

Spring 2013

TMA4275 LIFETIME ANALYSIS

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GOALS

After finishing this course you should

- know the most common concepts and distributions from lifetime modeling
- be able to use graphical methods for description and comparison of lifetime data
- be able to use statistical methods for statistical inference (estimation, confidence interval, hypothesis testing) of lifetime data
- be able to analyze lifetime data by using computer software (MINITAB)

RELIABILITY

Common technical definition of reliability:

The probability that a system or a component will perform its intended task, under given operational conditions, for a specified time period.

LIFE TIMES (SURVIVAL DATA)

- Time to failure of a component or a system
- Number of cycles to failure (fatigue testing)
- Time to next epileptic seizure for patient
- Times of failure and repair of a machine

WHY COLLECT AND ANALYZE LIFETIME/SURVIVAL/RELIABILITY DATA?

- **Assess reliability of a system/component/product**
- **Compare two or more products with respect to reliability**
- **Predict product reliability in the design phase**
- **Predict warranty claims for a product in the market**

SPECIAL ASPECTS OF LIFETIME ANALYSIS

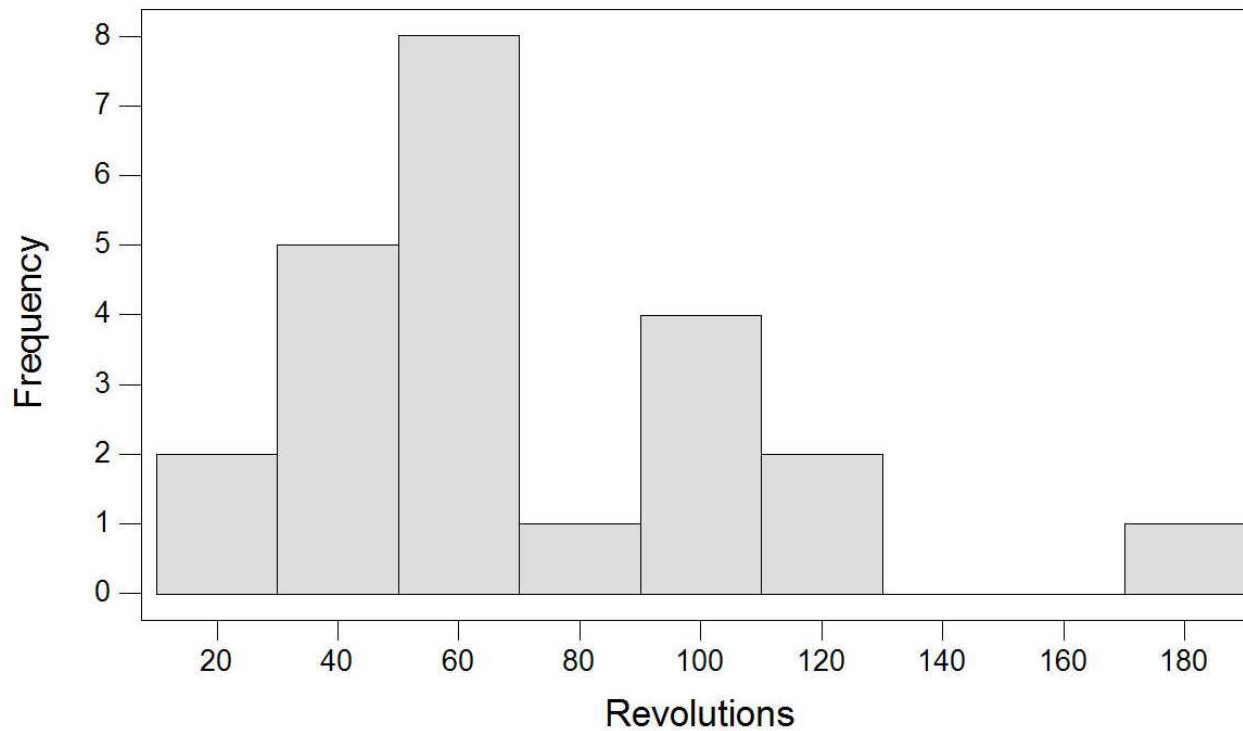
- Definition of starting time and failure time are difficult
- Definition of time scale (operation time, calendar time, number of cycles)
- Censored data (what do we do with units that have not failed within the observation period?)
- Effect of environmental conditions
- What if a unit fails of another cause than the one we would like to study? ("competing risks")
- Recurrent events – what if the system can fail several times; how to analyze recurring stages of a disease?

BALL BEARINGS FAILURE DATA

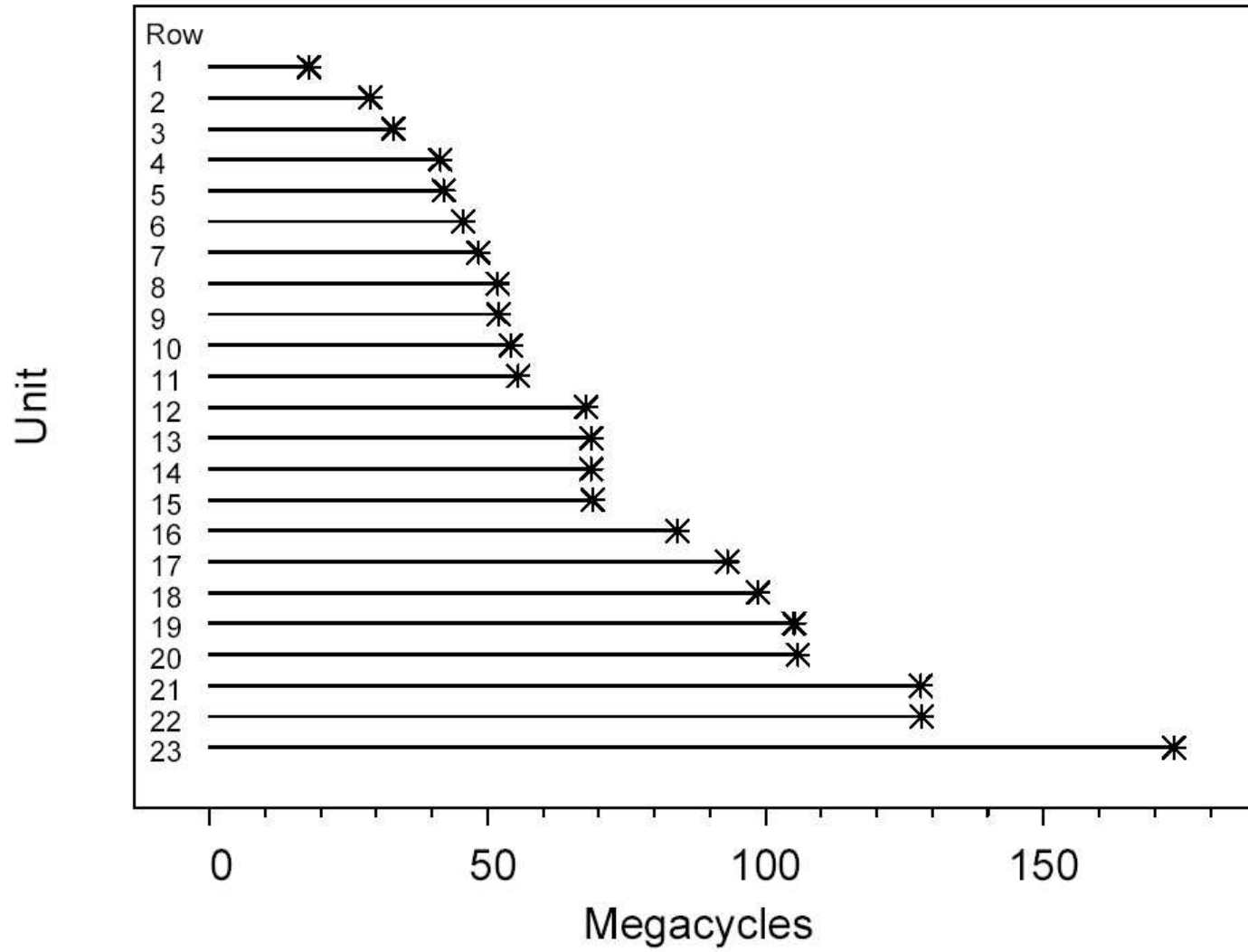
Data: Millions of revolutions to fatigue failure for 23 units

17,88	28,92	33,00	41,52	42,12	45,60	48,40	51,84
51,96	54,12	55,56	67,80	68,64	68,64	68,88	84,12
93,12	98,64	105,12	105,84	127,92	128,04	173,40	

Histogram of Revolutions



Lieblein and Zelen Ball Bearing Failure Data



IC Data (Meeker, 1987)

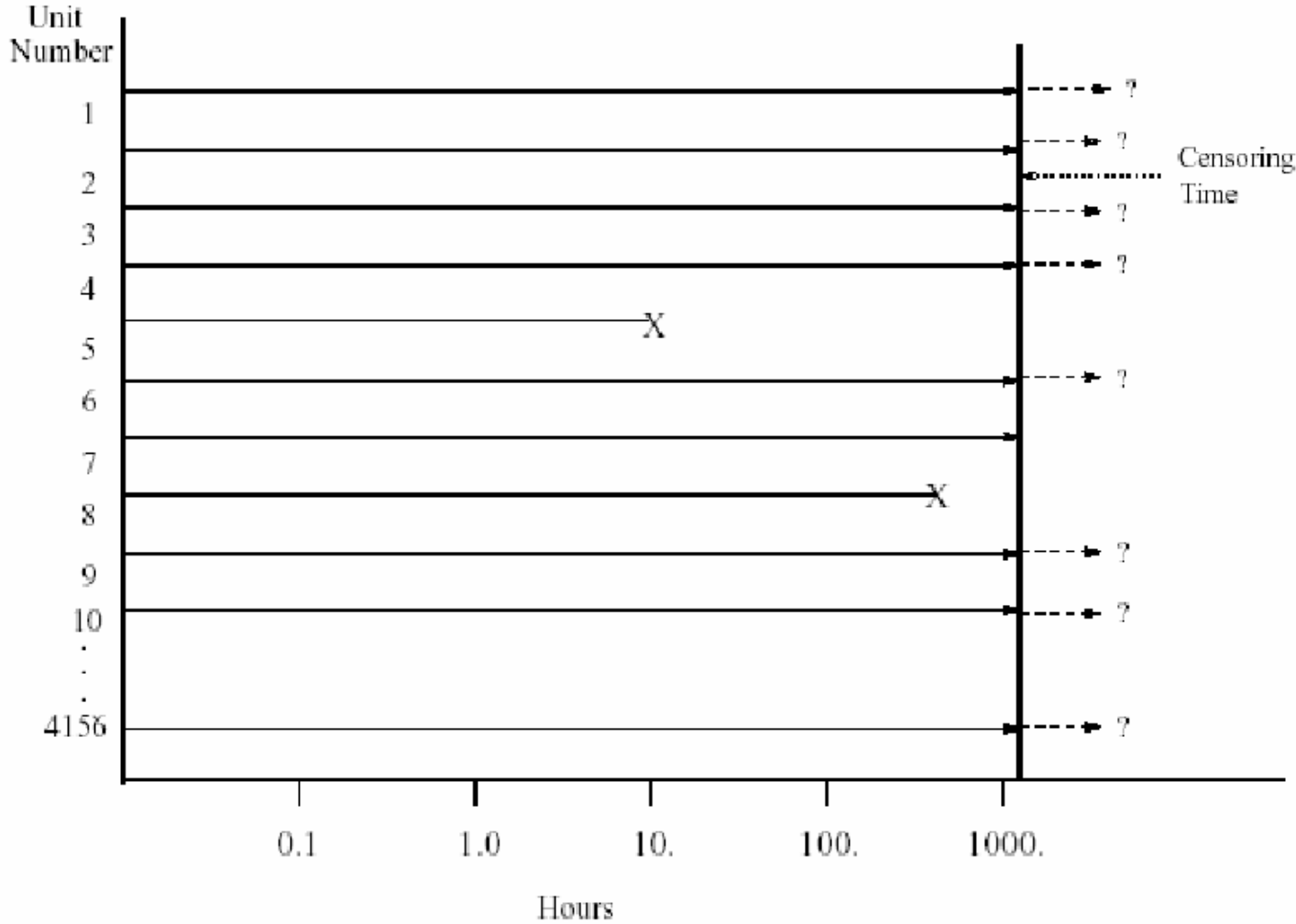
- Integrated circuit failure times in hours
 - $n = 4156$ ICs tested for 1,370 hours at 80° C and 80% relative humidity
 - There were 28 failures
 - When the test ended at 1,370 hours, 4128 units were still running

.10	.10	.15	.60	.80	.80
1.20	2.5	3.0	4.0	4.0	6.0
10.0	10.0	12.5	20.	20.	43.
43.	48.	48.	54.	74.	84.
94.	168.	263.	593.		

TYPICAL PROBLEMS:

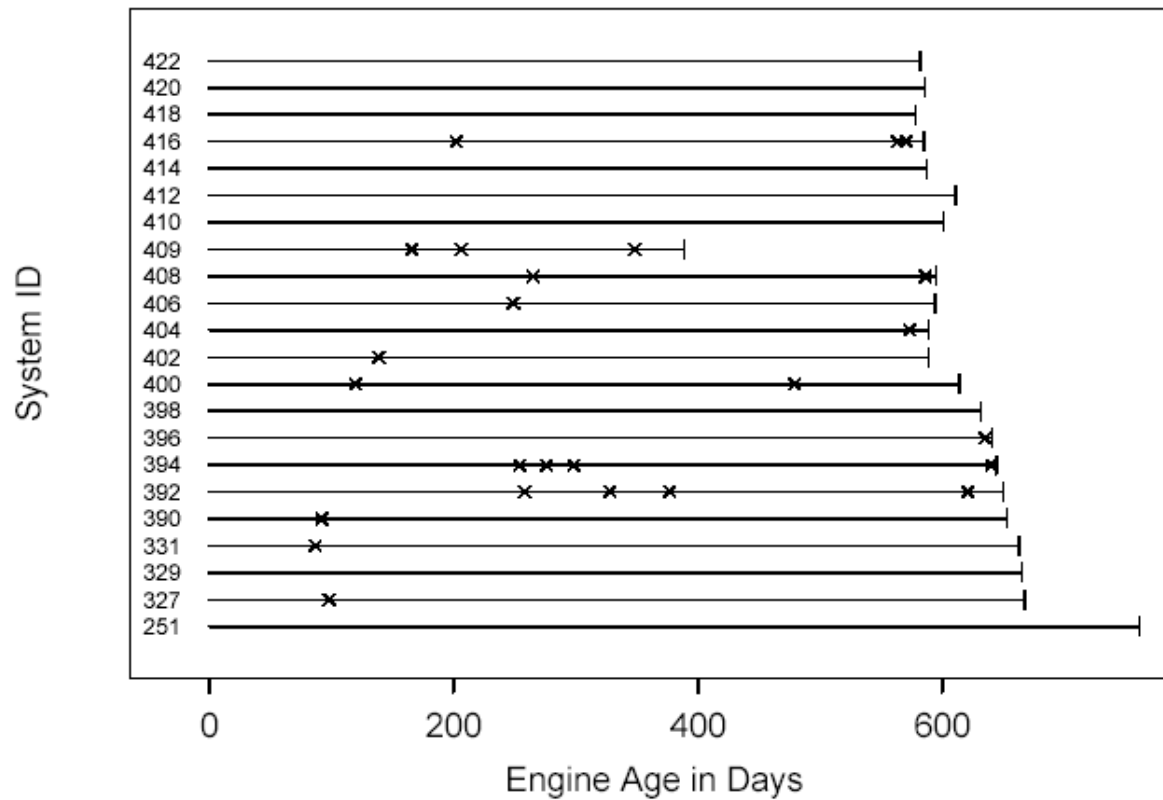
- How to estimate the distribution of the failure time when there are censored observations?
- Probability of failure before 100 hours?
- Failure rate by 100 hours?
- Proportion failed after 10^5 hours?

IC Data Failure Pattern



RECURRENT EVENTS/REPAIRABLE SYSTEMS

Valve Seat Replacement Times Event Plot
(Nelson and Doganaksoy 1989)

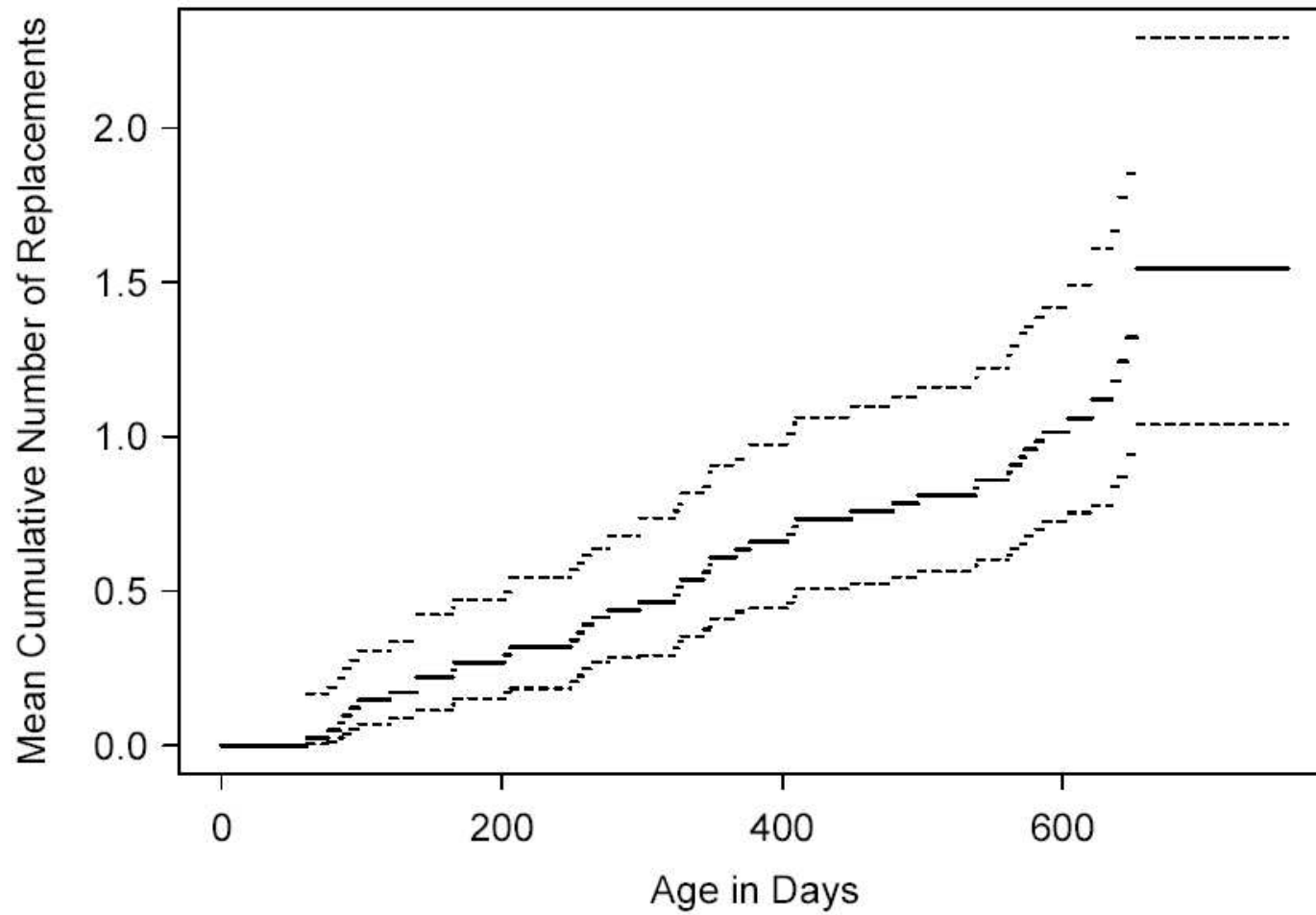


Valve Seat Replacement Times (Nelson and Doganaksoy 1989)

Data collected from valve seats from a fleet of 41 diesel engines (days of operation)

- Each engine has 16 valves
- Does the replacement rate increase with age?
- How many replacement valves will be needed in the future?
- Can valve life in these systems be modeled as a renewal process?

Estimate of Number of Valve Seat $\mu(t)$



MINITAB - Untitled

File Edit Manip Calc Stat Graph Editor Window Help

Basic Statistics
Regression
ANOVA
DOE
Control Charts
Quality Tools
Reliability/Survival
Multivariate
Time Series
Tables
Nonparametrics
EDA
Power and Sample Size

Distribution ID Plot-Right Cens...
Distribution Overview Plot-Right Cens...
Parametric Dist Analysis-Right Cens...
Nonparametric Dist Analysis-Right Cens...
Distribution ID Plot-Arbitrary Cens...
Distribution Overview Plot-Arbitrary Cens...
Parametric Dist Analysis-Arbitrary Cens...
Nonparametric Dist Analysis-Arbitrary Cens...
Accelerated Life Testing...
Regression with Life Data...
Probit Analysis...

Session

04.02

Welcome to Minitab
Retrieving worksheet
Worksheet was

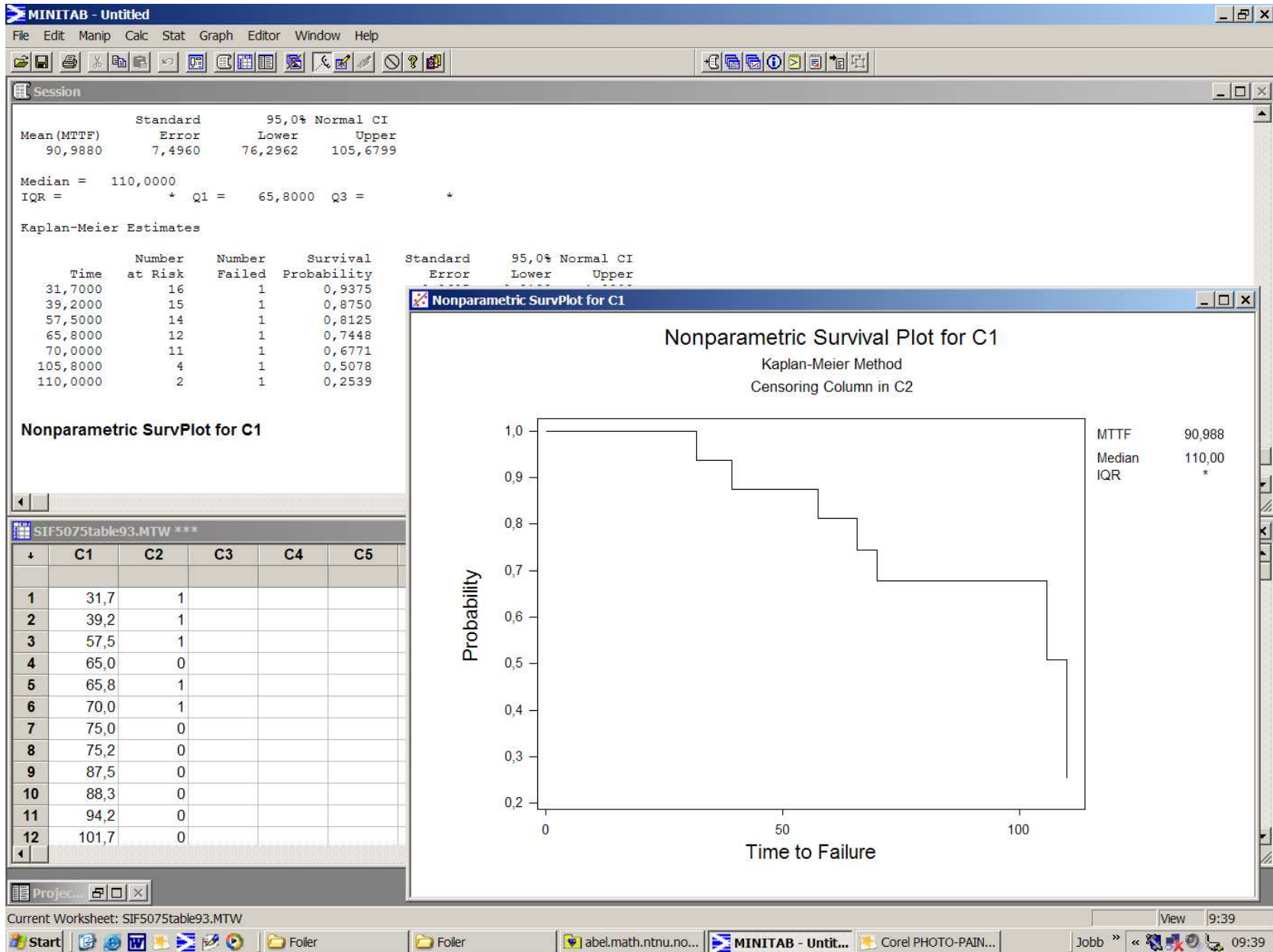
SIF5075table93.MTW ***

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15	C16	C17
1	31,7	1															
2	39,2	1															
3	57,5	1															
4	65,0	0															
5	65,8	1															
6	70,0	1															
7	75,0	0															
8	75,2	0															
9	87,5	0															
10	88,3	0															
11	94,2	0															
12	101,7	0															

Perform a nonparametric distribution analysis on uncensored/right censored data

13:39

Start | Win | Folder | Corel PHOTO-P... | WinEdt 5.3 (U... | Yap 0.99a - [For... | MINITAB - Un... | Jobb » | 13:40

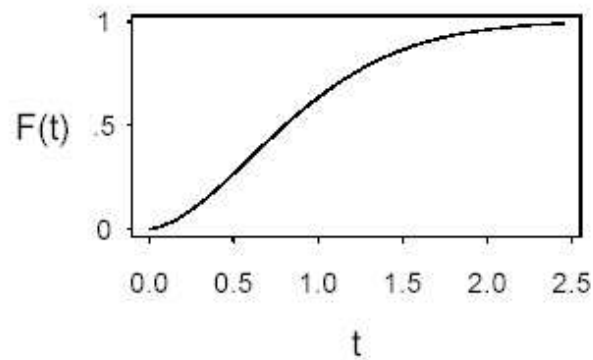


Typical Failure-time cdf, pdf, hf, and sf

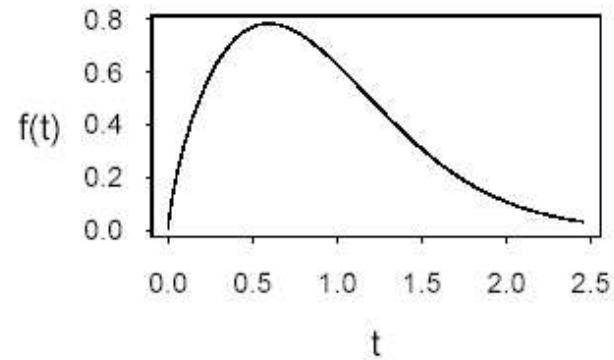
$$F(t) = 1 - \exp(-t^{1.7}); \quad f(t) = 1.7 \times t^{.7} \times \exp(-t^{1.7})$$

$$S(t) = \exp(-t^{1.7}); \quad h(t) = 1.7 \times t^{.7}$$

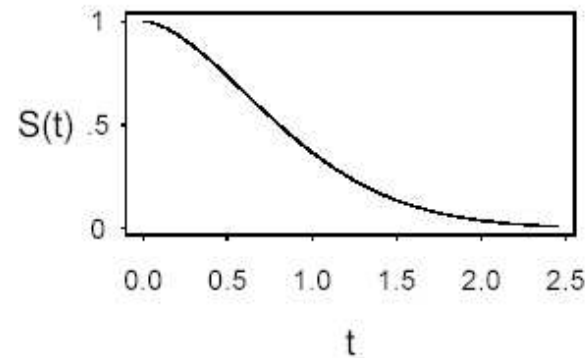
Cumulative Distribution Function



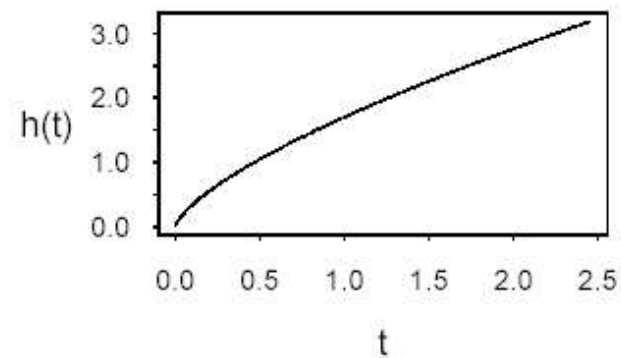
Probability Density Function



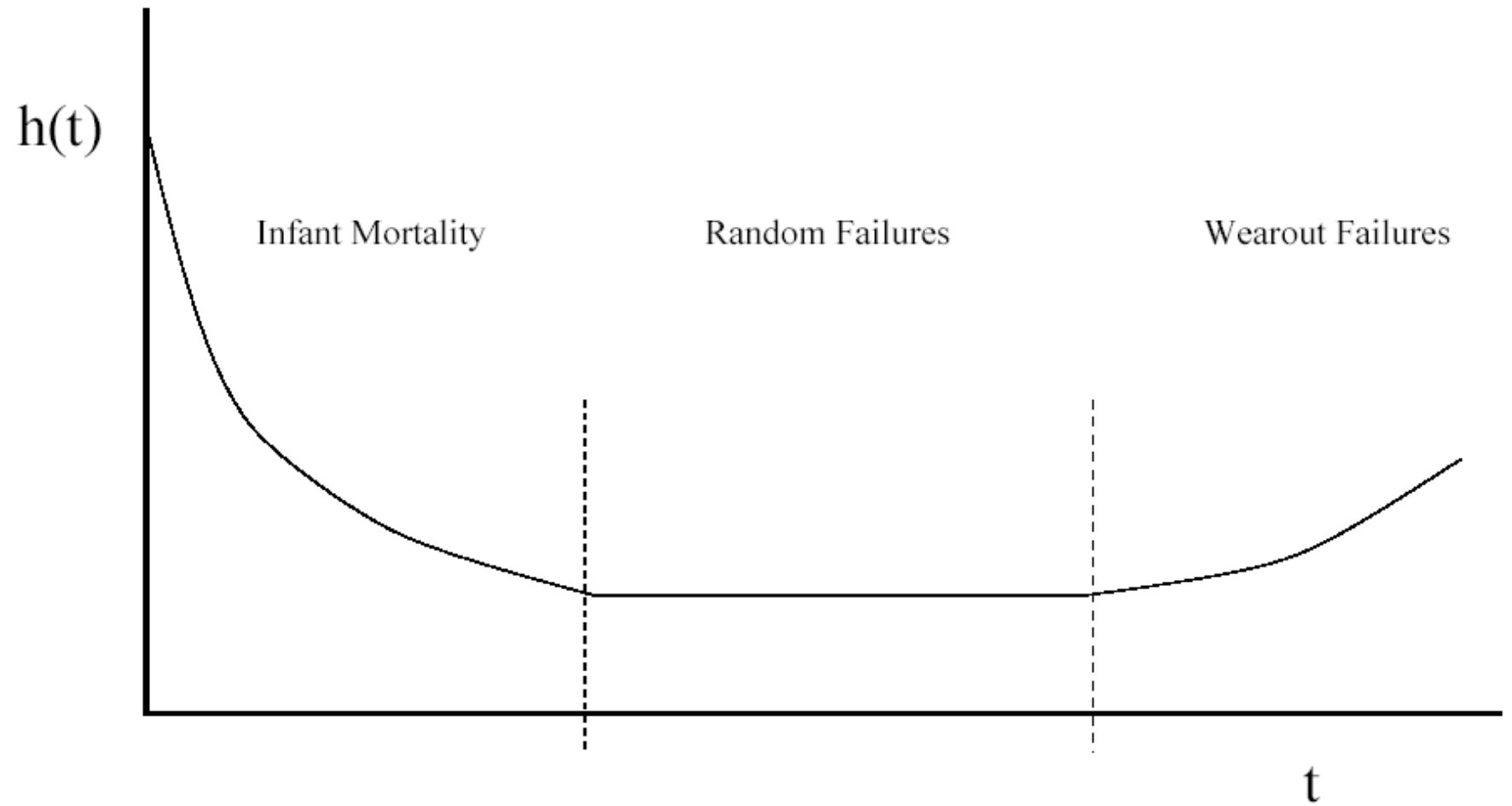
Survival Function



Hazard Function



Bathtub Curve Hazard Function

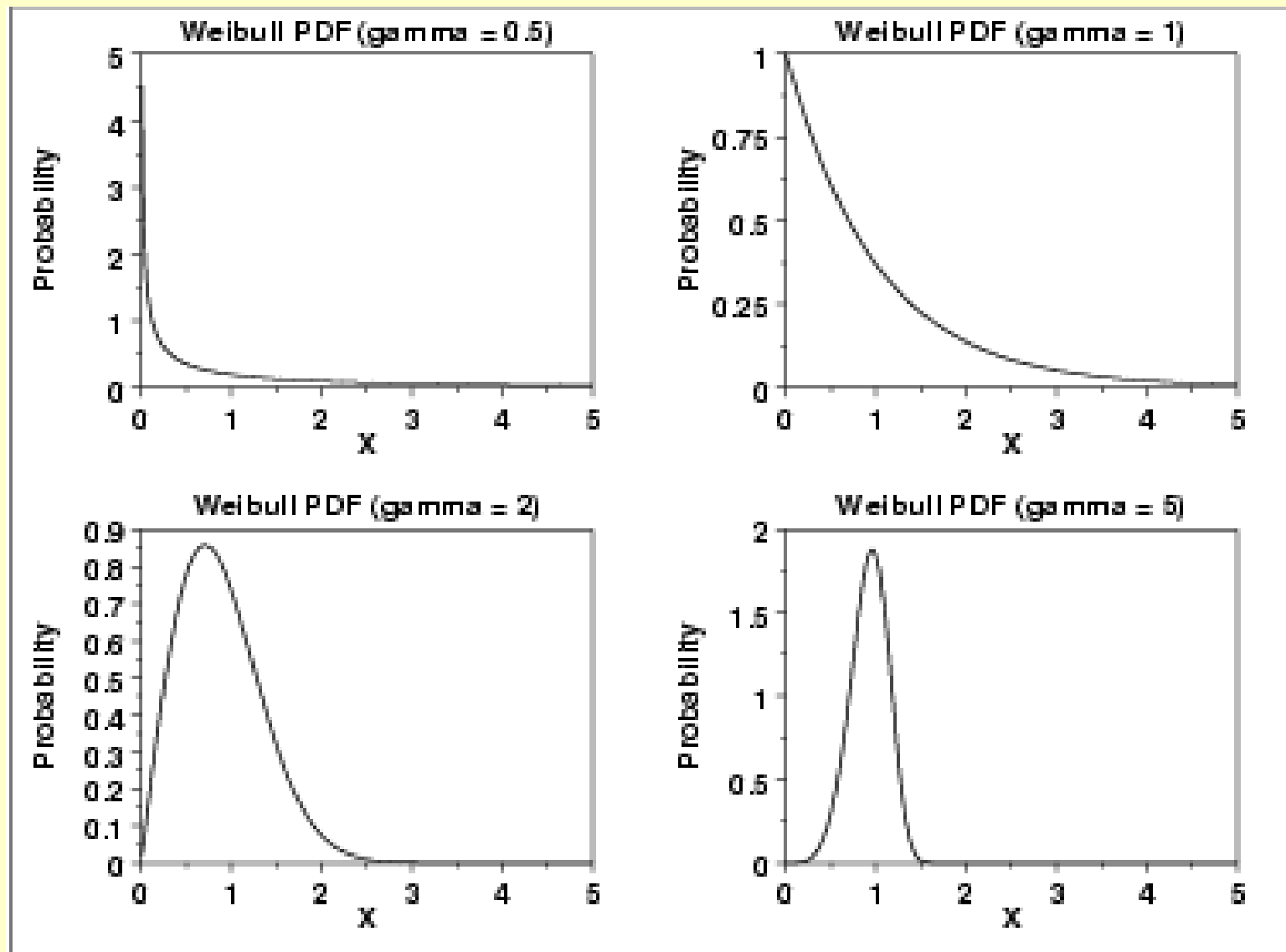


10 Dødelighetstabeller ¹. 2001

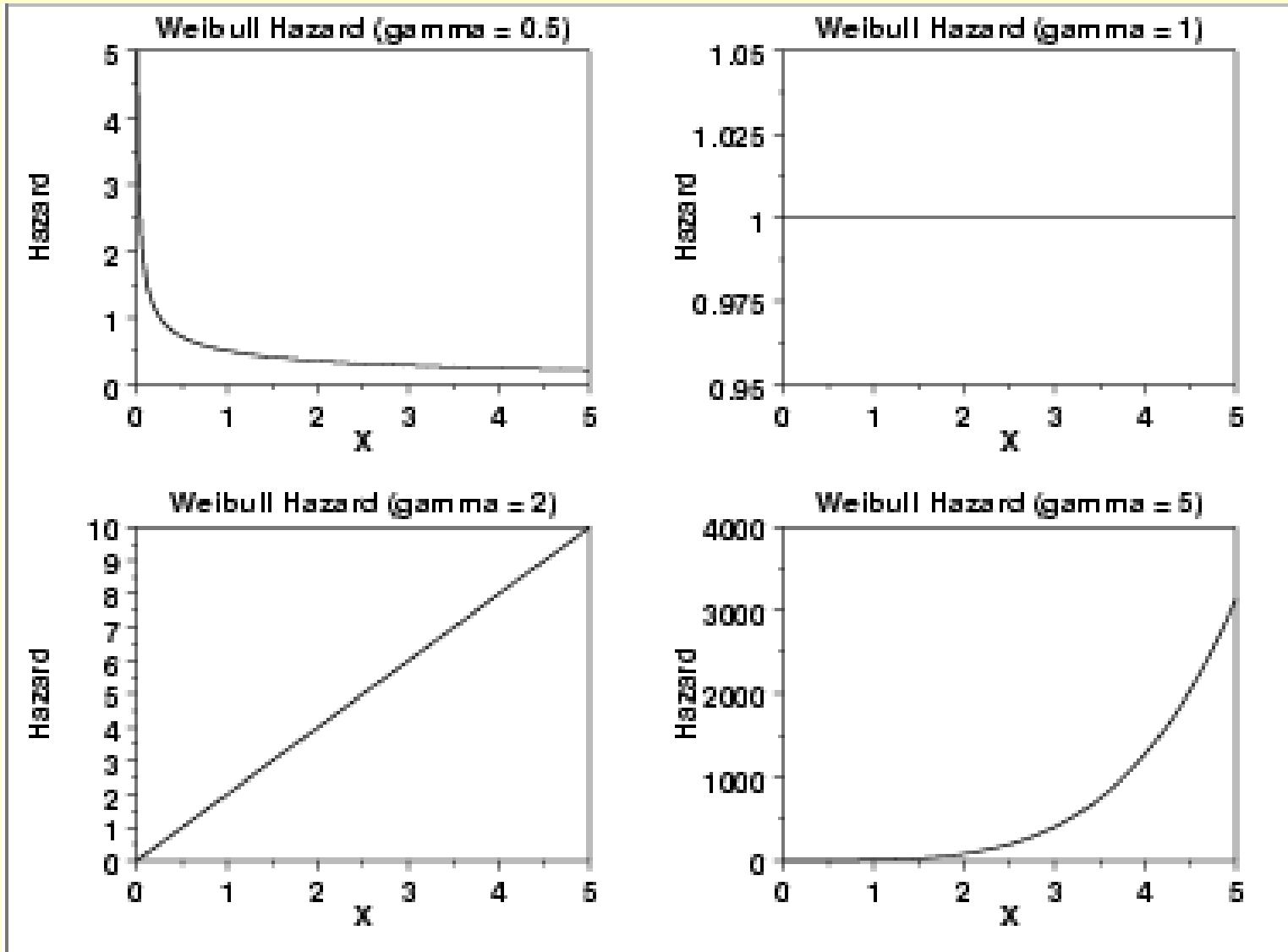
Alder x	Levende ved alder x		Døde i alder x til x+1		Forventet gjestående levetid ved alder x		Dødssannsynlighet for alder x, Promille, (Uglattet).	
	lx		dx		e0x		qx	
	Menn	Kvinner	Menn	Kvinner	Menn	Kvinner	Menn	Kvinner
0	100 000	100 000	424	343	76,21	81,53	4,24	3,43
1	99 576	99 657	39	27	75,53	80,81	0,39	0,28
2	99 537	99 630	33	7	74,56	79,83	0,33	0,07
3	99 504	99 623	32	17	73,58	78,84	0,33	0,17
4	99 472	99 606	13	10	72,61	77,85	0,13	0,10
5	99 459	99 596	6	10	71,62	76,86	0,06	0,10
6	99 453	99 586	22	17	70,62	75,87	0,22	0,17
7	99 431	99 569	10	3	69,64	74,88	0,10	0,03
8	99 422	99 566	9	13	68,64	73,88	0,09	0,13
9	99 412	99 552	9	3	67,65	72,89	0,09	0,03
10	99 403	99 549	12	3	66,66	71,89	0,12	0,03
11	99 390	99 546	3	10	65,66	70,89	0,03	0,10
12	99 387	99 536	16	3	64,67	69,90	0,16	0,03
13	99 371	99 532	10	11	63,68	68,90	0,10	0,11
14	99 361	99 522	7	7	62,68	67,91	0,07	0,07
15	99 354	99 514	32	11	61,69	66,92	0,32	0,11
16	99 322	99 503	33	23	60,71	65,92	0,33	0,23
17	99 289	99 480	77	39	59,73	64,94	0,77	0,39
18	99 212	99 441	90	35	58,77	63,96	0,91	0,35
19	99 122	99 407	123	34	57,83	62,99	1,24	0,34
20	98 999	99 373	155	60	56,90	62,01	1,57	0,60
21	98 844	99 313	142	15	55,99	61,04	1,44	0,15

40	96 600	98 433	147	85	38,05	42,49	1,53	0,86
41	96 453	98 348	144	110	37,11	41,53	1,49	1,12
42	96 309	98 239	208	100	36,16	40,58	2,16	1,02
43	96 101	98 138	181	110	35,24	39,62	1,89	1,12
44	95 919	98 029	205	112	34,31	38,66	2,14	1,15
45	95 715	97 916	190	153	33,38	37,70	1,98	1,57
46	95 525	97 763	256	172	32,44	36,76	2,68	1,76
47	95 268	97 591	256	160	31,53	35,83	2,68	1,64
48	95 013	97 431	324	191	30,61	34,88	3,41	1,96
49	94 689	97 240	310	197	29,72	33,95	3,28	2,03
50	94 379	97 042	324	233	28,81	33,02	3,43	2,40
51	94 055	96 810	387	265	27,91	32,10	4,11	2,74
52	93 668	96 545	332	255	27,02	31,18	3,54	2,64
53	93 336	96 290	461	293	26,12	30,27	4,94	3,04
54	92 875	95 997	504	343	25,25	29,36	5,42	3,58
55	92 371	95 653	546	342	24,38	28,46	5,91	3,57
56	91 825	95 311	583	362	23,52	27,56	6,35	3,80
57	91 242	94 949	647	400	22,67	26,66	7,09	4,22
58	90 595	94 549	593	435	21,83	25,77	6,55	4,60
59	90 002	94 115	713	554	20,97	24,89	7,92	5,89
60	89 289	93 560	797	543	20,13	24,04	8,93	5,81
61	88 492	93 017	853	543	19,31	23,17	9,64	5,83
62	87 639	92 475	911	626	18,49	22,31	10,39	6,77
63	86 728	91 848	1 200	781	17,68	21,45	13,84	8,50
64	85 528	91 068	1 359	795	16,92	20,63	15,89	8,73
65	84 168	90 273	1 356	763	16,19	19,81	16,11	8,45
66	82 812	89 509	1 349	883	15,44	18,98	16,29	9,86
67	81 463	88 627	1 572	897	14,69	18,16	19,30	10,12
68	79 891	87 730	1 746	1 070	13,97	17,34	21,86	12,19
69	78 145	86 660	1 869	1 056	13,27	16,55	23,91	12,19

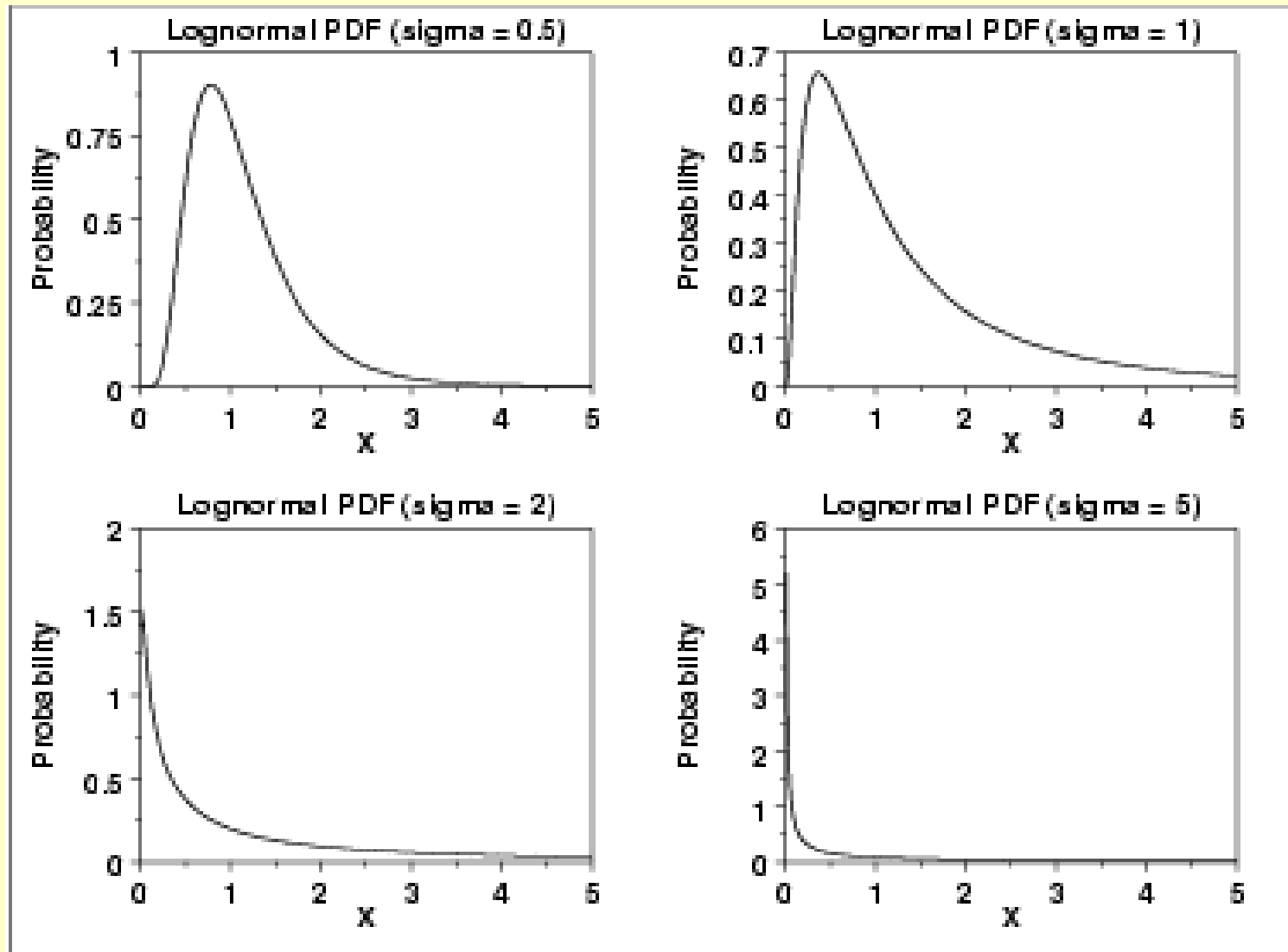
WEIBULL-DENSITIES ($\alpha = \gamma$, $\lambda = 1$ in terminology of book)



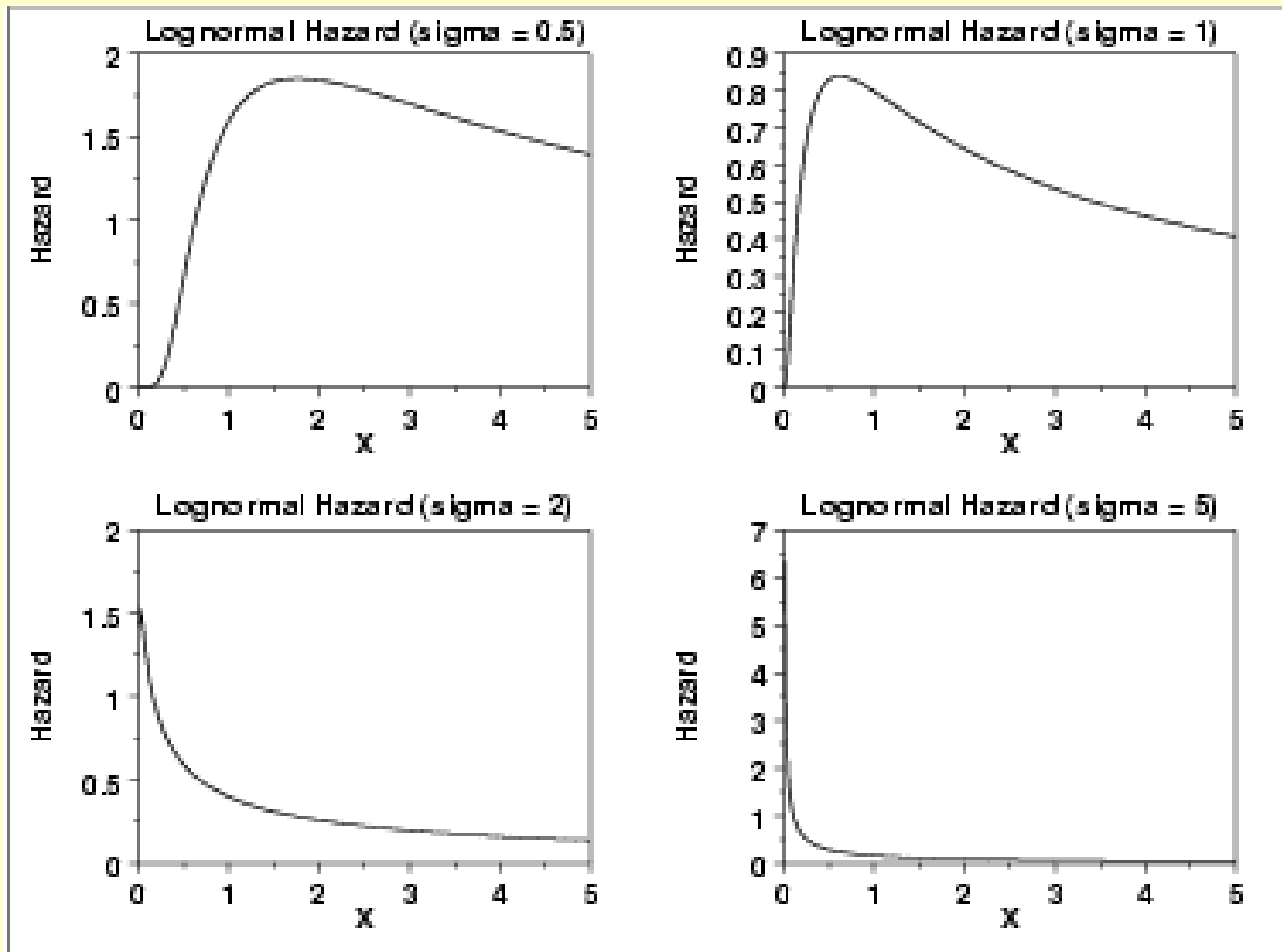
WEIBULL-HAZARDS ($\alpha = \gamma, \lambda = 1$ in terminology of book)



LOGNORMAL DENSITIES ($\tau = \sigma$, $\nu = 0$ in terminology of book)



LOGNORMAL HAZARDS ($\tau = \sigma$, $\nu = 0$ in terminology of book)

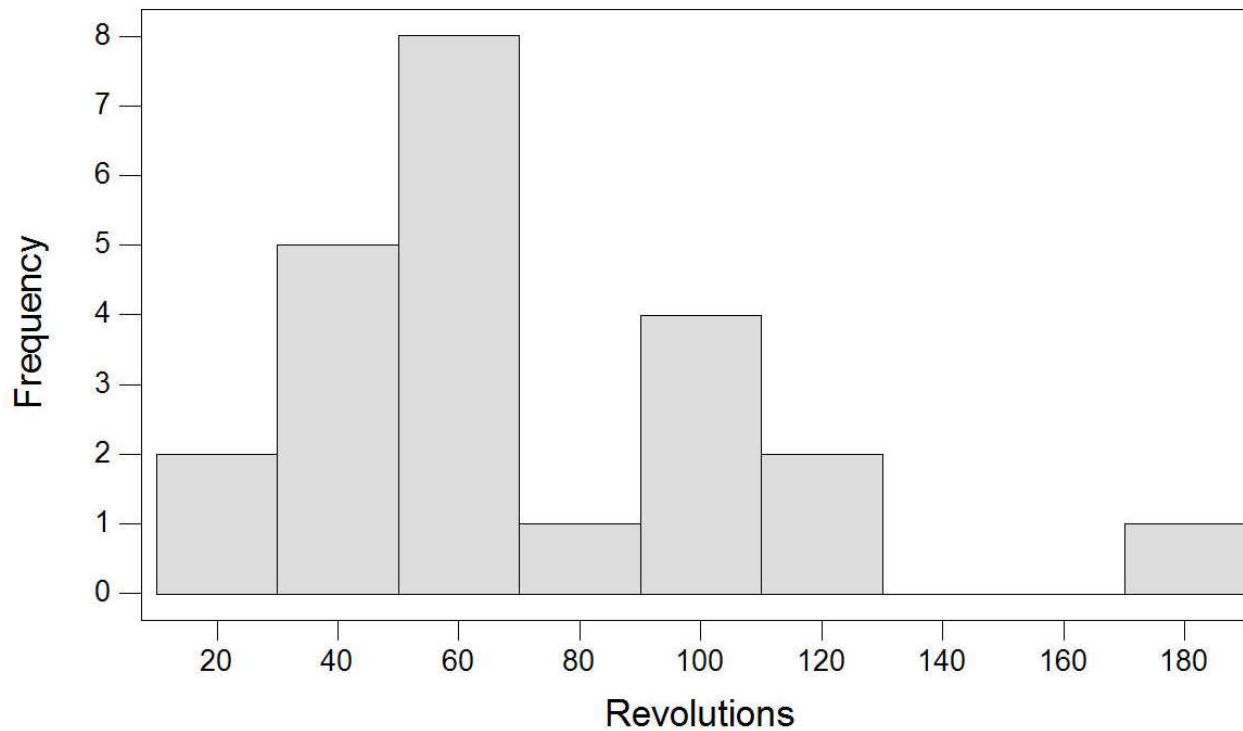


RECALL BALL BEARINGS FAILURE DATA

Data: Millions of revolutions to fatigue failure for 23 units

17,88	28,92	33,00	41,52	42,12	45,60	48,40	51,84
51,96	54,12	55,56	67,80	68,64	68,64	68,88	84,12
93,12	98,64	105,12	105,84	127,92	128,04	173,40	

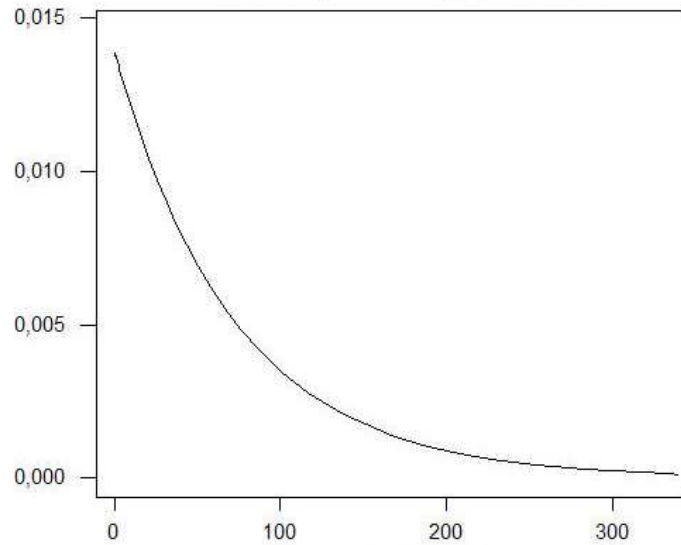
Histogram of Revolutions



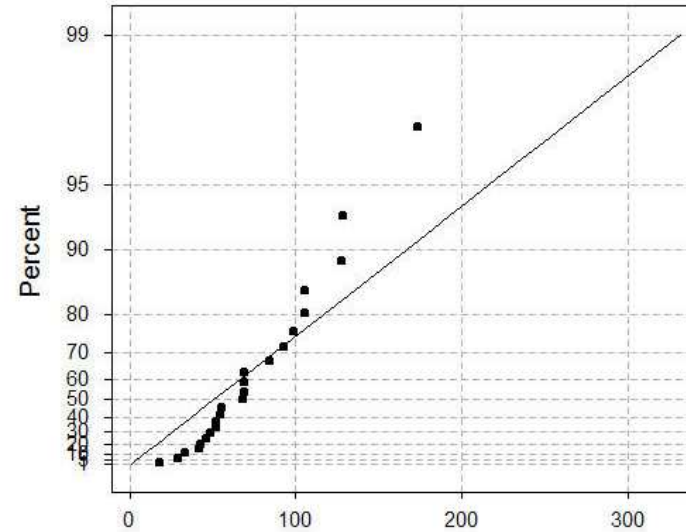
Ball Bearings Failure Data

ML Estimates - Complete Data

Probability Density Function



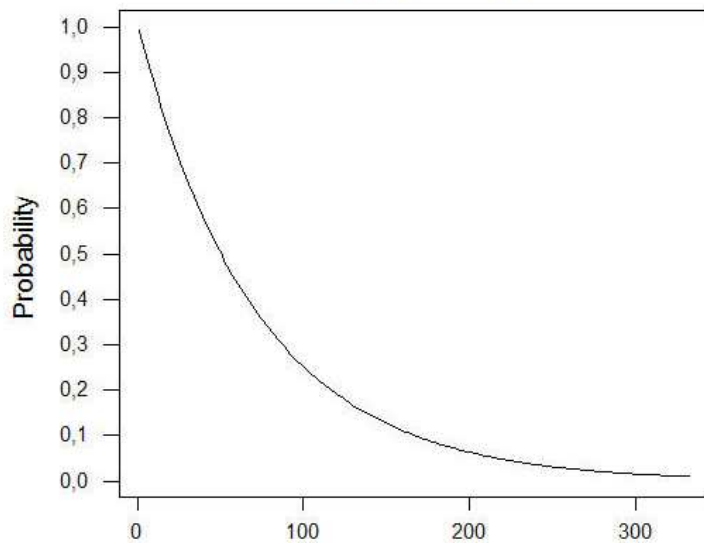
Exponential Probability



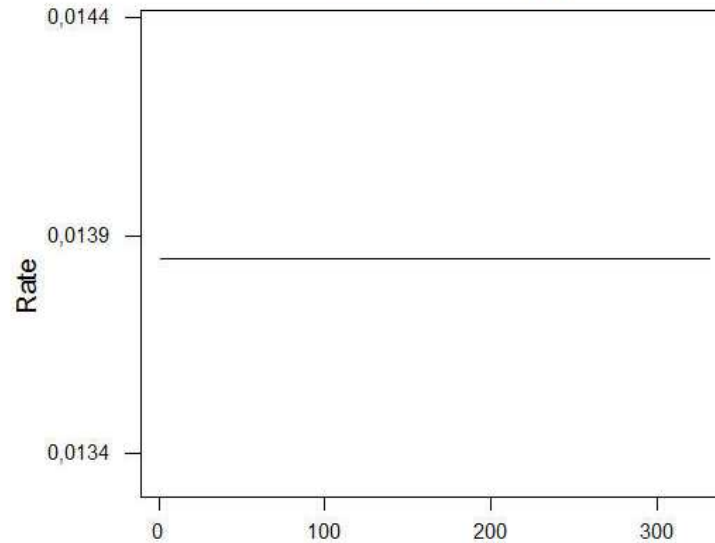
Shape	1
Scale	72,221
MTTF	72,221
Failure	23
Censor	0

Goodness of Fit
AD* 3,341

Survival Function



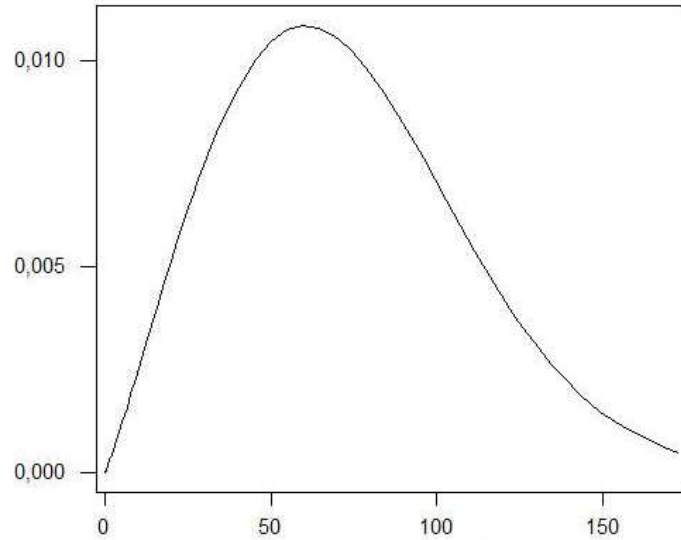
Hazard Function



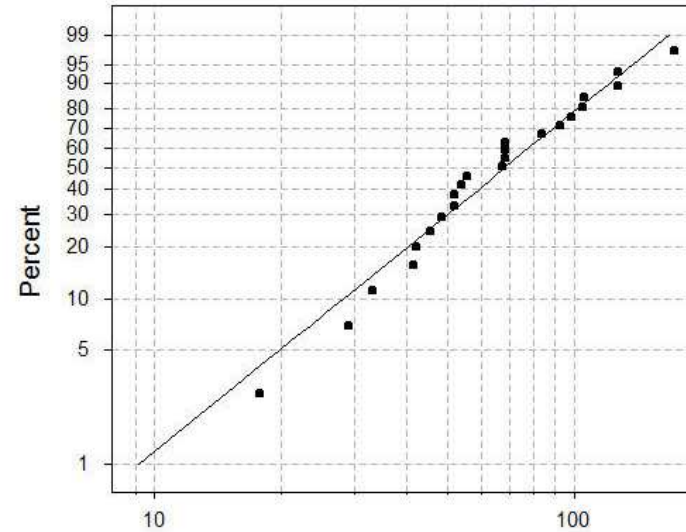
Ball Bearings Failure Data

ML Estimates - Complete Data

Probability Density Function



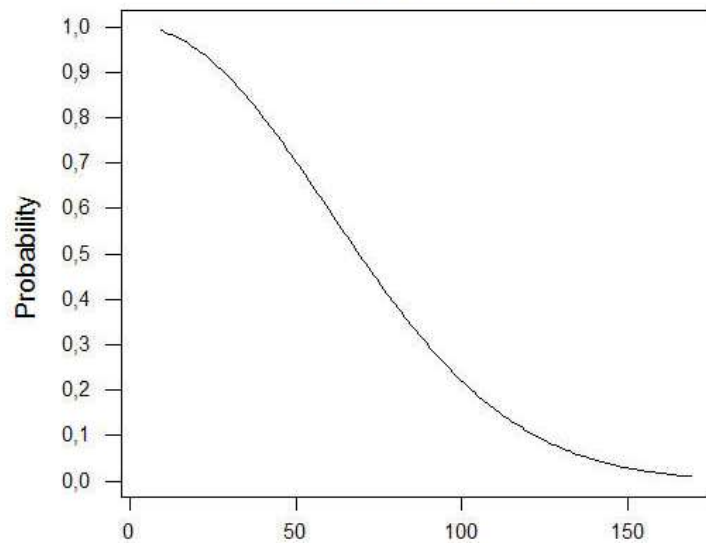
Weibull Probability



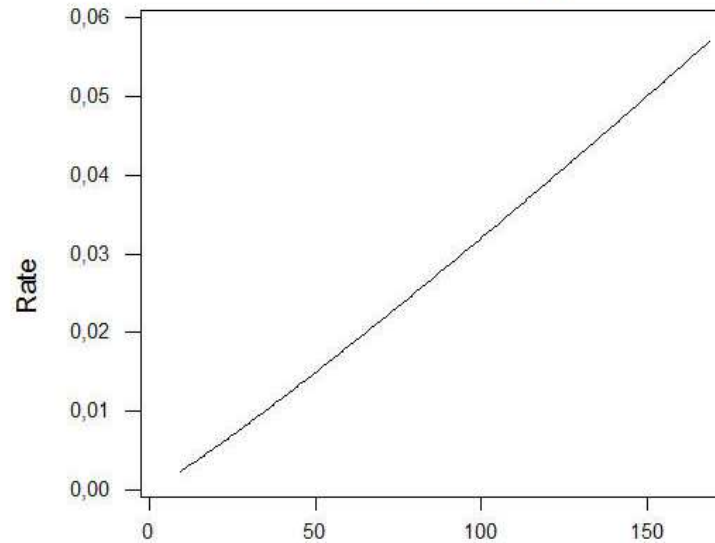
Shape	2,1018
Scale	81,875
MTTF	72,515
Failure	23
Censor	0

Goodness of Fit	
AD*	0,802

Survival Function



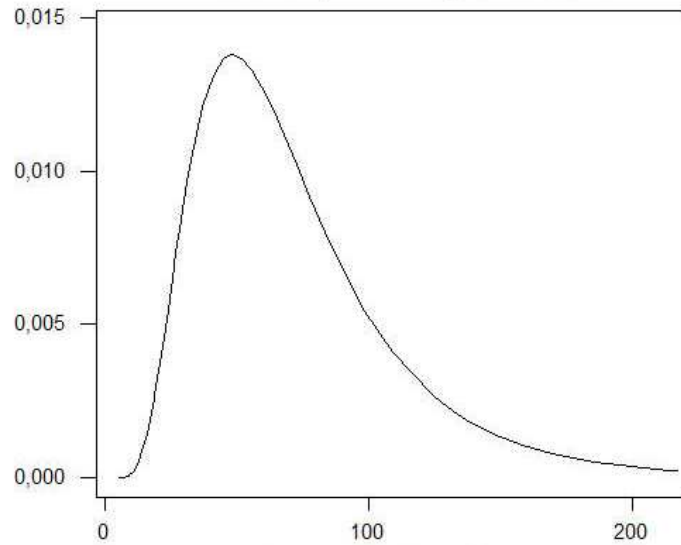
Hazard Function



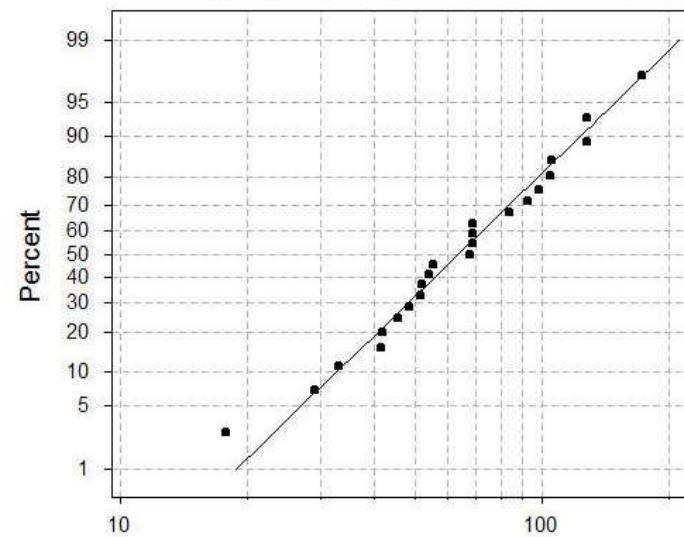
Ball Bearings Failure Data

ML Estimates - Complete Data

Probability Density Function



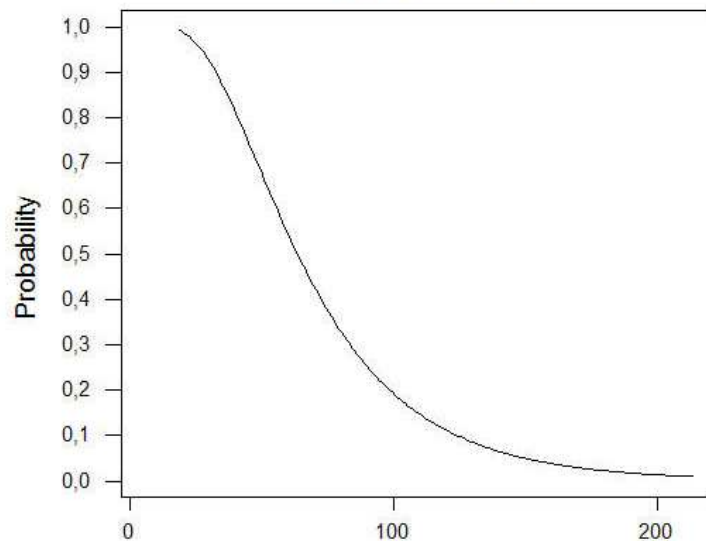
Lognormal base e Probability



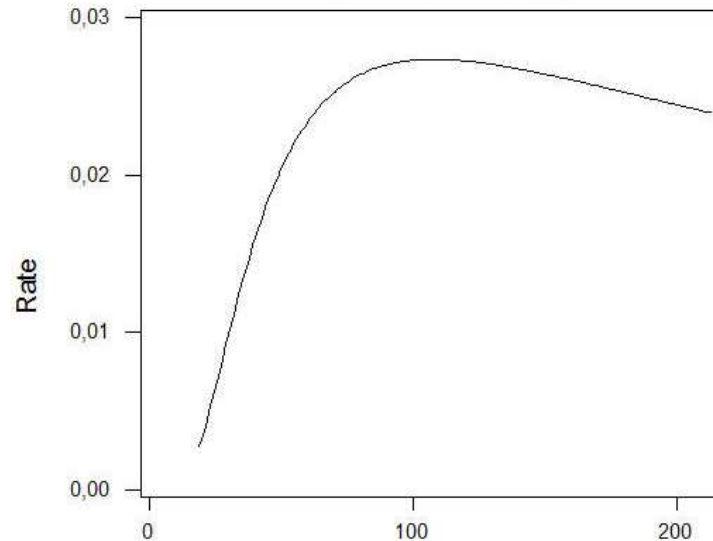
Location	4,1504
Scale	0,5217
MTTF	72,709
Failure	23
Censor	0

Goodness of Fit	
AD*	0,647

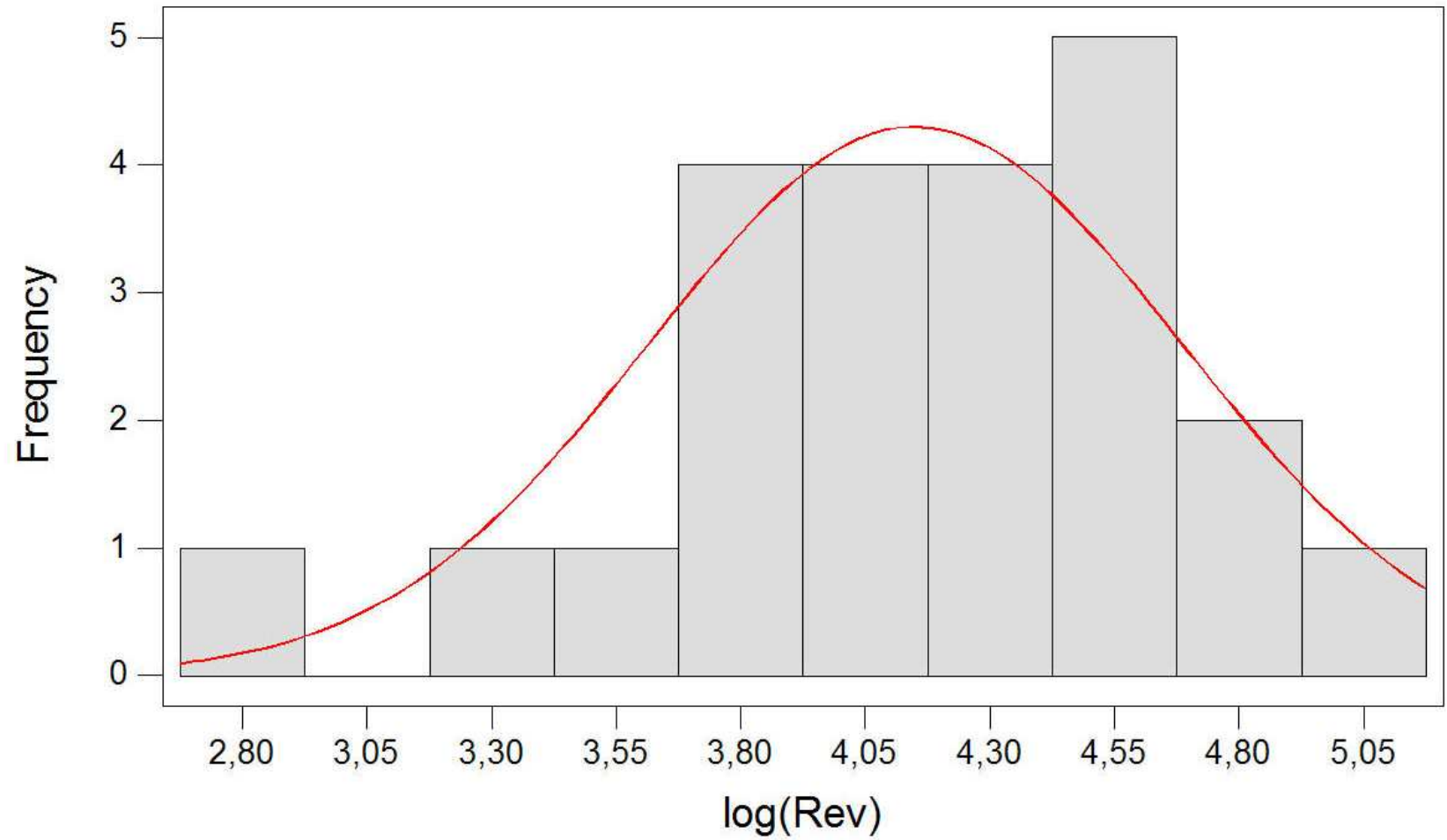
Survival Function



Hazard Function



Histogram of $\log(\text{Rev})$, with Normal Curve



AIRCONDITION FAILURES ON BOEING AIRPLANES (PROSCHAN, 1963)

MINITAB - Untitled

File Edit Manip Calc Stat Graph Editor Window Help

Worksheet 1 ***

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15	C16	C17
	T	log(T)															
1	413	6,02345															
2	14	2,63906															
3	58	4,06044															
4	37	3,61092															
5	100	4,60517															
6	65	4,17439															
7	9	2,19722															
8	169	5,12990															
9	447	6,10256															
10	184	5,21494															
11	36	3,58352															
12	201	5,30330															
13	118	4,77068															
14	34	3,52636															
15	31	3,43399															
16	18	2,89037															
17	18	2,89037															
18	67	4,20469															
19	57	4,04305															
20	62	4,12713															
21	7	1,94591															
22	22	3,09104															
23	34	3,52636															
24																	
25																	
26																	
27																	
28																	
29																	
30																	
31																	
32																	

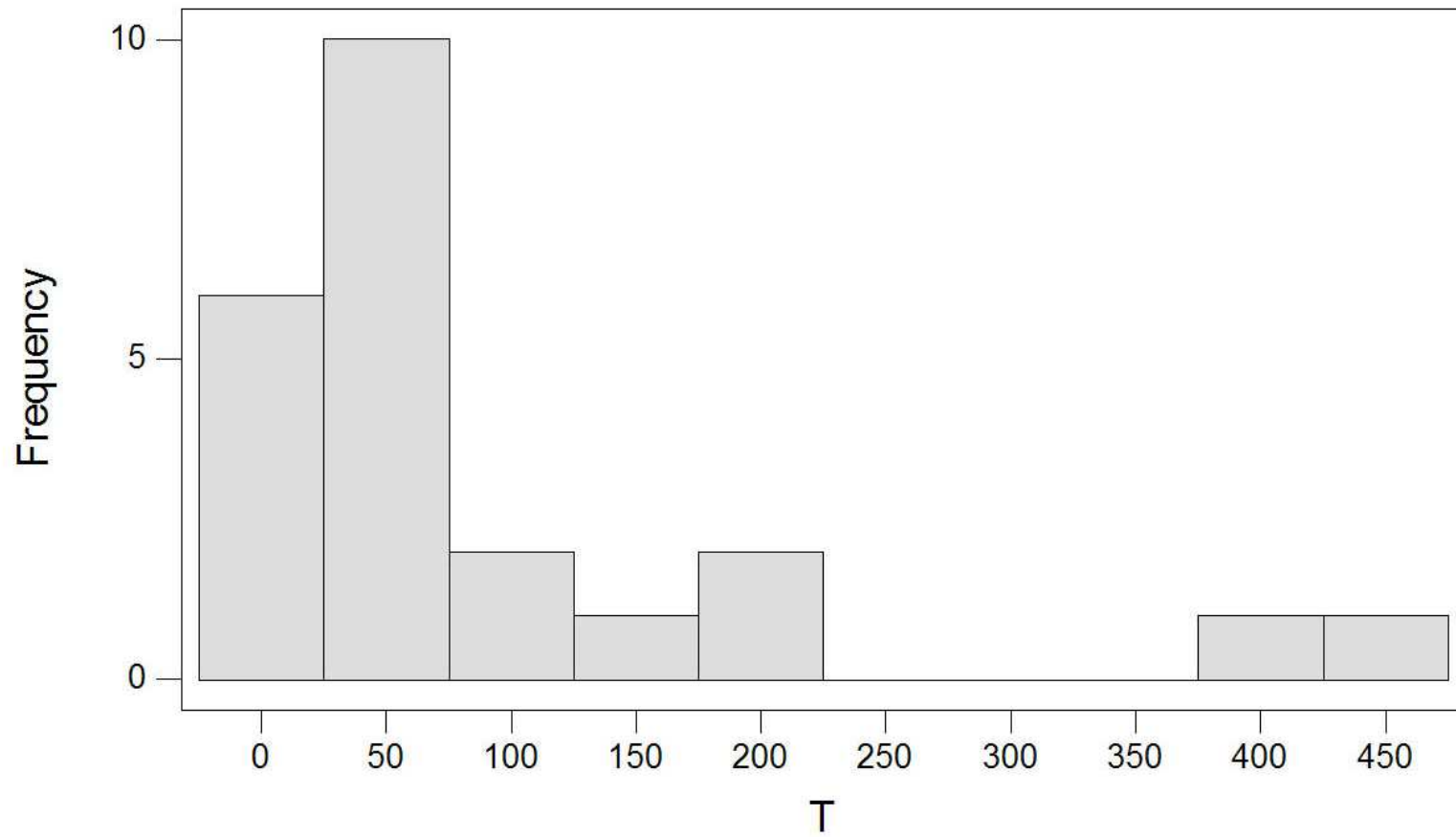
Projec...

Current Worksheet: Worksheet 1

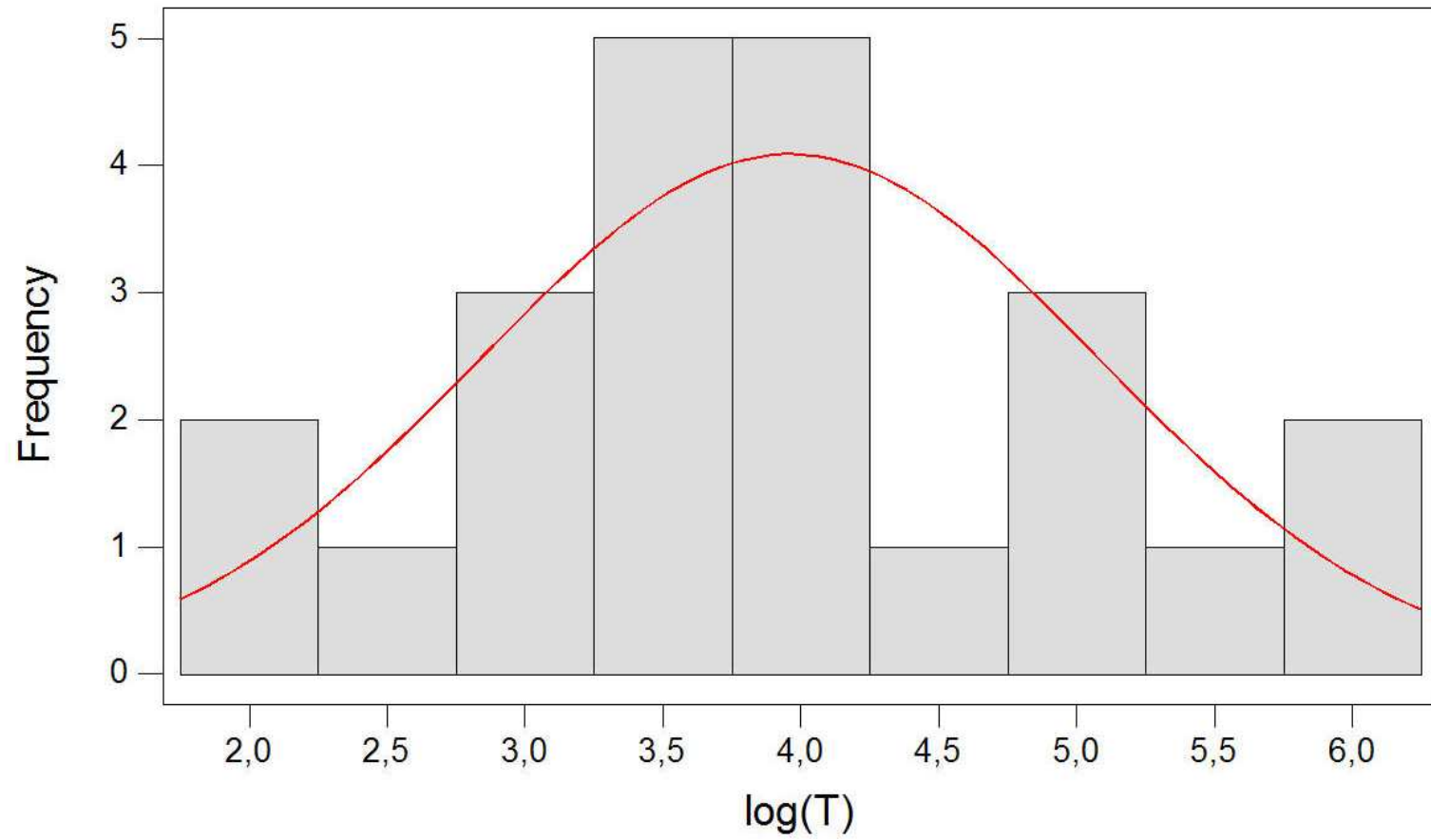
22:33

Start 1.3.6.6.9. Lo... http://www.p... SIO3020 - For... Foler WinEdt 5.3 (... Yap 0.99a - [F... Acrobat Read... Corel PHOTO... MINITAB - ... 22:33

Histogram of T

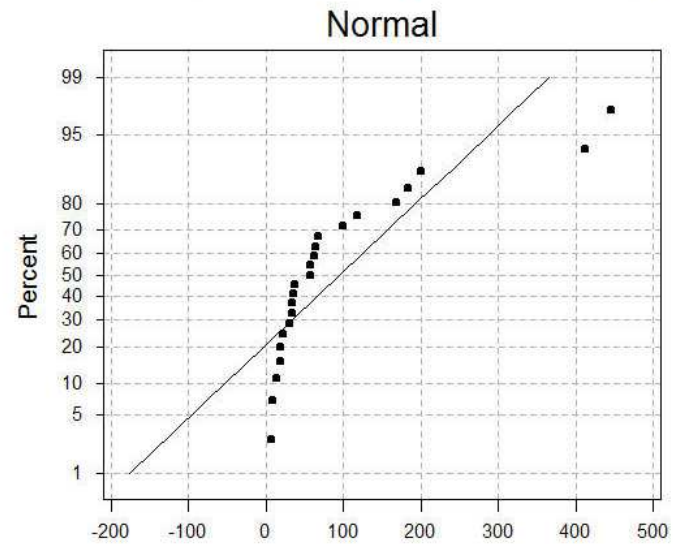
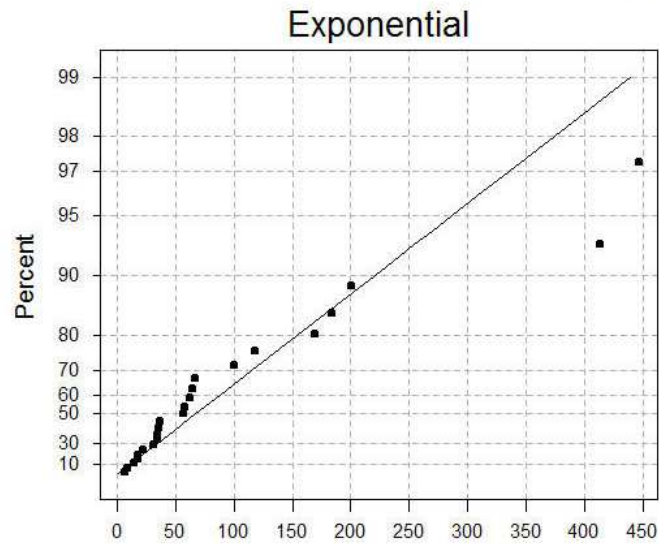
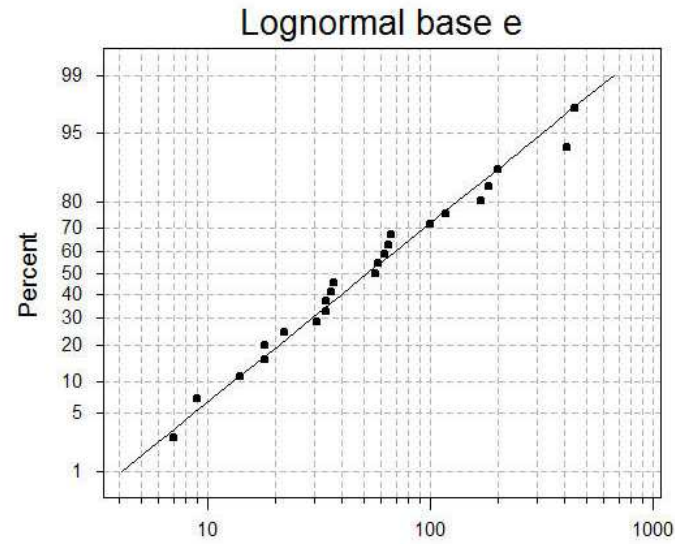
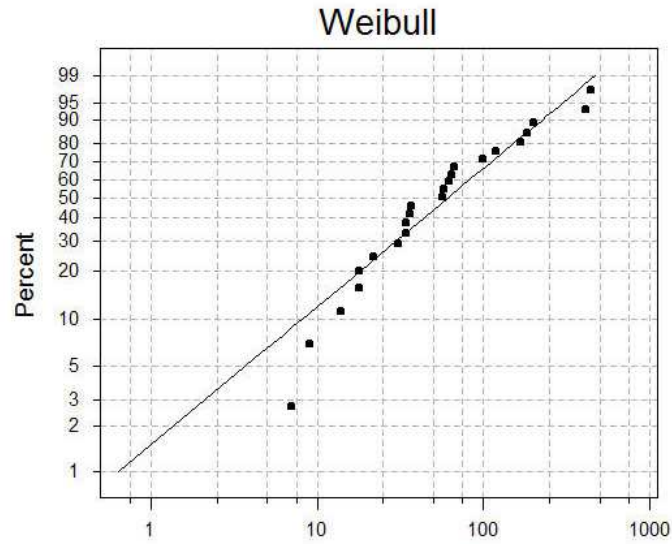


Histogram of $\log(T)$, with Normal Curve



Four-way Probability Plot for T

ML Estimates - Complete Data



Anderson-Darling (adj)

Weibull

0,999

Lognormal base e

0,675

Exponential

1,110

Normal

2,957

Motivation for the exponential distribution

- Simplest distribution used in the analysis of reliability data.
- Has the important characteristic that its hazard function is constant (does not depend on time t).
- Popular distribution for some kinds of electronic components (e.g., capacitors or robust, high-quality integrated circuits).
- This distribution would not be appropriate for a population of electronic components having failure-causing quality-defects.
- Might be useful to describe failure times for components that exhibit physical wearout only after expected technological life of the system in which the component would be installed.

Motivation for the Weibull distribution

- The theory of extreme values shows that the Weibull distribution can be used to model the minimum of a large number of independent positive random variables from a certain class of distributions.
 - Failure of the weakest link in a chain with many links with failure mechanisms (e.g. fatigue) in each link acting approximately independent.
 - Failure of a system with a large number of components in series and with approximately independent failure mechanisms in each component.
- The more common justification for its use is empirical: the Weibull distribution can be used to model failure-time data with a decreasing or an increasing hazard function.

Motivation for lognormal distribution

- The lognormal distribution is a common model for failure times.
- It can be justified for a random variable that arises from the product of a number of identically distributed independent positive random quantities (remember central limit theorem for sum of normals).
- It has been suggested as an appropriate model for failure time caused by a degradation process with combinations of random rates that combine multiplicatively.
- Widely used to describe time to fracture from fatigue crack growth in metals.
- Useful in modeling failure time of a population electronic components with a decreasing hazard function (due to a small proportion of defects in the population).
- Useful for describing the failure-time distribution of certain degradation processes.