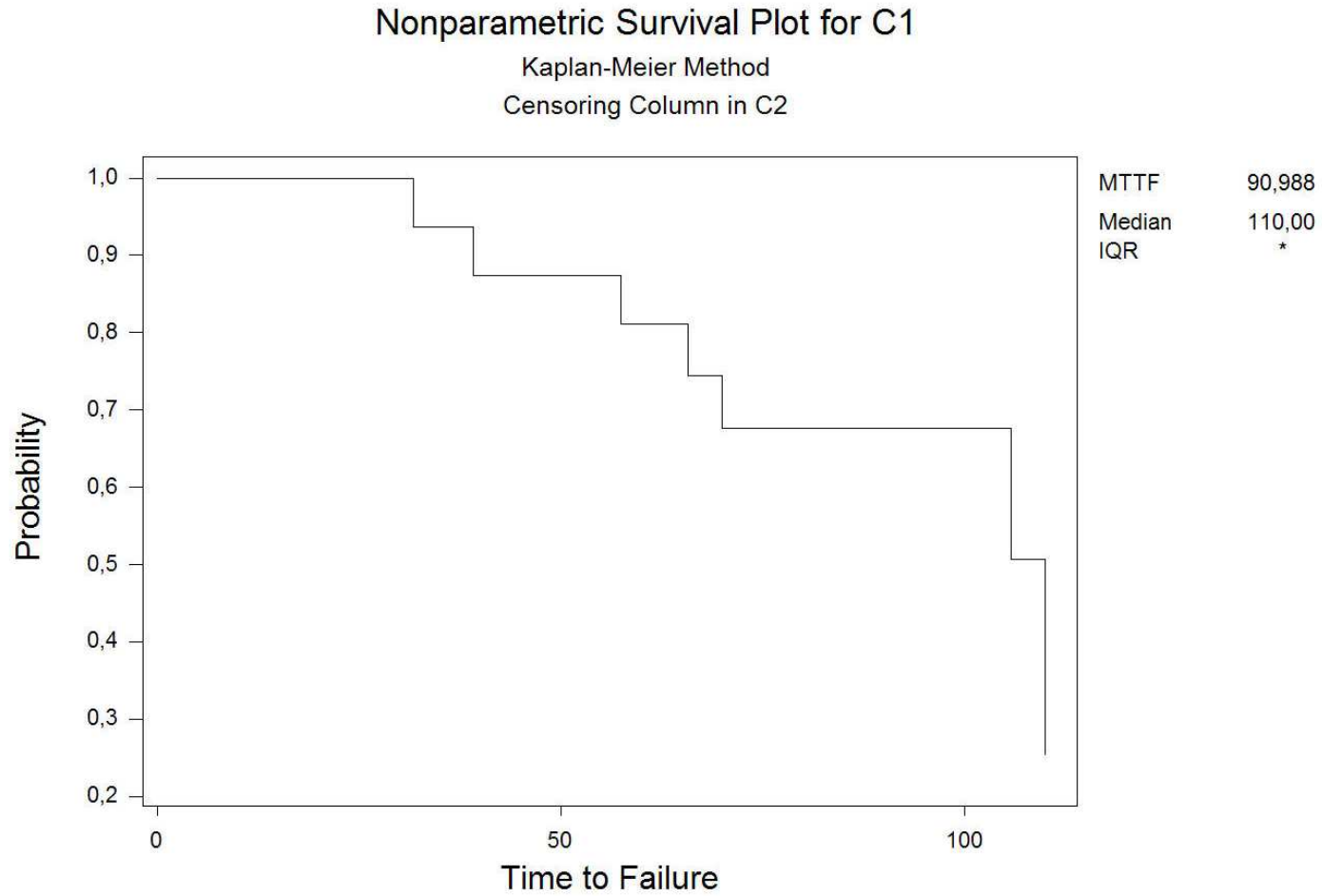


KAPLAN-MEIER ESTIMATOR – CENSORED DATA (Example 11.5)

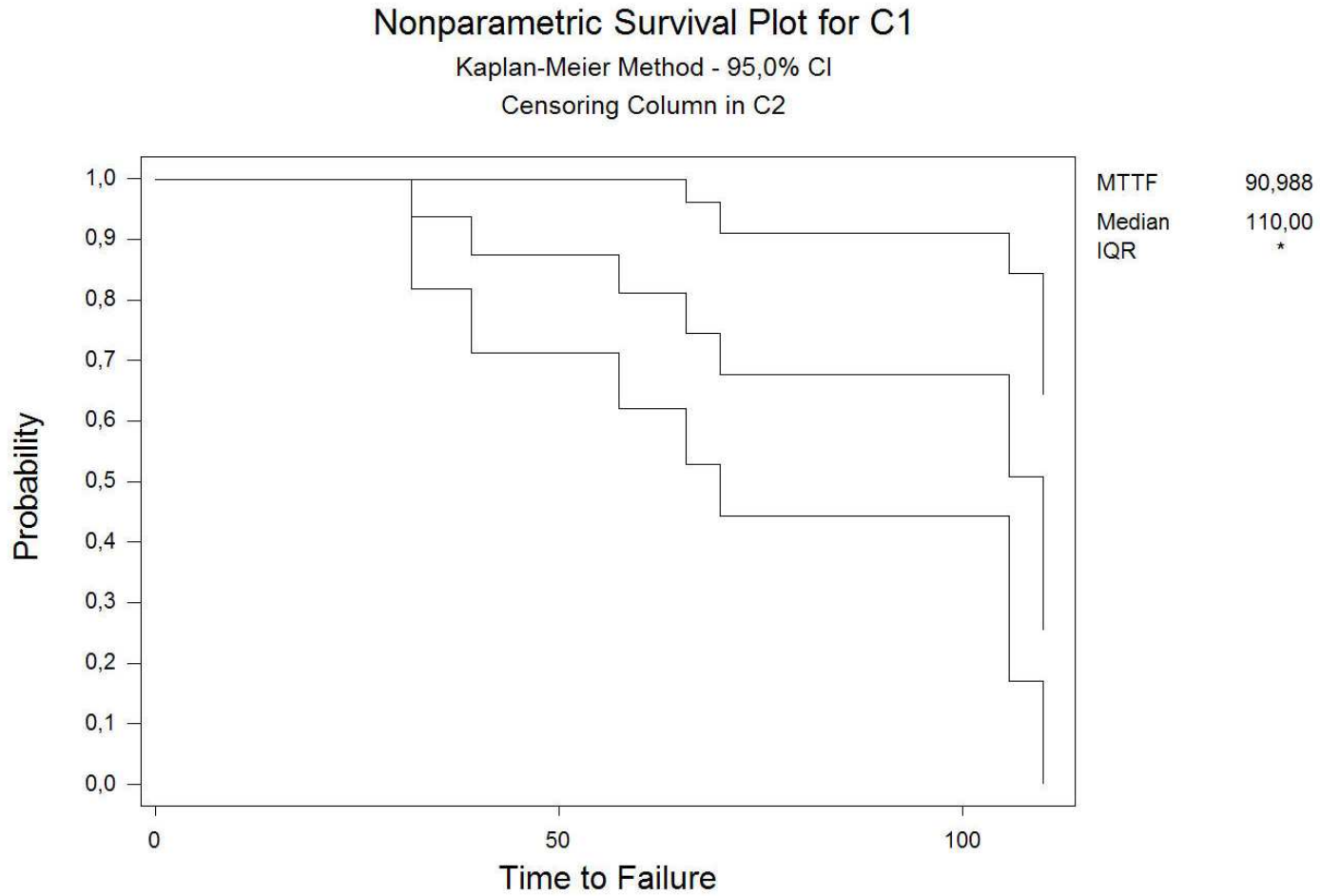
| Row | C1 | C2 |
|-----|-------|----|
| 1 | 31,7 | 1 |
| 2 | 39,2 | 1 |
| 3 | 57,5 | 1 |
| 4 | 65,0 | 0 |
| 5 | 65,8 | 1 |
| 6 | 70,0 | 1 |
| 7 | 75,0 | 0 |
| 8 | 75,2 | 0 |
| 9 | 87,5 | 0 |
| 10 | 88,3 | 0 |
| 11 | 94,2 | 0 |
| 12 | 101,7 | 0 |
| 13 | 105,8 | 1 |
| 14 | 109,2 | 0 |
| 15 | 110,0 | 1 |
| 16 | 130,0 | 0 |

| Time | Number at Risk | Number Failed | Survival Probability | Standard Error | 95,0% Normal CI Lower | Upper |
|----------|-------------------|------------------|-------------------------|-------------------|--------------------------|--------|
| 31,7000 | 16 | 1 | 0,9375 | 0,0605 | 0,8189 | 1,0000 |
| 39,2000 | 15 | 1 | 0,8750 | 0,0827 | 0,7130 | 1,0000 |
| 57,5000 | 14 | 1 | 0,8125 | 0,0976 | 0,6213 | 1,0000 |
| 65,8000 | 12 | 1 | 0,7448 | 0,1105 | 0,5283 | 0,9613 |
| 70,0000 | 11 | 1 | 0,6771 | 0,1194 | 0,4431 | 0,9111 |
| 105,8000 | 4 | 1 | 0,5078 | 0,1718 | 0,1711 | 0,8445 |
| 110,0000 | 2 | 1 | 0,2539 | 0,1990 | 0,0000 | 0,6440 |

KM-plot (Example 11.5)



KM-plot with confidence limits (Example 11.5)



Kaplan-Meier Estimator – Leukemia Data

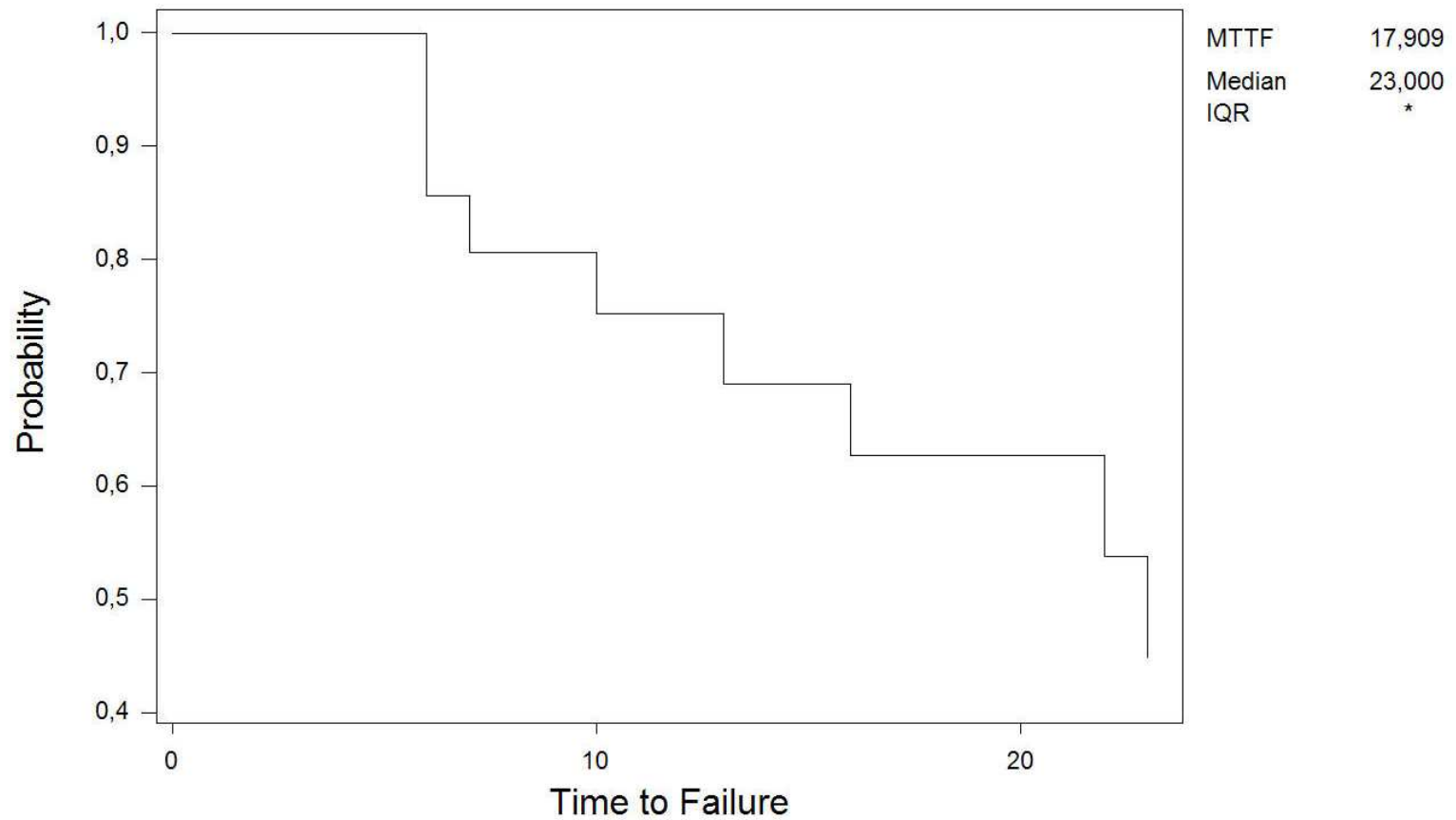
Treatment group (6MP)

| Time | Number at Risk | Number Failed | Survival Probability | Standard Error |
|---------|-------------------|------------------|-------------------------|-------------------|
| 6,0000 | 21 | 3 | 0,8571 | 0,0764 |
| 7,0000 | 17 | 1 | 0,8067 | 0,0869 |
| 10,0000 | 15 | 1 | 0,7529 | 0,0963 |
| 13,0000 | 12 | 1 | 0,6902 | 0,1068 |
| 16,0000 | 11 | 1 | 0,6275 | 0,1141 |
| 22,0000 | 7 | 1 | 0,5378 | 0,1282 |
| 23,0000 | 6 | 1 | 0,4482 | 0,1346 |

KM-plot: Leukemia Data (Treatment group)

Nonparametric Survival Plot for TMP

Kaplan-Meier Method
Censoring Column in CMP



NELSON ESTIMATOR

Data from Example 11.5

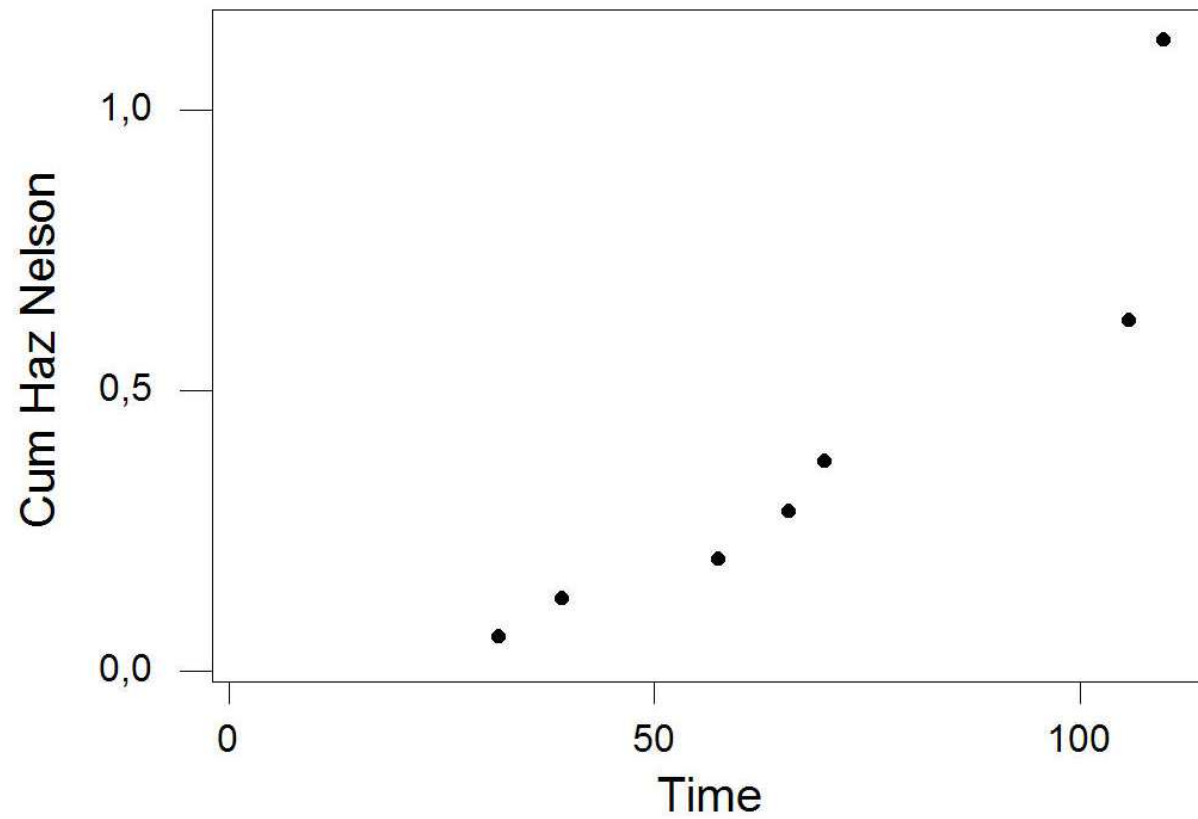
| Row | Time | Numb at risk | 1/Numb at risk | Cum Haz Nelson | Survival Nelson |
|-----|-------|--------------|----------------|----------------|-----------------|
| 1 | 31,7 | 16 | 0,062500 | 0,06250 | 0,939413 |
| 2 | 39,2 | 15 | 0,066667 | 0,12917 | 0,878827 |
| 3 | 57,5 | 14 | 0,071429 | 0,20060 | 0,818244 |
| 4 | 65,8 | 12 | 0,083333 | 0,28393 | 0,752820 |
| 5 | 70,0 | 11 | 0,090909 | 0,37484 | 0,687401 |
| 6 | 105,8 | 4 | 0,250000 | 0,62484 | 0,535348 |
| 7 | 110,0 | 2 | 0,500000 | 1,12484 | 0,324705 |

Survival KM

| | | | | | |
|----------|----------|----------|----------|----------|----------|
| 0,937500 | 0,875000 | 0,812500 | 0,744792 | 0,677083 | 0,507813 |
| 0,253906 | | | | | |

Nelson-plot: Example 11.5.

Nelson Plot



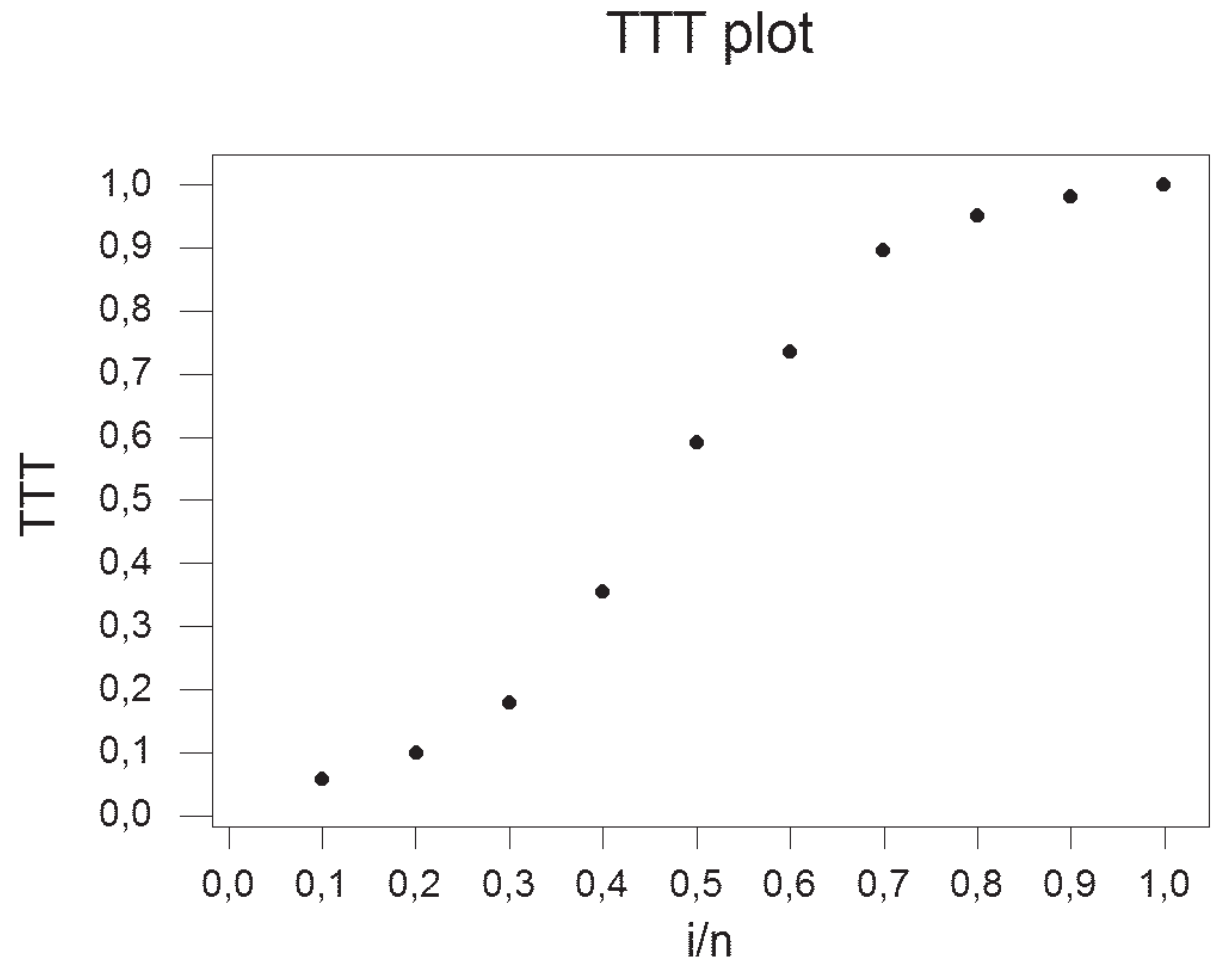
TTT-plot

See Table 11.5

Complete (i.e. uncensored) data.

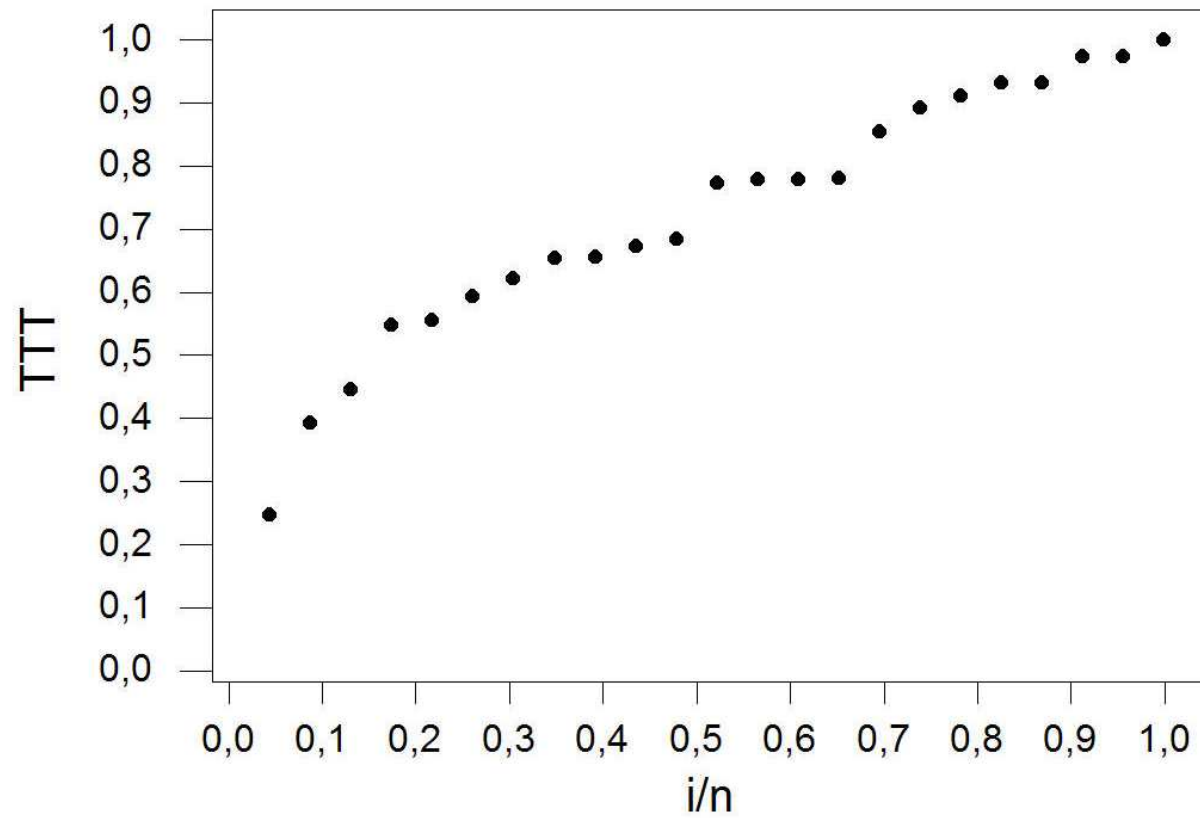
| Row | Time | TTT interval | TTT cum | i/n | TTT |
|-----|-------|------------------|---------|-----|---------|
| 1 | 6,3 | $10*6,3 = 63,0$ | 63,0 | 0,1 | 0,05943 |
| 2 | 11,0 | $9*4,7 = 42,3$ | 105,3 | 0,2 | 0,09934 |
| 3 | 21,5 | $8*10,5 = 84,0$ | 189,3 | 0,3 | 0,17858 |
| 4 | 48,4 | $7*27,9 = 188,3$ | 377,6 | 0,4 | 0,35623 |
| 5 | 90,1 | $6*41,7 = 250,2$ | 627,8 | 0,5 | 0,59226 |
| 6 | 120,2 | $5*30,1 = 150,5$ | 778,3 | 0,6 | 0,73425 |
| 7 | 163,0 | $4*42,8 = 171,2$ | 949,5 | 0,7 | 0,89575 |
| 8 | 182,5 | $3*19,5 = 58,5$ | 1008,0 | 0,8 | 0,95094 |
| 9 | 198,0 | $2*15,5 = 31,0$ | 1039,0 | 0,9 | 0,98019 |
| 10 | 219,0 | $1*21,0 = 21,0$ | 1060,0 | 1,0 | 1,00000 |

TTT-plot (Data from Table 11.5)



BALL BEARINGS FAILURE DATA

TTT plot



BALL BEARINGS FAILURE DATA

| Row | Time | Censor | No at risk | Total time | Cum total time | Plot total time | i/n | TTT |
|-----|--------|--------|------------------|------------|----------------------|-----------------------|---------|---------|
| 1 | 17,88 | 1 | 23 | 411,24 | 411,24 | 411,24 | 0,04348 | 0,24757 |
| 2 | 28,92 | 1 | 22 | 242,88 | 654,12 | 654,12 | 0,08696 | 0,39379 |
| 3 | 33,00 | 1 | 21 | 85,68 | 739,80 | 739,80 | 0,13043 | 0,44537 |
| 4 | 41,52 | 1 | 20 | 170,40 | 910,20 | 910,20 | 0,17391 | 0,54796 |
| 5 | 42,12 | 1 | 19 | 11,40 | 921,60 | 921,60 | 0,21739 | 0,55482 |
| 6 | 45,60 | 1 | 18 | 62,64 | 984,24 | 984,24 | 0,26087 | 0,59253 |
| 7 | 48,40 | 1 | 17 | 47,60 | 1031,84 | 1031,84 | 0,30435 | 0,62119 |
| 8 | 51,84 | 1 | 16 | 55,04 | 1086,88 | 1086,88 | 0,34783 | 0,65432 |
| 9 | 51,96 | 1 | 15 | 1,80 | 1088,68 | 1088,68 | 0,39130 | 0,65540 |
| 10 | 54,12 | 1 | 14 | 30,24 | 1118,92 | 1118,92 | 0,43478 | 0,67361 |
| 11 | 55,56 | 1 | 13 | 18,72 | 1137,64 | 1137,64 | 0,47826 | 0,68488 |
| 12 | 67,80 | 1 | 12 | 146,88 | 1284,52 | 1284,52 | 0,52174 | 0,77330 |
| 13 | 68,64 | 1 | 11 | 9,24 | 1293,76 | 1293,76 | 0,56522 | 0,77887 |
| 14 | 68,64 | 1 | 10 | 0,00 | 1293,76 | 1293,76 | 0,60870 | 0,77887 |
| 15 | 68,88 | 1 | 9 | 2,16 | 1295,92 | 1295,92 | 0,65217 | 0,78017 |
| 16 | 84,12 | 1 | 8 | 121,92 | 1417,84 | 1417,84 | 0,69565 | 0,85357 |
| 17 | 93,12 | 1 | 7 | 63,00 | 1480,84 | 1480,84 | 0,73913 | 0,89149 |
| 18 | 98,64 | 1 | 6 | 33,12 | 1513,96 | 1513,96 | 0,78261 | 0,91143 |
| 19 | 105,12 | 1 | 5 | 32,40 | 1546,36 | 1546,36 | 0,82609 | 0,93094 |
| 20 | 105,84 | 1 | 4 | 2,88 | 1549,24 | 1549,24 | 0,86957 | 0,93267 |
| 21 | 127,92 | 1 | 3 | 66,24 | 1615,48 | 1615,48 | 0,91304 | 0,97255 |
| 22 | 128,04 | 1 | 2 | 0,24 | 1615,72 | 1615,72 | 0,95652 | 0,97269 |
| 23 | 173,40 | 1 | 1 | 45,36 | 1661,08 | 1661,08 | 1,00000 | 1,00000 |

TTT AND BARLOW-PROSCHAN FOR CENSORED DATA

Example 11.15

| Row | Time | Censor | No at risk | Total time | Cum total time |
|-----|------|--------|------------|------------|----------------|
| 1 | 0,35 | 1 | 12 | 4,20 | 4,20 |
| 2 | 0,50 | 0 | 11 | 1,65 | 5,85 |
| 3 | 0,75 | 0 | 10 | 2,50 | 8,35 |
| 4 | 1,00 | 1 | 9 | 2,25 | 10,60 |
| 5 | 1,30 | 1 | 8 | 2,40 | 13,00 |
| 6 | 1,80 | 1 | 7 | 3,50 | 16,50 |
| 7 | 3,00 | 0 | 6 | 7,20 | 23,70 |
| 8 | 3,15 | 0 | 5 | 0,75 | 24,45 |
| 9 | 4,85 | 0 | 4 | 6,80 | 31,25 |
| 10 | 5,50 | 1 | 3 | 1,95 | 33,20 |
| 11 | 5,50 | 0 | 2 | 0,00 | 33,20 |
| 12 | 6,25 | 0 | 1 | 0,75 | 33,95 |

| Row | i/n | TTT |
|-----|-----|---------|
| 1 | 0,2 | 0,12651 |
| 2 | 0,4 | 0,31928 |
| 3 | 0,6 | 0,39157 |
| 4 | 0,8 | 0,49699 |
| 5 | 1,0 | 1,00000 |

BARLOW-PROSCHAN'S TEST:

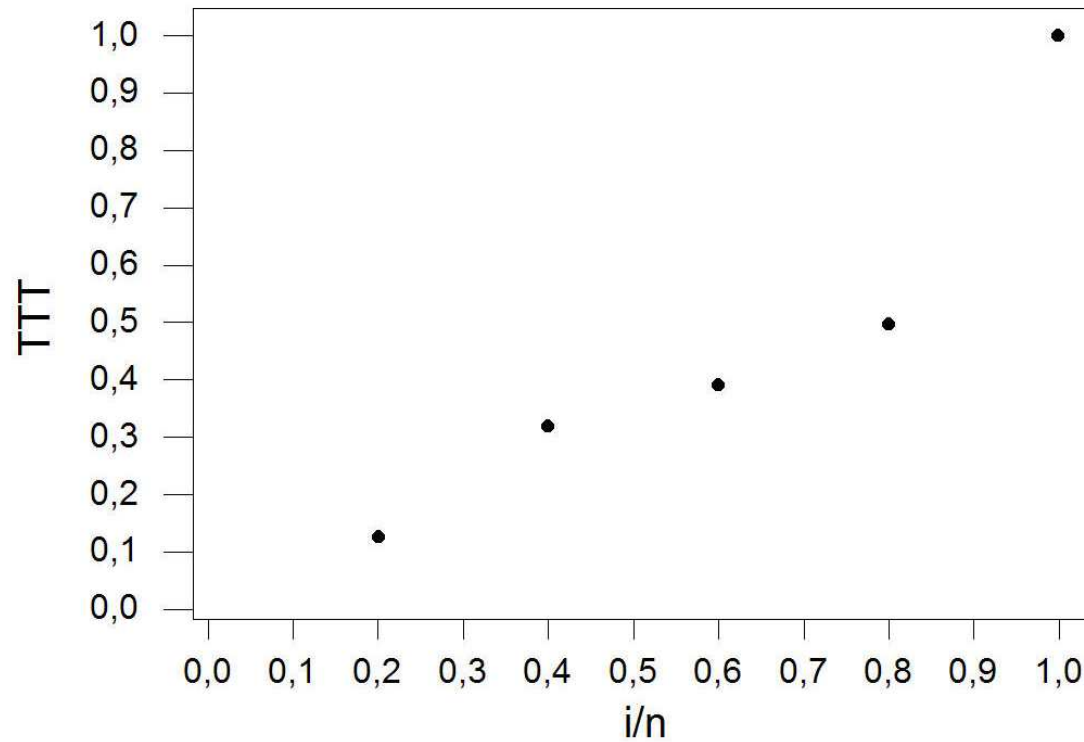
$$W = (4.2+10.6+13.0+16.5)/33.2 = 1.33$$

alt. $W = 0.13 + 0.32 + 0.39 + 0.50 = 1.34$
(k-1 = 4)

Example 11.15

NOTE! This is not the same plot as is suggested for censored data in the book (Chap. 11.3.8/p. 498)

TTT-plot censored data



| Row | Time | Censor | No at risk | Total time | Cum total time | Plot total time |
|-----|-------|--------|------------|------------|----------------|-----------------|
| 1 | 31,7 | 1 | 16 | 507,2 | 507,2 | 507,2 |
| 2 | 39,2 | 1 | 15 | 112,5 | 619,7 | 619,7 |
| 3 | 57,5 | 1 | 14 | 256,2 | 875,9 | 875,9 |
| 4 | 65,0 | 0 | 13 | 97,5 | 973,4 | 983,0 |
| 5 | 65,8 | 1 | 12 | 9,6 | 983,0 | 1029,2 |
| 6 | 70,0 | 1 | 11 | 46,2 | 1029,2 | 1274,3 |
| 7 | 75,0 | 0 | 10 | 50,0 | 1079,2 | 1286,1 |
| 8 | 75,2 | 0 | 9 | 1,8 | 1081,0 | |
| 9 | 87,5 | 0 | 8 | 98,4 | 1179,4 | |
| 10 | 88,3 | 0 | 7 | 5,6 | 1185,0 | |
| 11 | 94,2 | 0 | 6 | 35,4 | 1220,4 | |
| 12 | 101,7 | 0 | 5 | 37,5 | 1257,9 | |
| 13 | 105,8 | 1 | 4 | 16,4 | 1274,3 | |
| 14 | 109,2 | 0 | 3 | 10,2 | 1284,5 | |
| 15 | 110,0 | 1 | 2 | 1,6 | 1286,1 | |
| 16 | 130,0 | 0 | 1 | 20,0 | 1306,1 | |

| Row | i/n | TTT |
|-----|---------|---------|
| 1 | 0,14286 | 0,39437 |
| 2 | 0,28571 | 0,48184 |
| 3 | 0,42857 | 0,68105 |
| 4 | 0,57143 | 0,76433 |
| 5 | 0,71429 | 0,80025 |
| 6 | 0,85714 | 0,99082 |
| 7 | 1,00000 | 1,00000 |

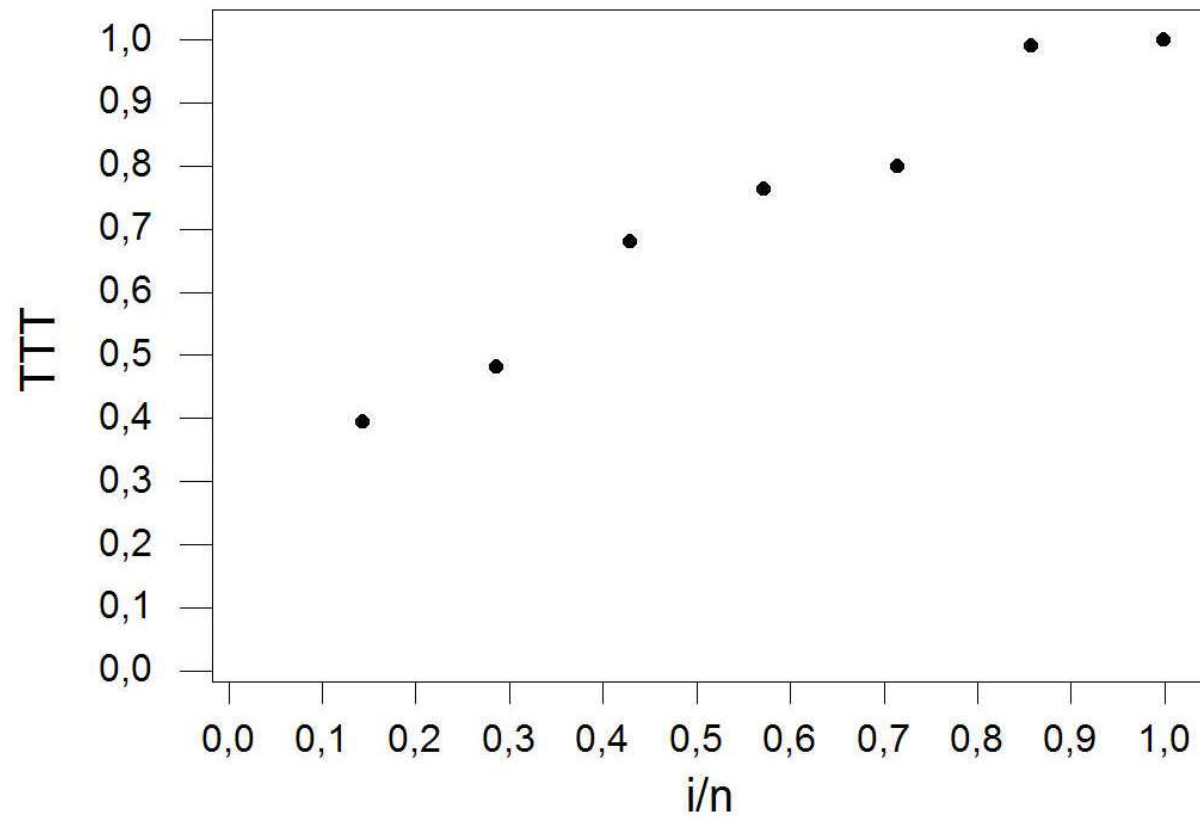
Data from Table 11.1/9.3

BARLOW-PROSCHAN'S TEST:

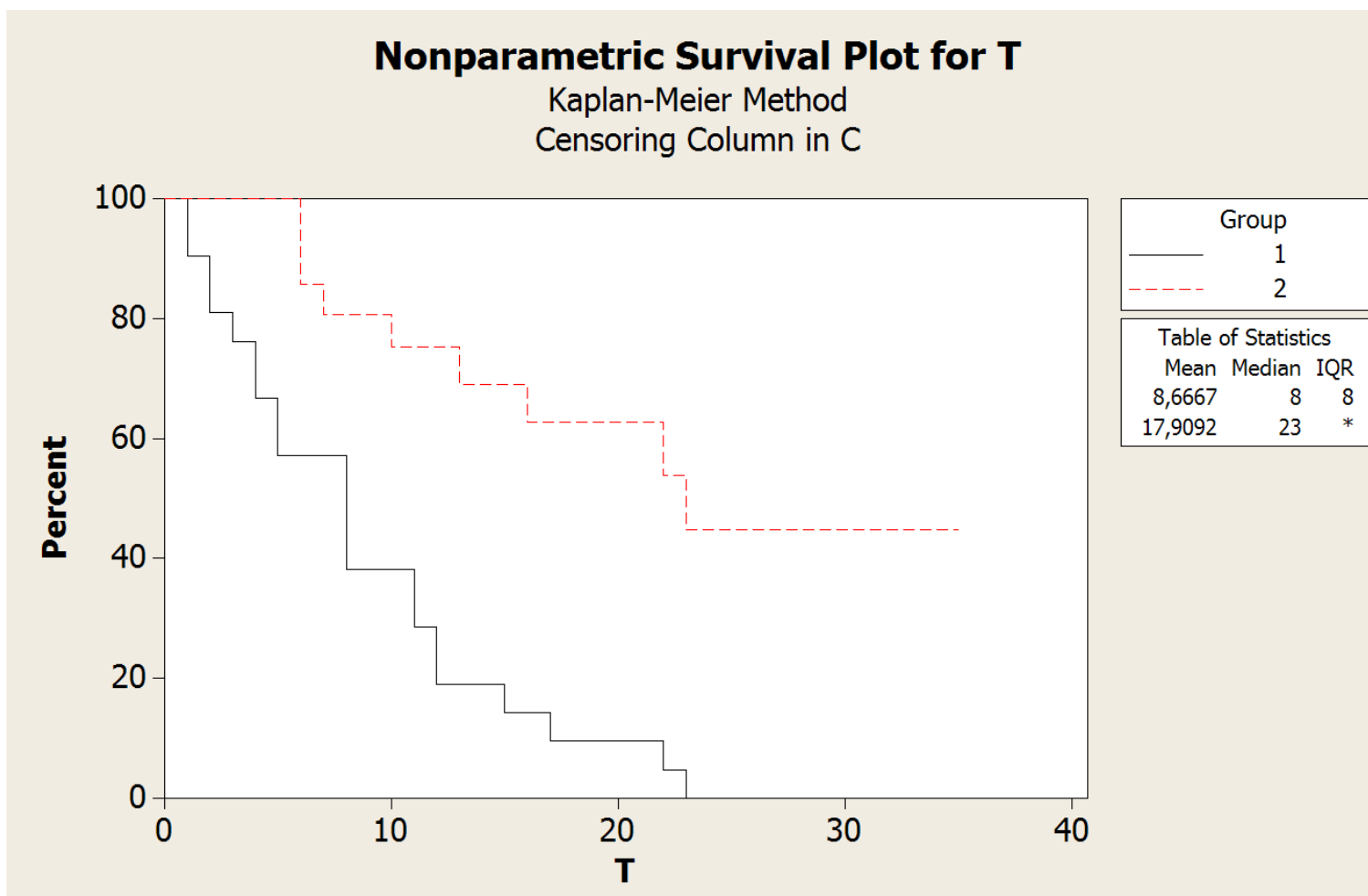
$$W = 0.39+0.48+0.68+0.76+0.80+0.99 = 4.10 \quad (k-1 = 6)$$

Table 11.1

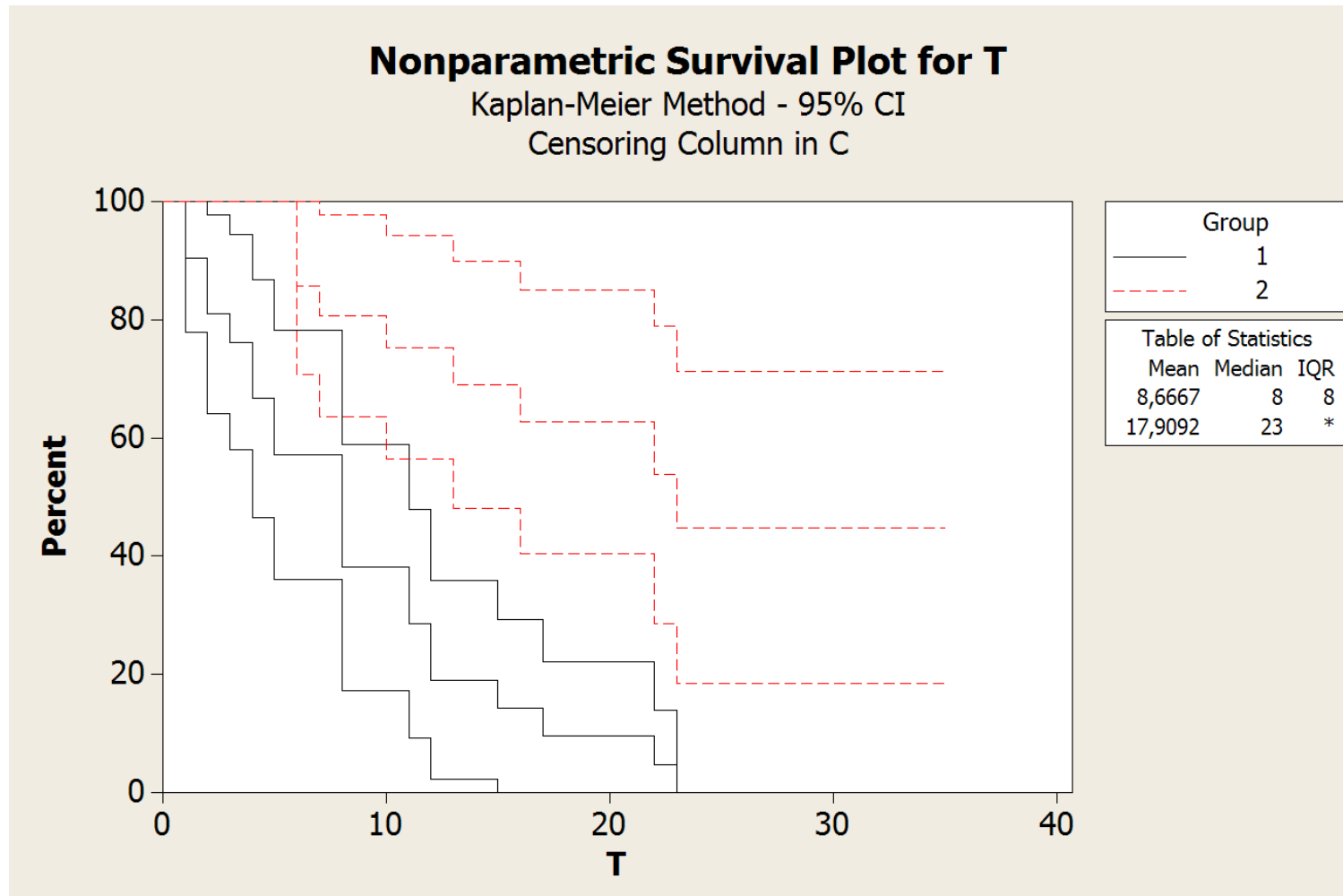
TTT-plot censored data



Example: Leukemia data (1=Placebo, 2=6MP)



Example: Leukemia data (1=Placebo, 2=6MP, with confidence limits)



Logrank Test for Leukemia Data

C = Control group (Placebo)

B = Treatment group (6MP)

| Time | RiskC | RiskB | Risk | FailC | FailB | Fail | EC | EB |
|-------|-------|-------|------|-------|-------|------|-----------------------|-----------------------|
| 1 | 21 | 21 | 42 | 2 | 0 | 2 | $(2/42) * 21 = 1$ | $(2/42) * 21 = 1$ |
| 2 | 19 | 21 | 40 | 2 | 0 | 2 | $(2/40) * 19 = 0.95$ | $(2/40) * 21 = 1.05$ |
| 3 | 17 | 21 | 38 | 1 | 0 | 1 | $(1/38) * 17 = 0.447$ | $(1/38) * 21 = 0.553$ |
| 4 | 16 | 21 | 37 | 2 | 0 | 2 | $(2/37) * 16 = 0.865$ | $(2/37) * 21 = 1.135$ |
| 13 | 4 | 12 | 16 | 0 | 1 | 1 | $(1/16) * 4 = 0.25$ | $(1/16) * 12 = 0.75$ |
| 23 | 1 | 6 | 7 | 1 | 1 | 2 | $(2/7) * 1 = 0.286$ | $(2/7) * 6 = 1.714$ |
| Total | | | | 21 | 9 | | 10.749 | 19.251 |

Test statistic:

$$\frac{(O_C - E_C)^2}{E_C} + \frac{(O_B - E_B)^2}{E_B}$$

$$= \frac{(21 - 10.749)^2}{10.749} + \frac{(9 - 19.251)^2}{19.251} = 5.46 + 9.77 = 15.33$$