SIMPLE EXAMPLE COX-REGRESSION

i	Y_i	x_i	δ_i
1	5	12	0
2	10	10	1
3	40	3	0
4	80	5	0
5	120	3	1
6	400	4	1
7	600	1	0

Model:

- $z(t|x) = z_0(t)\,\exp\{\beta x\}$

Partial likelihood:

$$L(\beta) = \frac{e^{10\beta}}{e^{10\beta} + e^{3\beta} + e^{5\beta} + e^{3\beta} + e^{4\beta} + e^{\beta}} \cdot \frac{e^{3\beta}}{e^{3\beta} + e^{4\beta} + e^{\beta}} \cdot \frac{e^{4\beta}}{e^{4\beta} + e^{\beta}}$$

$\widehat{\beta} = 0.765$

BRESLOW-ESTIMATOR

$$\hat{Z}_0(t) = \sum_{T_j \le t} \frac{1}{\sum_{i \in R_j} e^{\hat{\boldsymbol{\beta}}' \boldsymbol{x}_i}}$$

so

$$\begin{aligned}
\hat{Z}_0(10) &= \frac{1}{e^{10\hat{\beta}} + e^{3\hat{\beta}} + e^{5\hat{\beta}} + e^{3\hat{\beta}} + e^{4\hat{\beta}} + e^{\hat{\beta}}} = 4.57 \cdot 10^{-4} \\
\hat{Z}_0(120) &= 4.57 \cdot 10^{-4} + \frac{1}{e^{3\hat{\beta}} + e^{4\hat{\beta}} + e^{\hat{\beta}}} = 0.0304 \\
\hat{Z}_0(400) &= 0.0304 + \frac{1}{e^{4\hat{\beta}} + e^{\hat{\beta}}} = 0.0730
\end{aligned}$$

ESTIMATED SURVIVAL FUNCTION $P(T > t) = R(t; \boldsymbol{x})$

$$\widehat{R}(t; \boldsymbol{x}) = \exp\{-\widehat{Z}_0(t) e^{\widehat{\boldsymbol{\beta}}' \boldsymbol{x}}\}$$

so

$$\begin{aligned} \widehat{R}(10; x) &= \exp\{-4.57 \cdot 10^{-4} e^{0.765x}\} \\ \widehat{R}(120; x) &= \exp\{-0.0304 e^{0.765x}\} \\ \widehat{R}(400; x) &= \exp\{-0.0730 e^{0.765x}\} \end{aligned}$$

x	$\widehat{R}(10; x)$	$\widehat{R}(120; x)$	$\widehat{R}(400; x)$
0*	0.9995	0.9701	0.9296
1	0.9990	0.9368	0.8548
3	0.9955	0.7396	0.4846
5	0.9793	0.2483	0.0352
10	0.3829	0.0000	0.0000
KM**	0.8333	0.5556	0.2778

* Baseline survival function
**KM-estimator does not use the value of x

COX-SNELL RESIDUALS

$$\widehat{V}_i = \widehat{Z}_0(Y_i)e^{\widehat{\boldsymbol{\beta}}'\boldsymbol{x}_i}$$

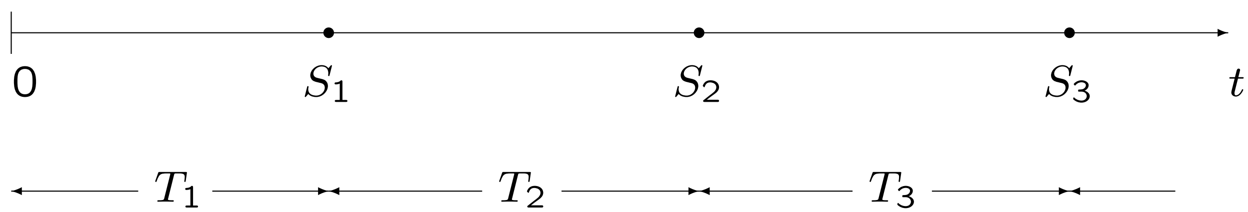
which should behave like *expon*(1)

i	Y_i	x_i	δ_i	$\widehat{V}_i$	
1	5	12	0	$\widehat{Z}_0(0)e^{0.765\cdot 12}$	$= 0.0000$
2	10	10	1	$\widehat{Z}_0(10)e^{0.765\cdot 10}$	$= 0.9593$
3	40	3	0	$\widehat{Z}_0(10)e^{0.765\cdot 3}$	$= 0.0045$
4	80	5	0	$\widehat{Z}_0(10)e^{0.765\cdot 5}$	$= 0.0209$
5	120	3	1	$\widehat{Z}_0(120)e^{0.765\cdot 3}$	$= 0.3017$
6	400	4	1	$\widehat{Z}_0(400)e^{0.765\cdot 4}$	$= 1.5567$
7	600	1	0	$\widehat{Z}_0(400)e^{0.765\cdot 1}$	$= 0.1569$

REPAIRABLE SYSTEMS/RECURRENT EVENTS/COUNTING PROCESSES

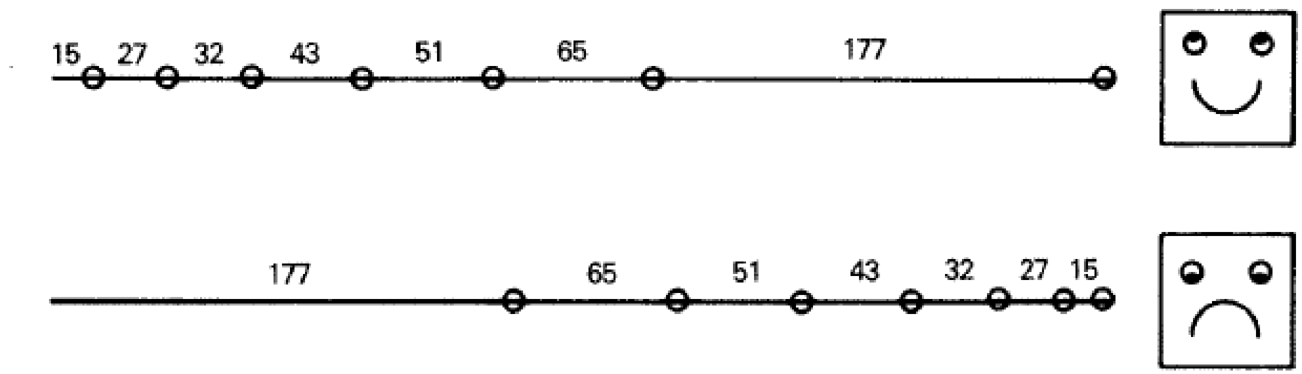
Definition of repairable system (Ascher and Feingold 1984):

"A repairable system is a system which, after failing to perform one or more of its functions satisfactorily, can be restored to fully satisfactory performance by any method, other than replacement of the entire system".



“HAPPY” AND “SAD” SYSTEMS

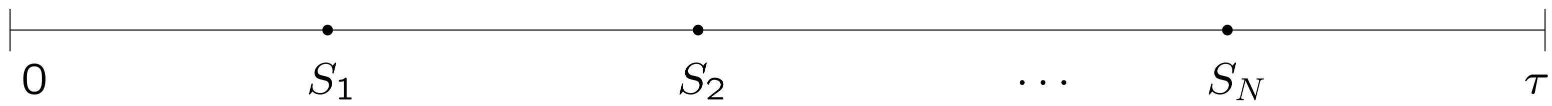
Ascher and Feingold presented the following example of a “happy” and “sad” system:



- *Their claim:* Reliability engineers do not recognize the difference between these cases since they always treat times between failures as i.i.d. and fit probability models like Weibull.

- *Their conclusion:* Use point process models to analyze repairable systems data!

Today: Recurrent events extensively studied



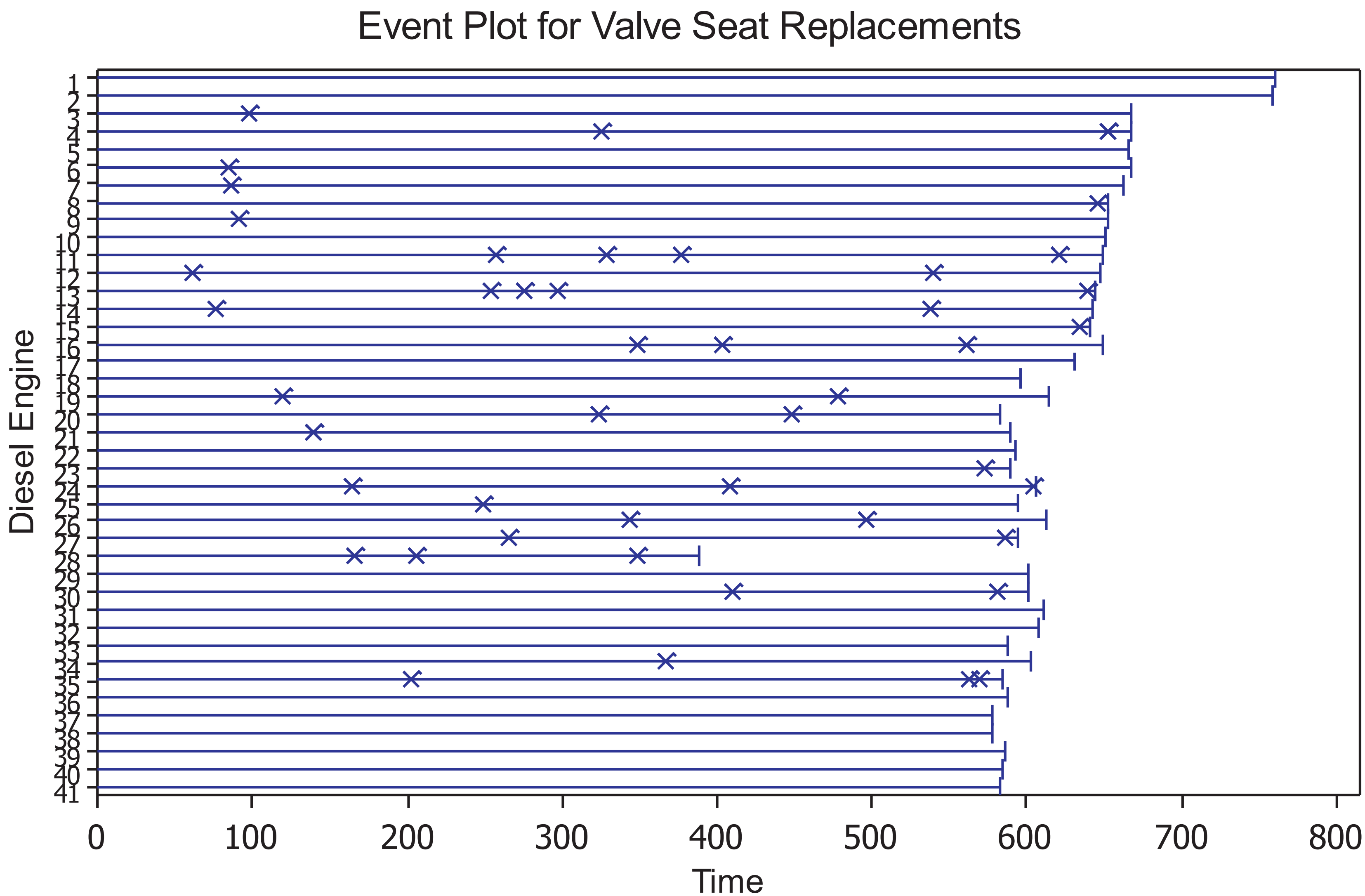
- Applications: engineering and reliability studies, public health, clinical trials, politics, finance, insurance, sociology, etc.

Reliability applications:

- breakdown or failure of a mechanical or electronic system
- discovery of a bug in an operating system software
- the occurrence of a crack in concrete structures
- the breakdown of a fiber in fibrous composites
- Warranty claims of manufactured products

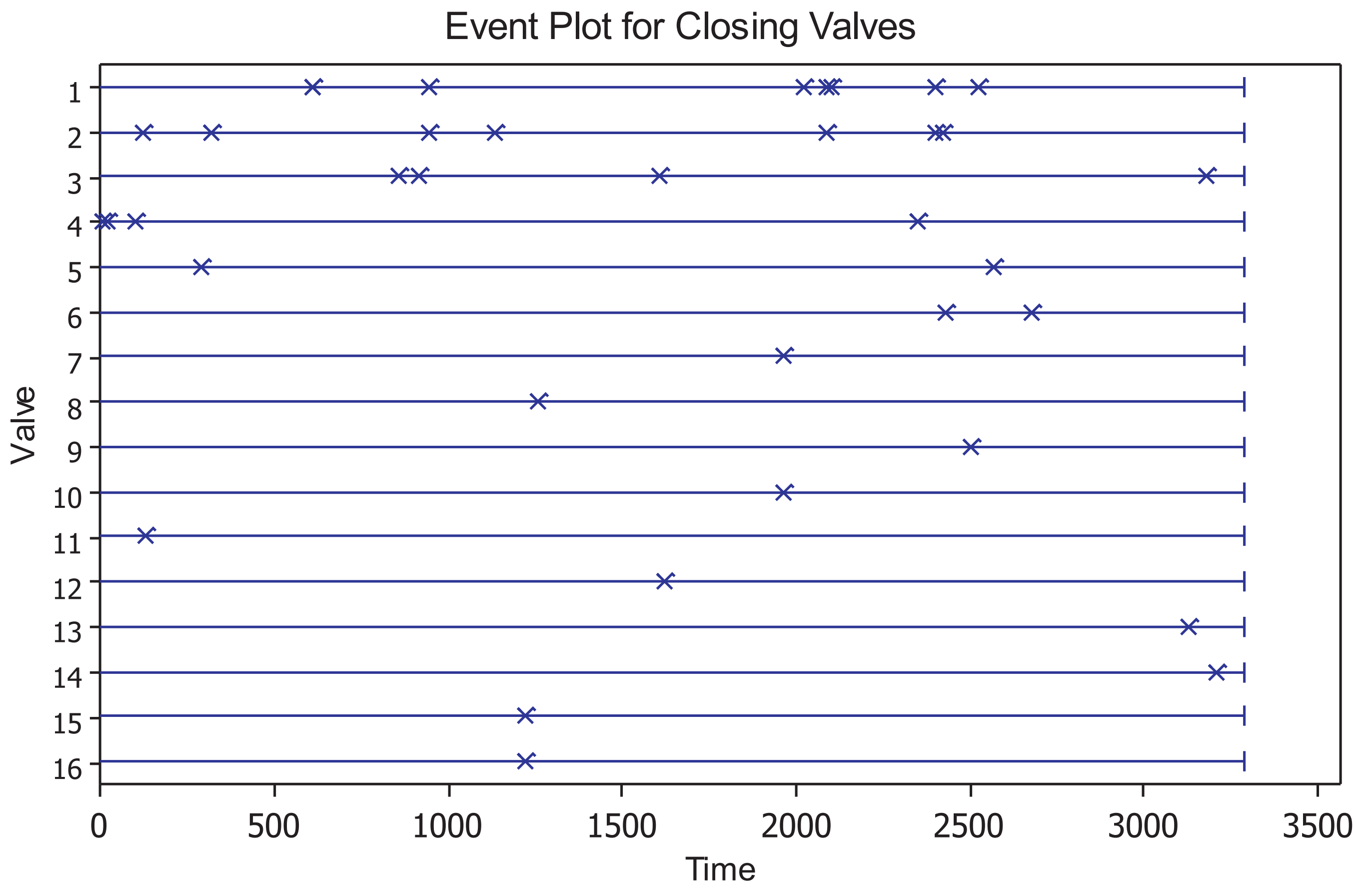
Nelson (1995): Valve seat data

- Times of valve-seat replacements in a fleet of 41 diesel engines



Bhattacharjee et al. (2003): Nuclear plant failure data

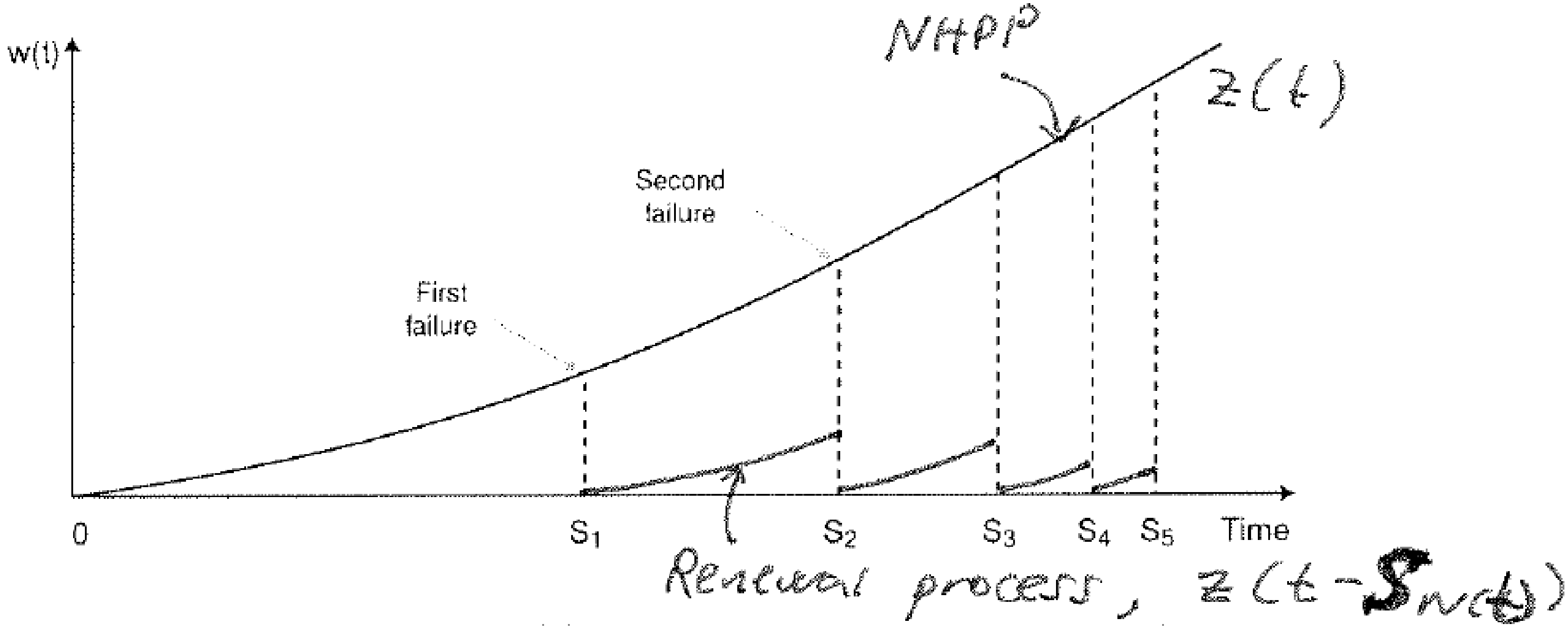
- Failure data for closing valves in safety systems at two nuclear reactor plants in Finland. Failures type: *External Leakage*, follow-up 9 years for 104 valves. 88 valves had no failures



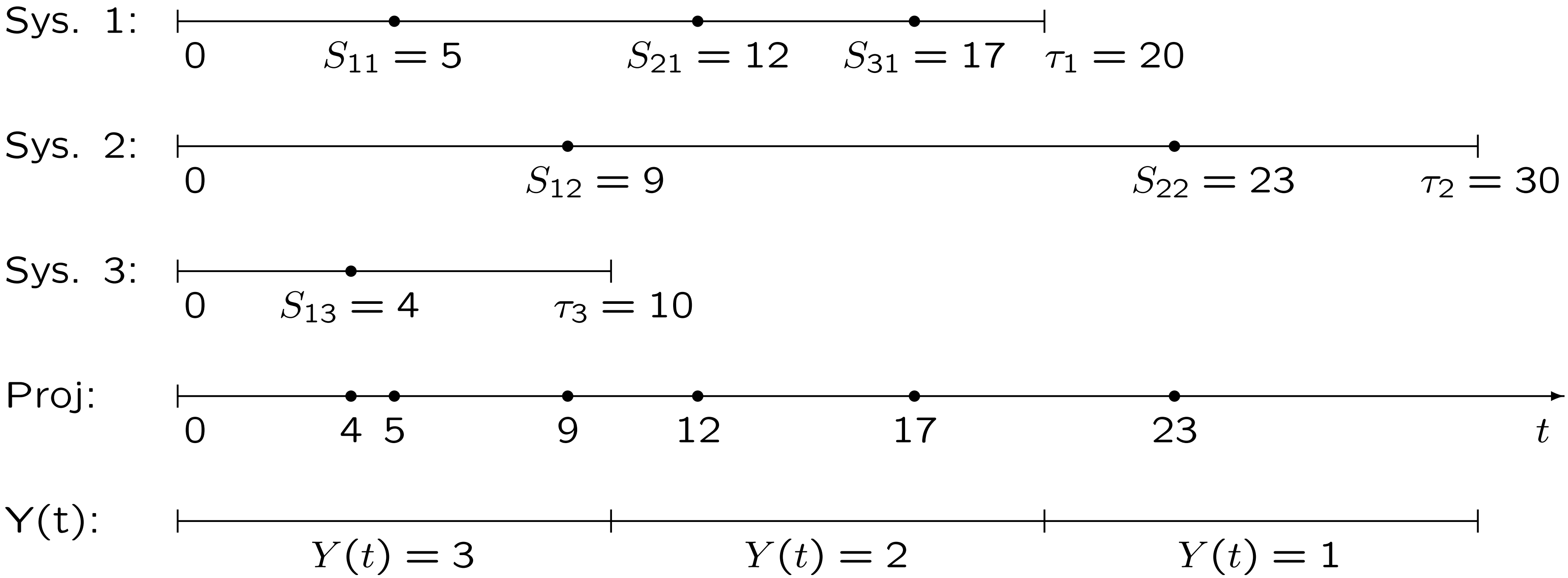
Aalen and Husebye (1991): Migratory motor complex (MMC) periods in 19 patients, 1-9 events per individual.

Individual	Observed periods (minutes)					
1	112	145	39	52	21	34
	33	51	(54)			
2	206	147	(30)			
3	284	59	186	(4)		
4	94	98	84	(87)		
5	67	(131)				
6	124	34	87	75	43	38
	58	142	75	(23)		
7	116	71	83	68	125	(111)
8	111	59	47	95	(110)	
9	98	161	154	55	(44)	
10	166	56	(122)			
11	63	90	63	103	51	(85)
12	47	86	68	144	(72)	
⋮			⋮			

CONDITIONAL ROCOF BY MINIMAL REPAIR (NHPP) AND PERFECT REPAIR (RENEWAL PROCESS)



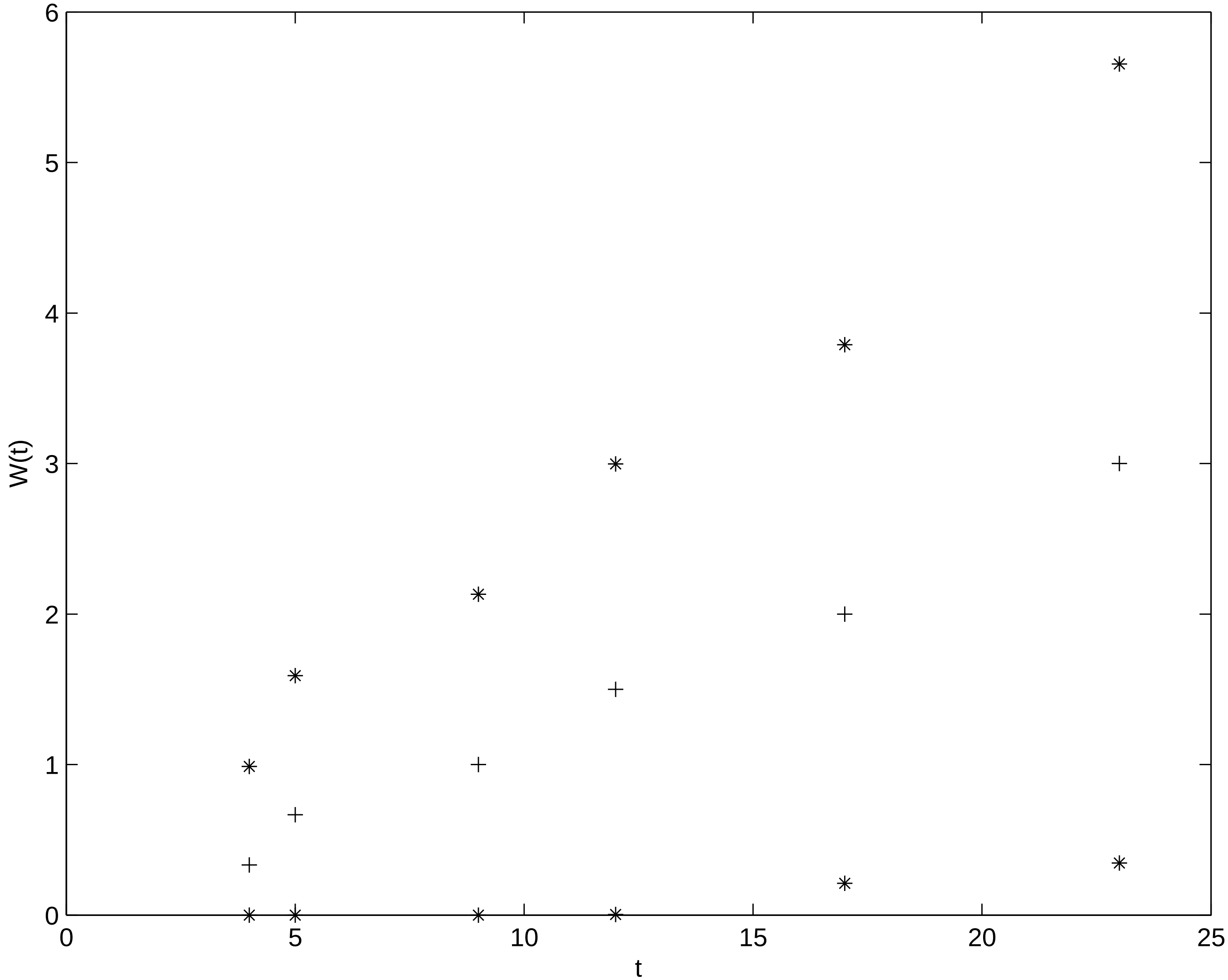
SIMPLE EXAMPLE WITH THREE SYSTEMS



COMPUTATIONS FOR THE NELSON-AALEN ESTIMATOR

t	$1/Y(t)$	$1/Y(t)^2$	$\hat{W}(t)$	$\widehat{Var\hat{W}}(t)$	$\widehat{SD\hat{W}}(t)$
4	1/3	1/9	1/3	1/9	0.3333
5	1/3	1/9	2/3	2/9	0.4714
9	1/3	1/9	1	1/3	0.5774
12	1/2	1/4	3/2	7/12	0.7638
17	1/2	1/4	2	5/6	0.9129
23	1	1	3	11/6	1.3540

ESTIMATED $W(t)$ with 95% confidence limits (Nelson-Aalen)

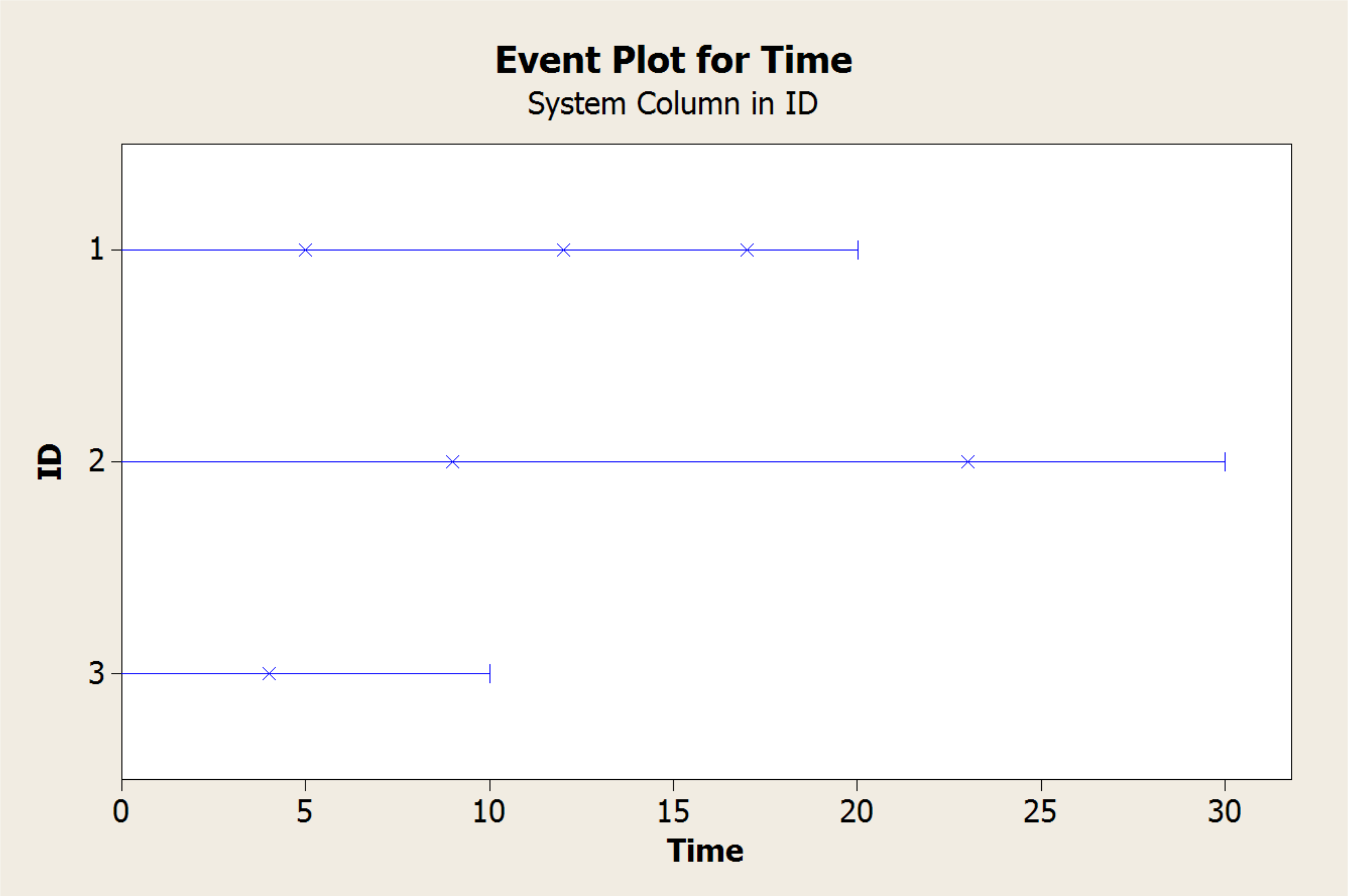


Simple Example With 3 Systems

SimpleNHPP.MTW ***

↓	C1	C2	C3
	ID	Time	
1	1	5	
2	1	12	
3	1	17	
4	1	20	
5	2	9	
6	2	23	
7	2	30	
8	3	4	
9	3	10	
10			

Simple Example With 3 Systems



Simple Example With 3 Systems

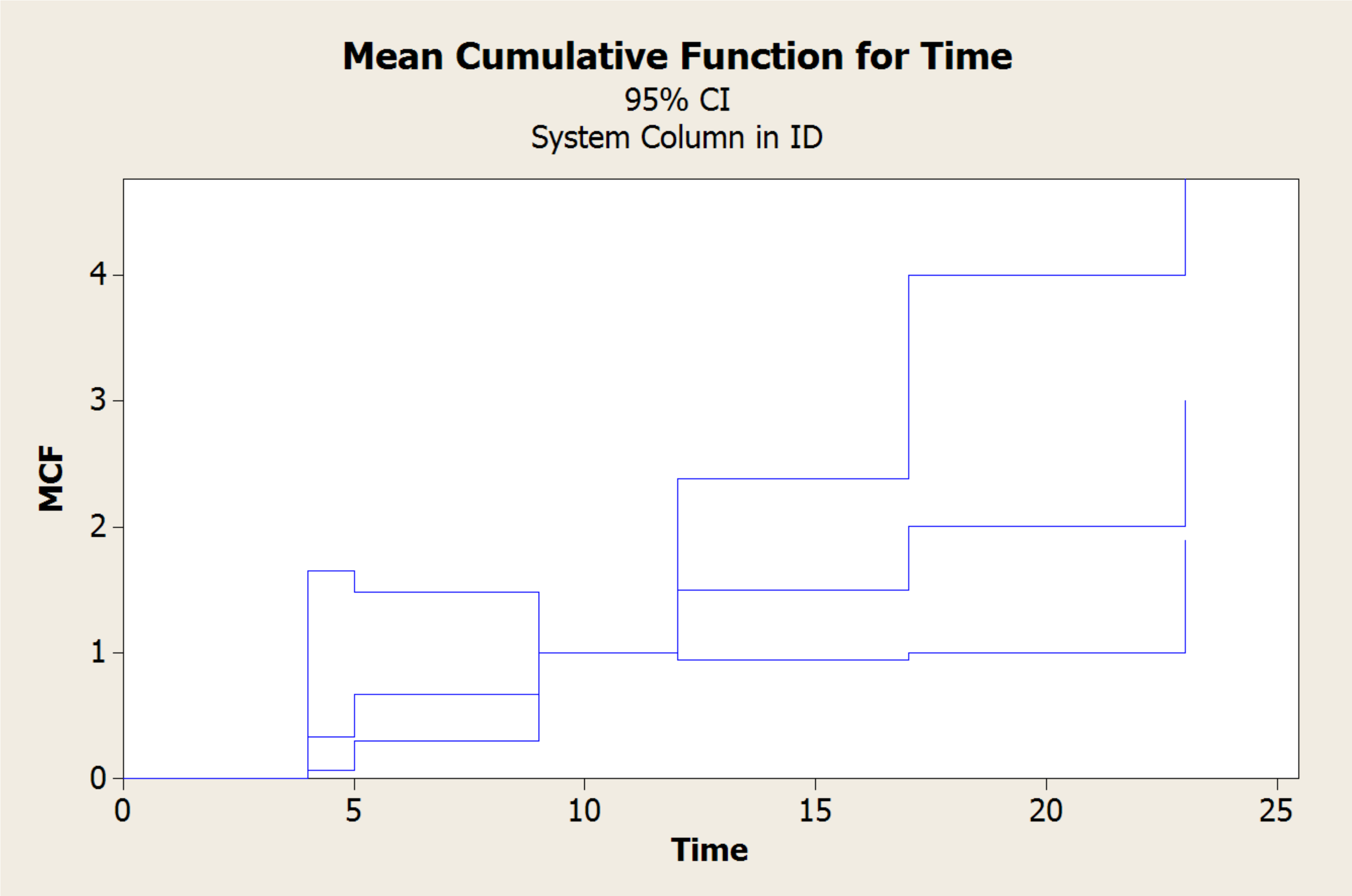
```
 Results for: SimpleNHPP.MTW
Nonparametric Growth Curve:  Time System:  ID

Nonparametric Estimates

Table of Mean Cumulative Function
```

Time	Mean Cumulative Function	Standard Error	95% Normal CI Lower	95% Normal CI Upper	System
4	0,33333	0,272166	0,06728	1,65151	3
5	0,66667	0,272166	0,29951	1,48392	1
9	1,00000	0,000000	1,00000	1,00000	2
12	1,50000	0,353553	0,94506	2,38079	1
17	2,00000	0,707107	1,00020	3,99922	1
23	3,00000	0,707107	1,89013	4,76158	2

Simple Example With 3 Systems



Nelson-Aalen estimator for Cumulative ROCOF $W(t)$

1. Order all failure times as $t_1 < t_2 < \ldots t_n$.

2. Let $d_j(t_i) = \#$ events in system j at t_i.

3. Let $d(t_i) = \sum_{j=1}^{m} d_j(t_i) = \#$ events in all systems at t_i.

4. Let $Y_j(t) = \begin{cases} 1 & \text{if system } j \text{ is under observation at time } t \\ 0 & \text{otherwise} \end{cases}$

5. Let $Y(t) = \sum_{j=1}^{m} Y_j(t) = \#$ systems under observation at time t .

Then

Under general assumptions: $\widehat{W}(t) = \sum_{t_i \le t} \frac{d(t_i)}{Y(t_i)}.$

Assuming NHPP: $\widehat{\text{Var}\ \widehat{W}(t)} = \sum_{t_i \le t} \frac{d(t_i)}{\{Y(t_i)\}^2}$

Under general assumptions (MINITAB): $\widehat{\text{Var}\ \widehat{W}(t)} = \sum_{j=1}^{m} \left\{ \sum_{t_i \le t} \frac{Y_j(t_i)}{Y(t_i)} \left[d_j(t_i) - \frac{d(t_i)}{Y(t_i)} \right] \right\}^2$

Illustration of last formula for Simple NHPP Example (Compare with MINITAB Output):

$$\begin{aligned}
\mathrm{Var}\,\widehat{\widehat{W}}(4) &= \left\{\frac{1}{3}\left[0-\frac{1}{3}\right]\right\}^2+\left\{\frac{1}{3}\left[0-\frac{1}{3}\right]\right\}^2+\left\{\frac{1}{3}\left[1-\frac{1}{3}\right]\right\}^2 \\
&= \frac{6}{81}=0.2722^2
\end{aligned}$$

$$\begin{aligned}
\mathrm{Var}\,\widehat{\widehat{W}}(5) &= \left\{\frac{1}{3}\left[0-\frac{1}{3}\right]+\frac{1}{3}\left[1-\frac{1}{3}\right]\right\}^2 \\
&+ \left\{\frac{1}{3}\left[0-\frac{1}{3}\right]+\frac{1}{3}\left[0-\frac{1}{3}\right]\right\}^2 \\
&+ \left\{\frac{1}{3}\left[1-\frac{1}{3}\right]+\frac{1}{3}\left[0-\frac{1}{3}\right]\right\}^2 \\
&= \frac{6}{81}=0.2722^2
\end{aligned}$$

$$
\begin{aligned}
\text{Var}\,\widehat{\widehat{W}}(9) &= \left\{\frac{1}{3}\left[0-\frac{1}{3}\right]+\frac{1}{3}\left[1-\frac{1}{3}\right]+\frac{1}{3}\left[0-\frac{1}{3}\right]\right\}^2 \\
&+ \left\{\frac{1}{3}\left[0-\frac{1}{3}\right]+\frac{1}{3}\left[0-\frac{1}{3}\right]+\frac{1}{3}\left[1-\frac{1}{3}\right]\right\}^2 \\
&+ \left\{\frac{1}{3}\left[1-\frac{1}{3}\right]+\frac{1}{3}\left[0-\frac{1}{3}\right]+\frac{1}{3}\left[0-\frac{1}{3}\right]\right\}^2 \\
&= 0
\end{aligned}
$$

$$
\begin{aligned}
\text{Var}\,\widehat{\widehat{W}}(12) &= \left\{\frac{1}{3}\left[0-\frac{1}{3}\right]+\frac{1}{3}\left[1-\frac{1}{3}\right]+\frac{1}{3}\left[0-\frac{1}{3}\right]+\frac{1}{2}\left[1-\frac{1}{2}\right]\right\}^2 \\
&+ \left\{\frac{1}{3}\left[0-\frac{1}{3}\right]+\frac{1}{3}\left[0-\frac{1}{3}\right]+\frac{1}{3}\left[1-\frac{1}{3}\right]+\frac{1}{2}\left[0-\frac{1}{2}\right]\right\}^2 \\
&+ \left\{\frac{1}{3}\left[1-\frac{1}{3}\right]+\frac{1}{3}\left[0-\frac{1}{3}\right]+\frac{1}{3}\left[0-\frac{1}{3}\right]\right\}^2 \\
&= \frac{1}{8} = 0.3536^2
\end{aligned}
$$

Simple Example With 3 Systems

Power Law NHPP Model: $W(t;\alpha,\theta) = (t/\theta)^{\alpha}$

Results for: SimpleNHPP.MTW

Parametric Growth Curve: Time

System: ID

Model: Power-Law Process
Estimation Method: Maximum Likelihood

Parameter Estimates

Parameter	Estimate	Standard Error	95% Normal CI Lower	95% Normal CI Upper
Shape	1,19423	0,445	0,323015	2,06545
Scale	11,3803	4,840	1,89335	20,8672

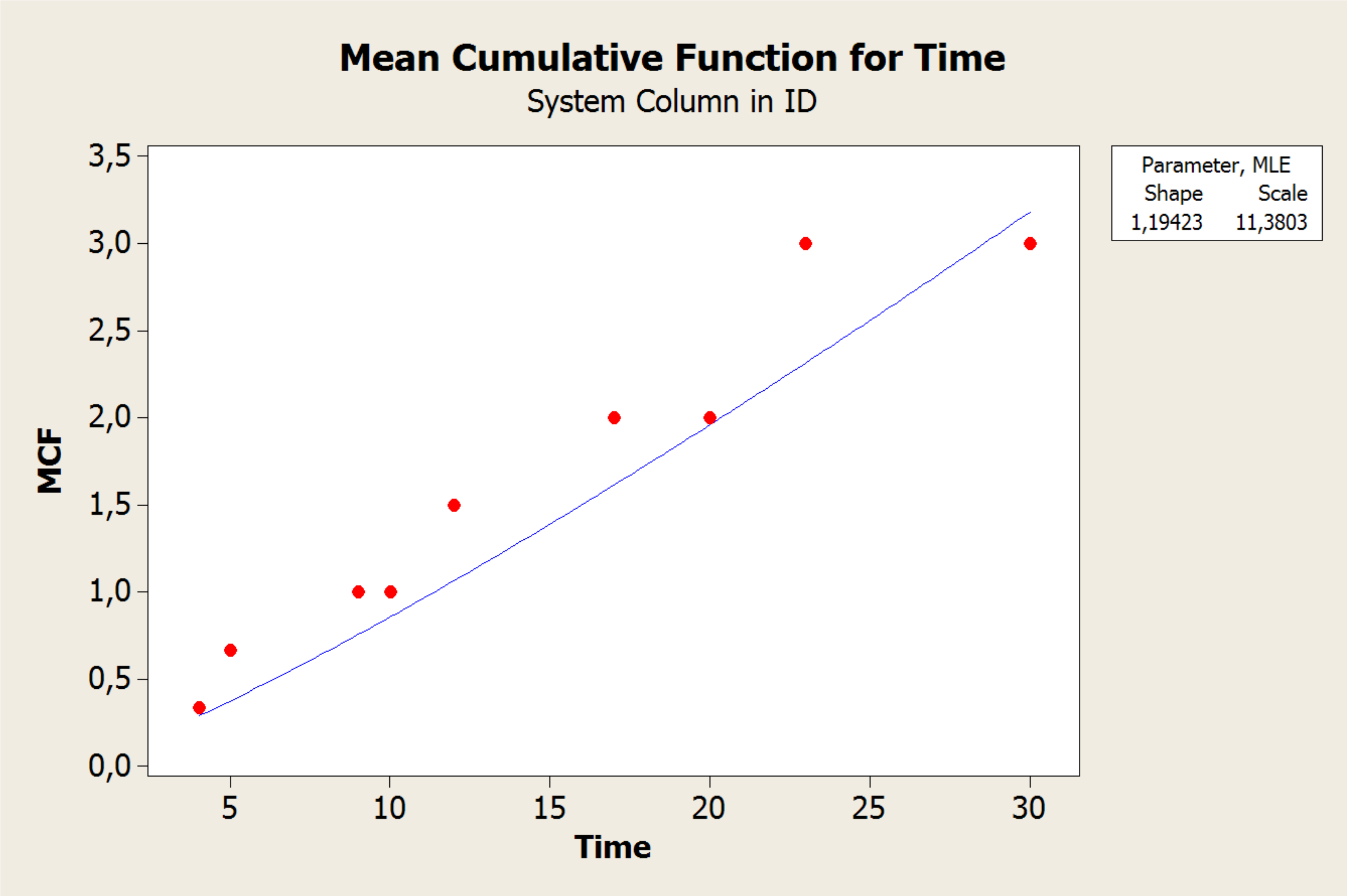
Test for Equal Shape Parameters
Bartlett's Modified Likelihood Ratio Chi-Square

Test Statistic	0,06
P-Value	0,972
DF	2

Trend Tests

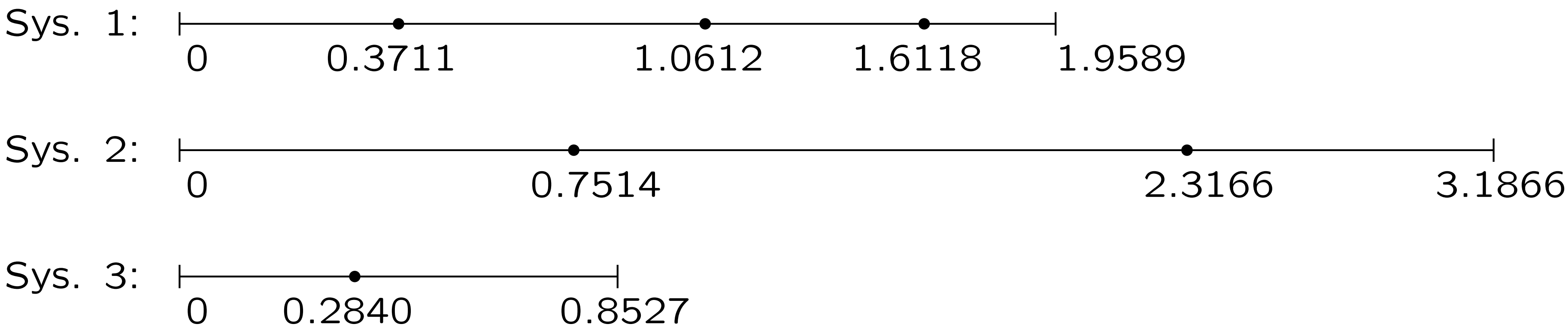
	MIL-Hdbk-189 TTT-based	MIL-Hdbk-189 Pooled	Laplace's TTT-based	Laplace's Pooled	Anderson-Darling
Test Statistic	9,03	8,89	0,28	0,31	0,28
P-Value	0,599	0,576	0,781	0,756	0,954
DF	12	12			

Simple Example With 3 Systems

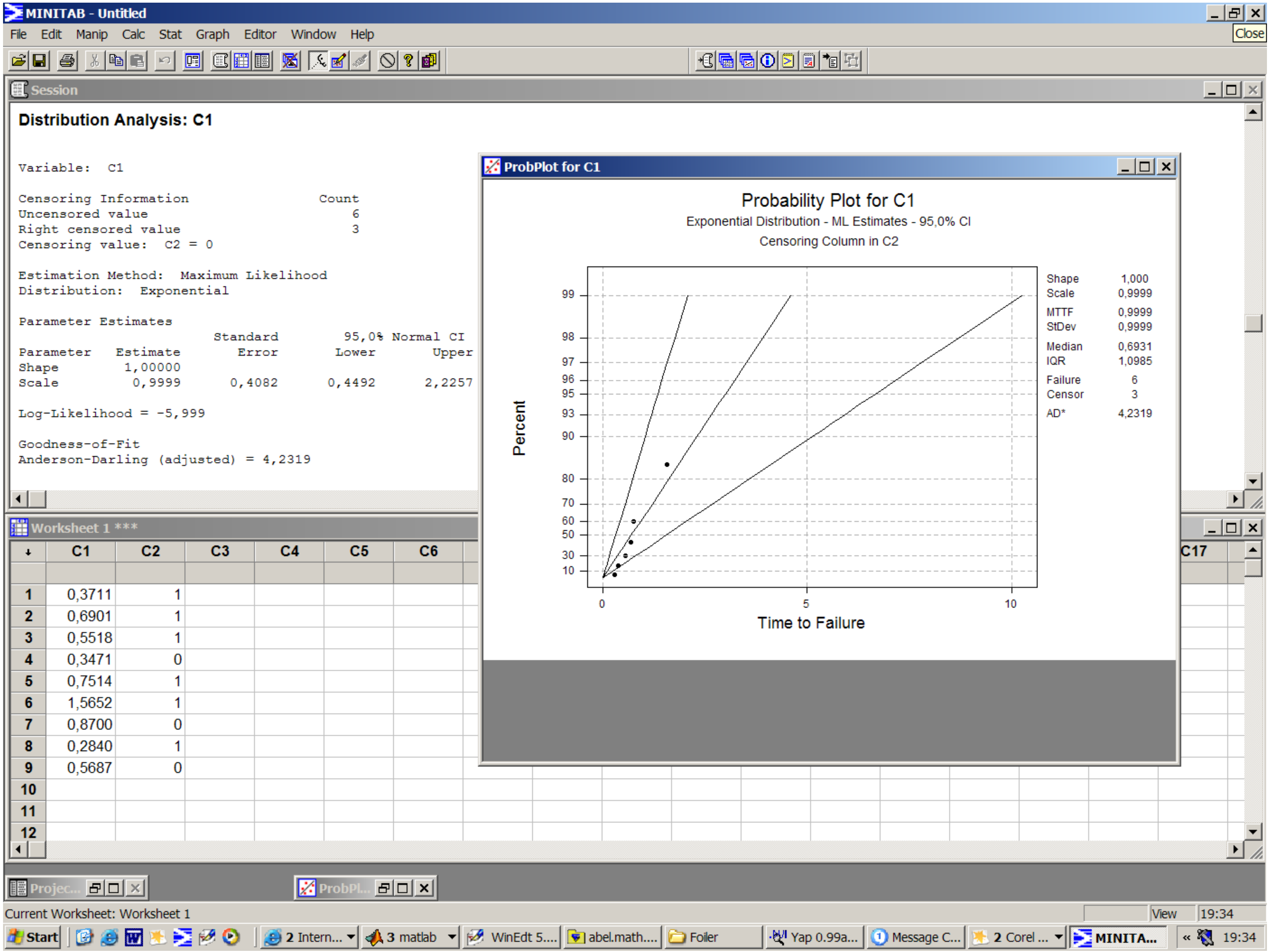


RESIDUAL PROCESS: "SIMPLE EXAMPLE".
Data points (and endpoints on axes) are transformed with the estimated cumulative ROCOF,

$$\widehat{W}(t) = 0.0538 \cdot t^{1.20}$$



Times between events, plus censored times at the end of each axis, are on the next slide anlysed by MINITAB as a set of censored exponential variables.


MINITAB - Untitled
File Edit Manip Calc Stat Graph Editor Window Help
Session
Distribution Analysis: C1
Variable: C1
Censoring Information Count
Uncensored value 6
Right censored value 3
Censoring value: C2 = 0
Estimation Method: Maximum Likelihood
Distribution: Exponential
Parameter Estimates
Standard 95,0% Normal CI
Parameter Estimate Error Lower Upper
Shape 1,00000
Scale 0,9999 0,4082 0,4492 2,2257
Log-Likelihood = -5,999
Goodness-of-Fit
Anderson-Darling (adjusted) = 4,2319
ProbPlot for C1
Probability Plot for C1
Exponential Distribution - ML Estimates - 95,0% CI
Censoring Column in C2
Percent
Time to Failure
Shape 1,000
Scale 0,9999
MTTF 0,9999
StDev 0,9999
Median 0,6931
IQR 1,0985
Failure 6
Censor 3
AD* 4,2319
Worksheet 1 ***
C1 C2 C3 C4 C5 C6 C17
1 0,3711 1
2 0,6901 1
3 0,5518 1
4 0,3471 0
5 0,7514 1
6 1,5652 1
7 0,8700 0
8 0,2840 1
9 0,5687 0
Current Worksheet: Worksheet 1
19:34

Valve Seat Replacement Times (Nelson and Doganaksoy 1989)

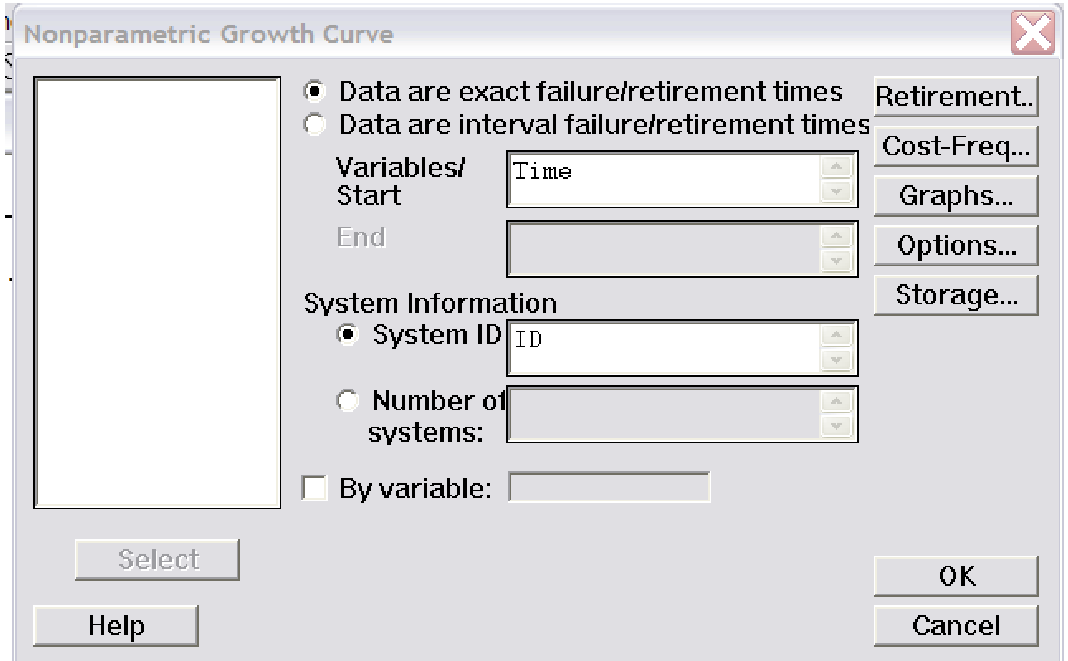
Data collected from valve seats from a fleet of 41 diesel engines (days of operation)

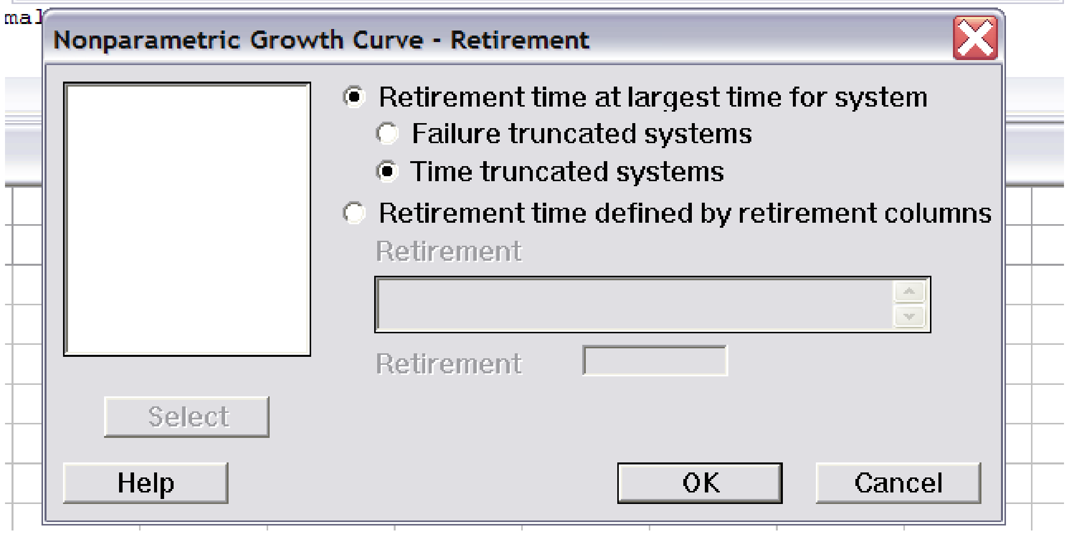
- Each engine has 16 valves
- Does the replacement rate increase with age?
- How many replacement valves will be needed in the future?
- Can valve life in these systems be modeled as a renewal process?

VALVESEAT DATA

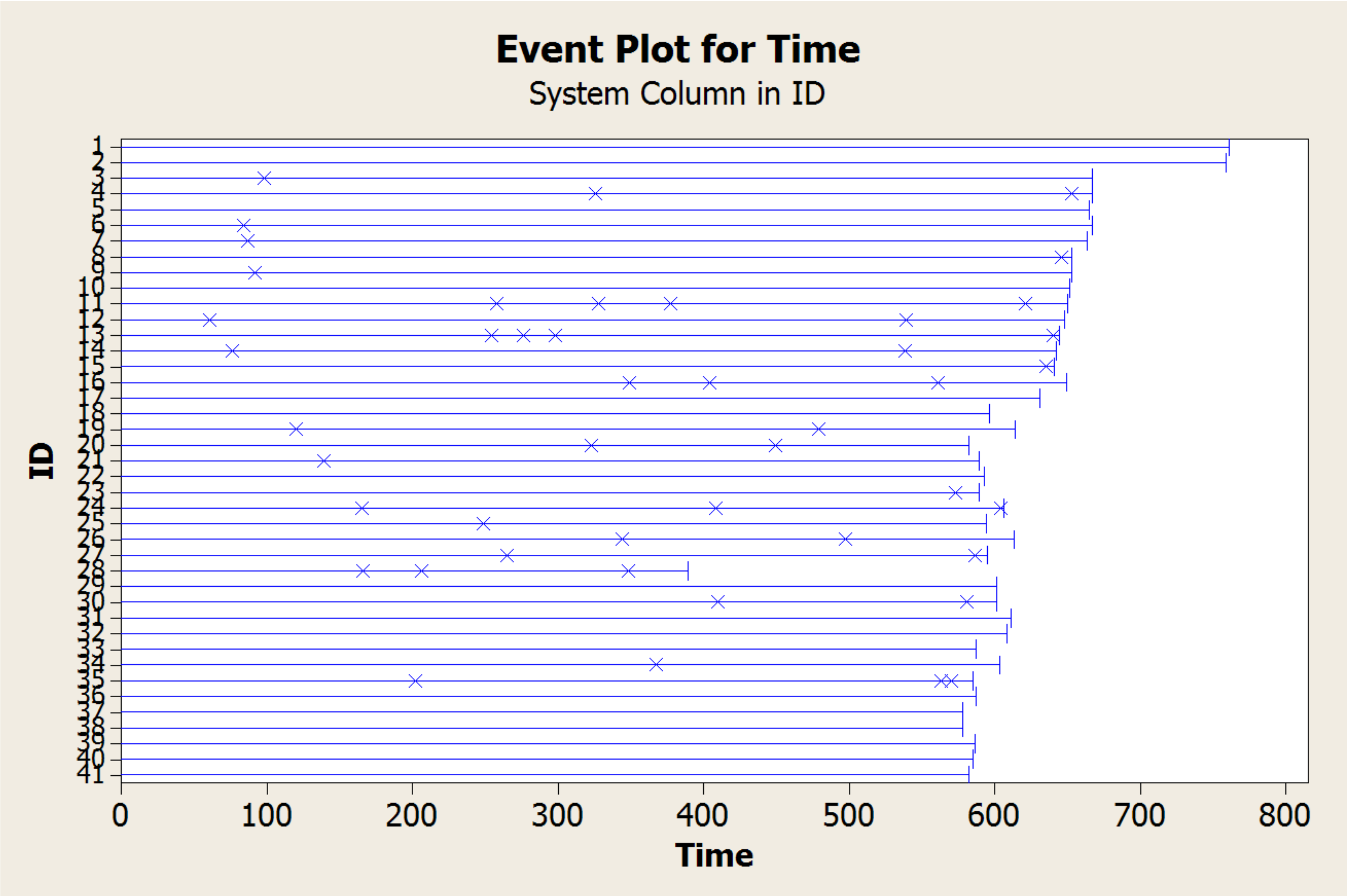
TMA4275valveseat.MTW ***

↓	C1	C2
	ID	Time
1	1	761
2	2	759
3	3	98
4	3	667
5	4	326
6	4	653
7	4	653
8	4	667
9	5	665
10	6	84
11	6	667
12	7	87
13	7	663
14	8	646
15	8	653
16	9	92
17	9	653
18	10	651
19	11	258
20	11	328
21	11	377
22	11	621
23	11	650
24	12	61
25	12	539
26	12	648

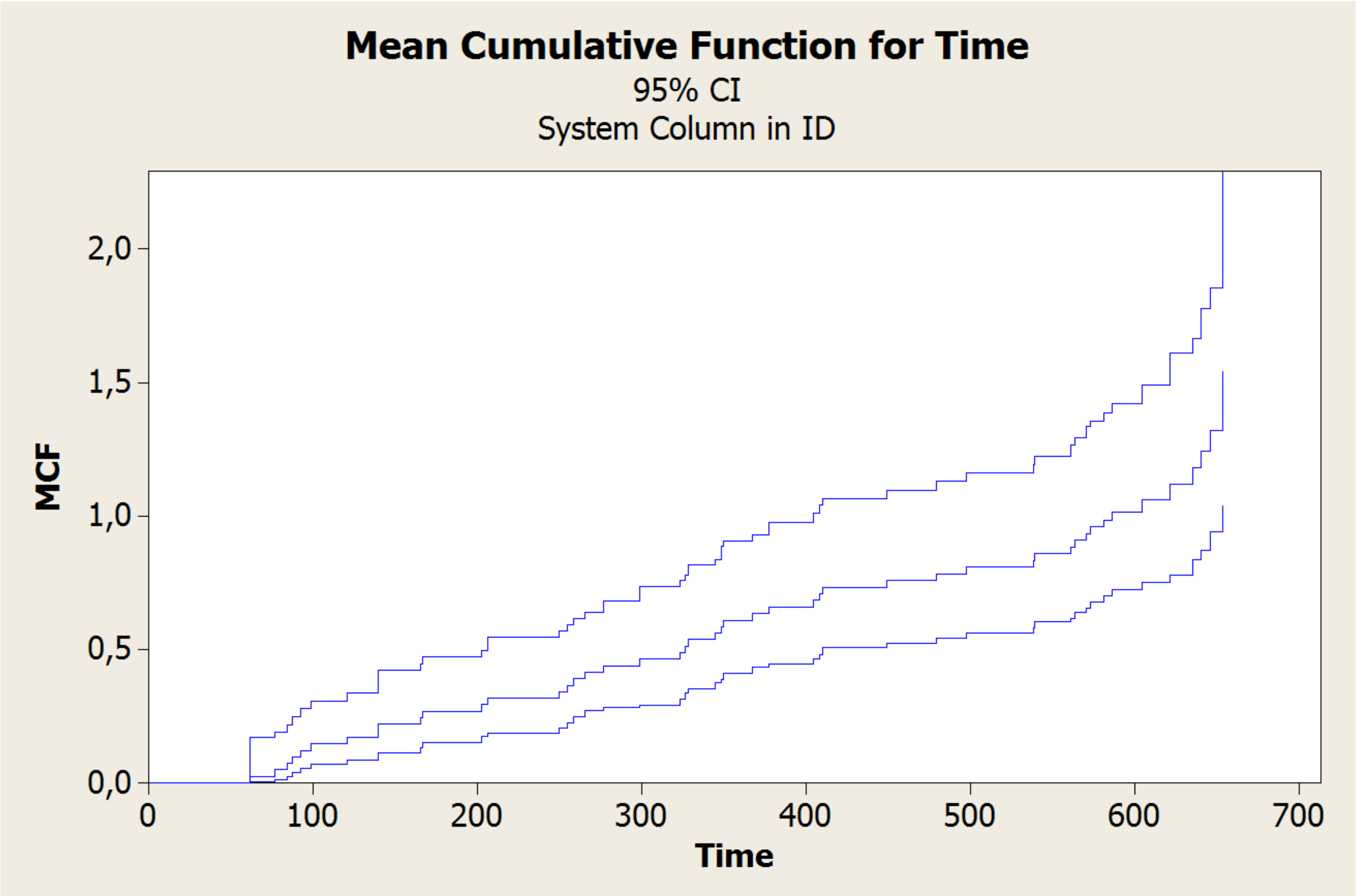

Nonparametric Growth Curve
Data are exact failure/retirement times
Data are interval failure/retirement times
Variables/
Start
Time
End
System Information
System ID
ID
Number of
systems:
By variable:
Select
Help
Retirement...
Cost-Freq...
Graphs...
Options...
Storage...
OK
Cancel


Nonparametric Growth Curve - Retirement
Retirement time at largest time for system
Failure truncated systems
Time truncated systems
Retirement time defined by retirement columns
Retirement
Retirement
Select
Help
OK
Cancel

VALVESEAT DATA



VALVESEAT DATA



VALVESEAT DATA

Nonparametric Growth Curve: Time

System: ID

Nonparametric Estimates

Table of Mean Cumulative Function

Time	Mean Cumulative Function	Standard Error	95% Normal CI Lower	95% Normal CI Upper	System
61	0,02439	0,024091	0,00352	0,16903	12
76	0,04878	0,033641	0,01262	0,18848	14
84	0,07317	0,040670	0,02462	0,21750	6
87	0,09756	0,046340	0,03846	0,24750	7
92	0,12195	0,051105	0,05364	0,27726	9
98	0,14634	0,055199	0,06987	0,30650	3
120	0,17073	0,058764	0,08696	0,33519	19
139	0,19512	0,061891	0,10479	0,36333	21
139	0,21951	0,073270	0,11411	0,42226	21
165	0,24390	0,075417	0,13305	0,44711	24
166	0,26829	0,077317	0,15251	0,47196	28
202	0,29268	0,078988	0,17246	0,49672	35
206	0,31707	0,087527	0,18458	0,54467	28
249	0,34146	0,088680	0,20525	0,56807	25
254	0,36585	0,089656	0,22631	0,59143	13
258	0,39024	0,090461	0,24775	0,61468	11
265	0,41463	0,091101	0,26955	0,63780	27
276	0,43902	0,097858	0,28363	0,67955	13
298	0,46341	0,109607	0,29150	0,73671	13
323	0,48780	0,109740	0,31387	0,75812	20
326	0,51220	0,109740	0,33656	0,77949	4
328	0,53659	0,114907	0,35266	0,81643	11
344	0,56098	0,114654	0,37581	0,83737	26
348	0,58537	0,124250	0,38615	0,88737	28
349	0,60976	0,123782	0,40960	0,90772	16
367	0,63415	0,123194	0,43334	0,92801	34
377	0,65854	0,131842	0,44480	0,97498	11
404	0,68354	0,135939	0,46289	1,00936	16

Times of Unscheduled Maintenance Actions for a USS Grampus Diesel Engine

- Unscheduled maintenance actions caused by failure of imminent failure.

- Unscheduled maintenance actions are inconvenient and expensive.

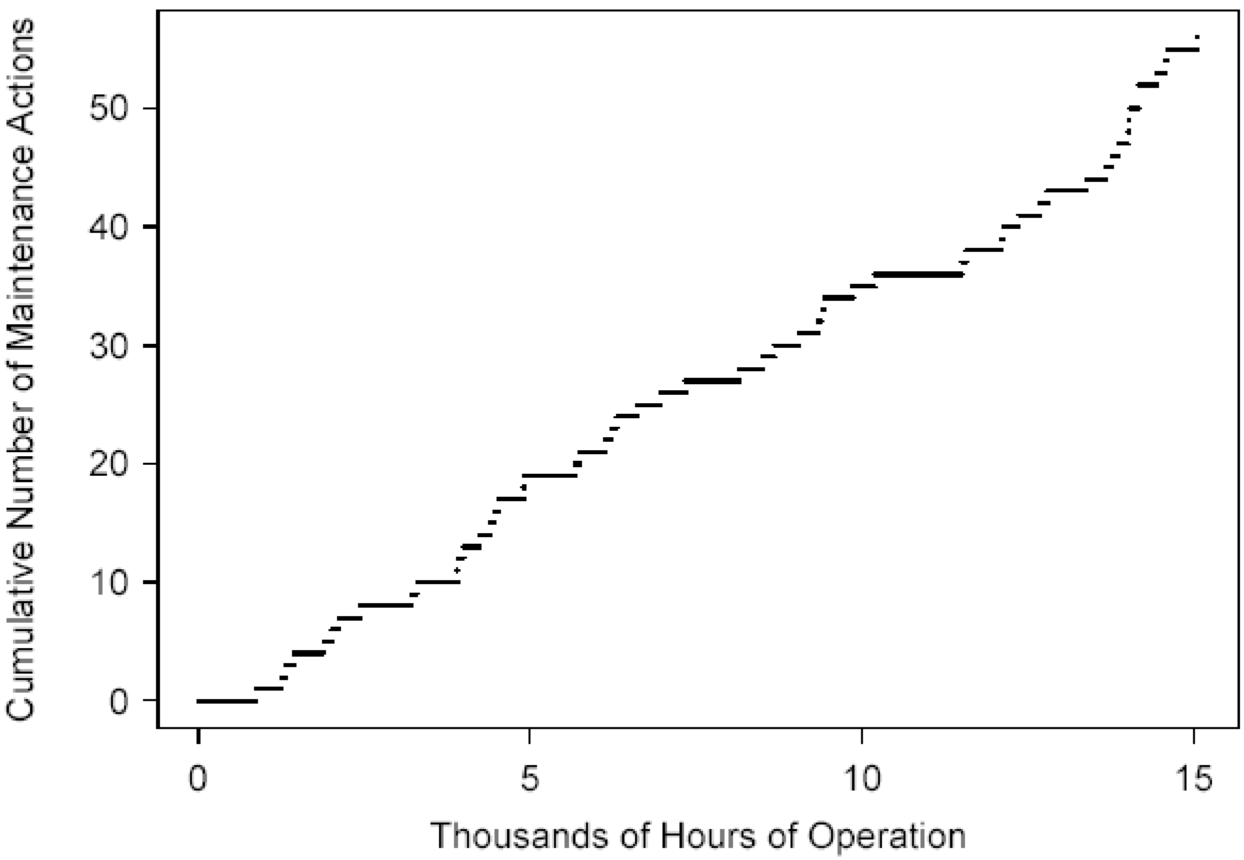
- Data available for 16,000 operating hours.

- Data from Lee (1980).

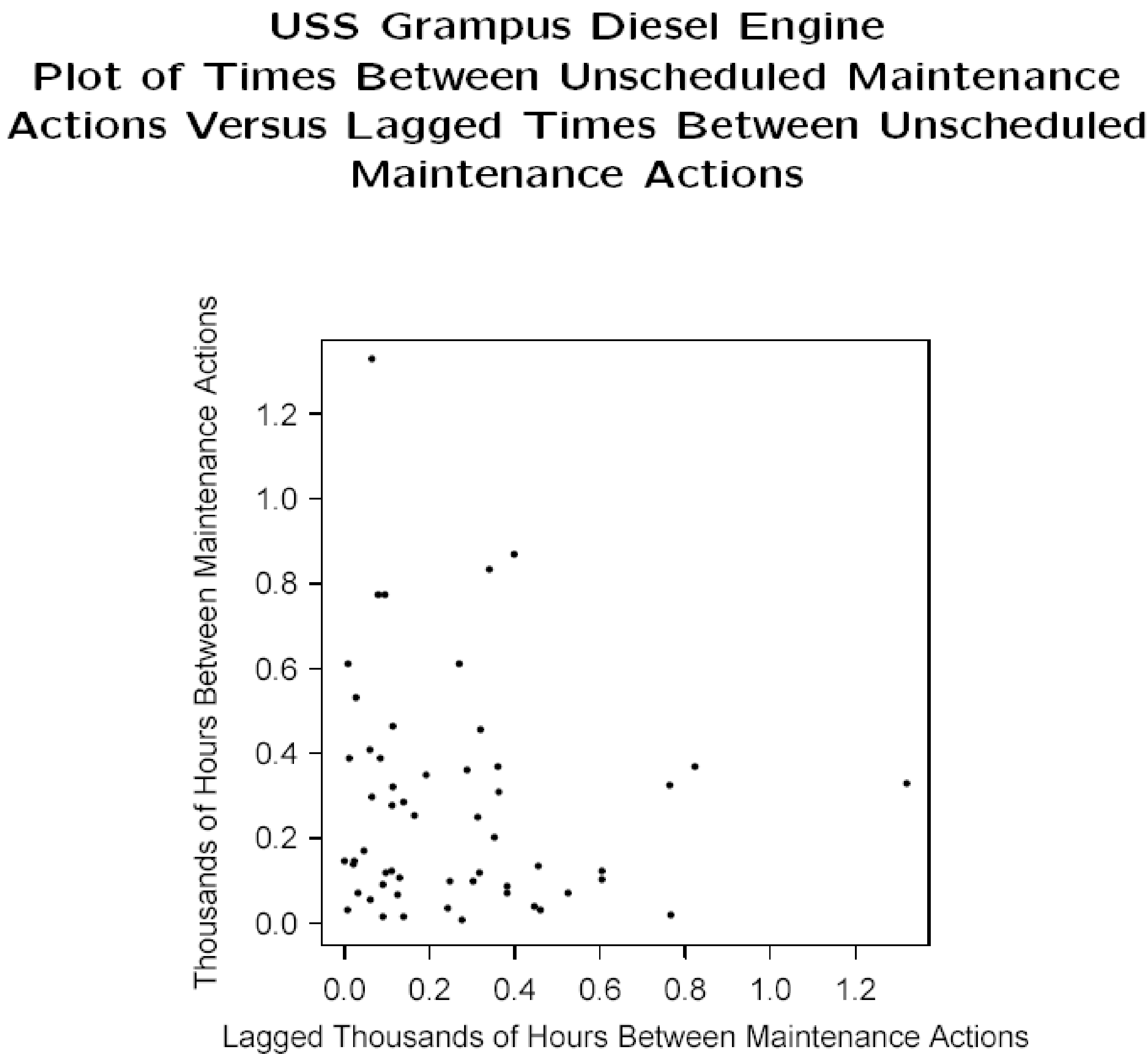
- Is the system deteriorating (i.e., are failures occurring more rapidly as the system ages)?

- Can the occurrence of unscheduled maintenance actions be modeled by an HPP?

Cumulative Number of Unscheduled Maintenance Actions Versus Operating Hours for a USS Grampus Diesel Engine Lee (1980)



Grampus- data: Plot of (T_i, T_{i+1}) to investigate whether times between failures can be assumed independent. The figure does not indicate a correlation between successive times.



The Likelihood for the NHPP - Single Unit

- With **interval** recurrence data.

  Suppose that the unit has been observed for a period $(0, t_a]$ and the data are the number of recurrences $d_1, \ldots, d_m$ in the nonoverlapping intervals $(t_0, t_1]$, $(t_1, t_2]$, $\ldots$, $(t_{m-1}, t_m]$ (with $t_0 = 0$, $t_m = t_a$).

$$
\begin{aligned}
L(\boldsymbol{\theta}) &= \Pr\left[N(t_0, t_1) = d_1, \ldots, N(t_{m-1}, t_m) = d_m\right] \\
&= \prod_{j=1}^{m} \Pr\left[N(t_{j-1}, t_j) = d_j\right] \\
&= \prod_{j=1}^{m} \frac{\left[\mu(t_{j-1}, t_j; \boldsymbol{\theta})\right]^{d_j}}{d_j!} \exp\left[-\mu(t_{j-1}, t_j; \boldsymbol{\theta})\right] \\
&= \prod_{j=1}^{m} \frac{\left[\mu(t_{j-1}, t_j; \boldsymbol{\theta})\right]^{d_j}}{d_j!} \times \exp\left[-\mu(t_0, t_a; \boldsymbol{\theta})\right]
\end{aligned}
$$

The Likelihood for the NHPP (Continued)

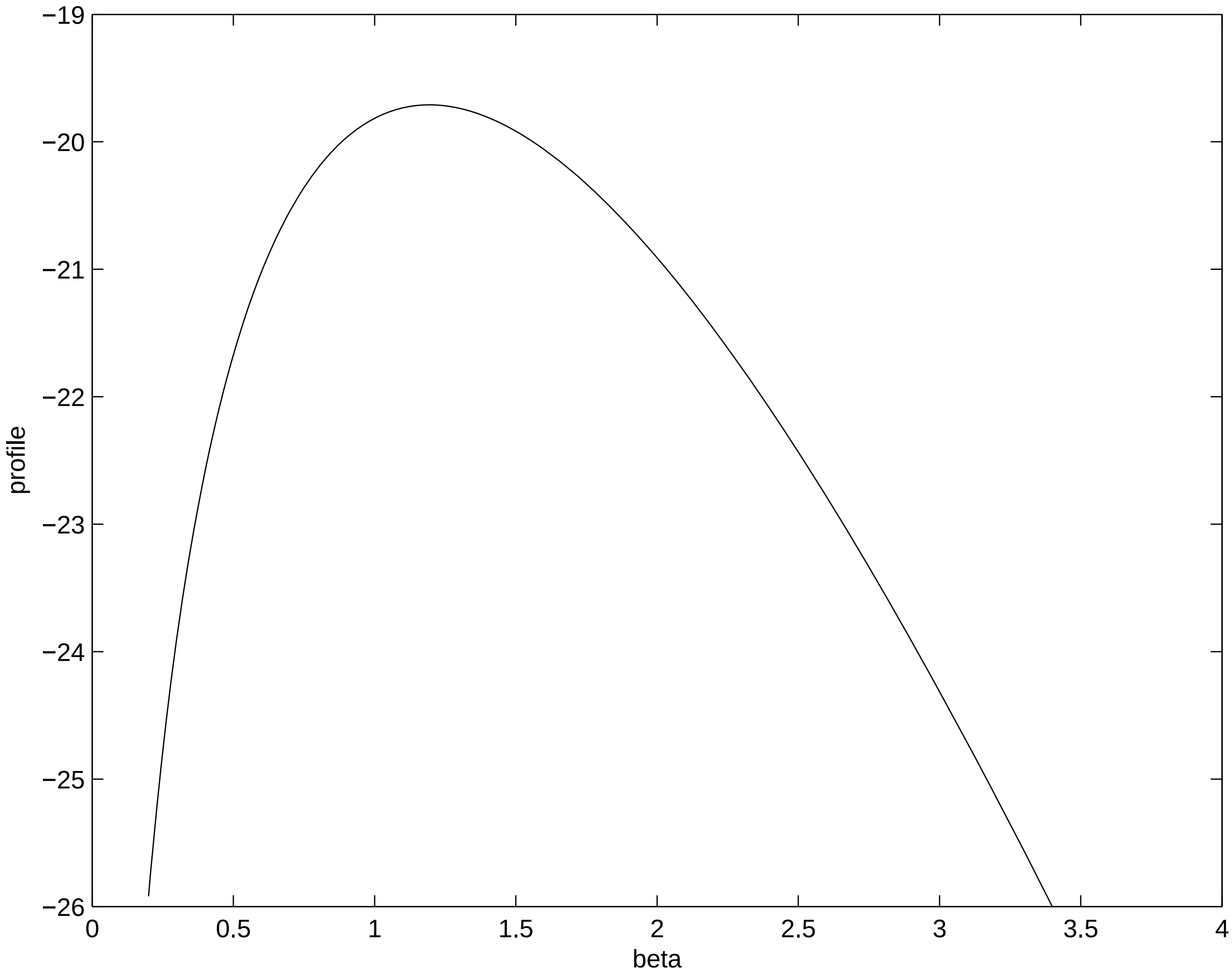
- If the number of intervals m increases and there are **exact** recurrences at $t_1 \leq \ldots \leq t_r$ (here $r = \sum_{j=1}^{m} d_j$, $t_0 \leq t_1$, $t_r \leq t_a$), then using a limiting argument it follows that the likelihood in terms of the density approximation is

$$L(\boldsymbol{\theta}) = \prod_{j=1}^{r} \nu(t_j; \boldsymbol{\theta}) \times \exp\left[-\mu(0, t_a; \boldsymbol{\theta})\right]$$

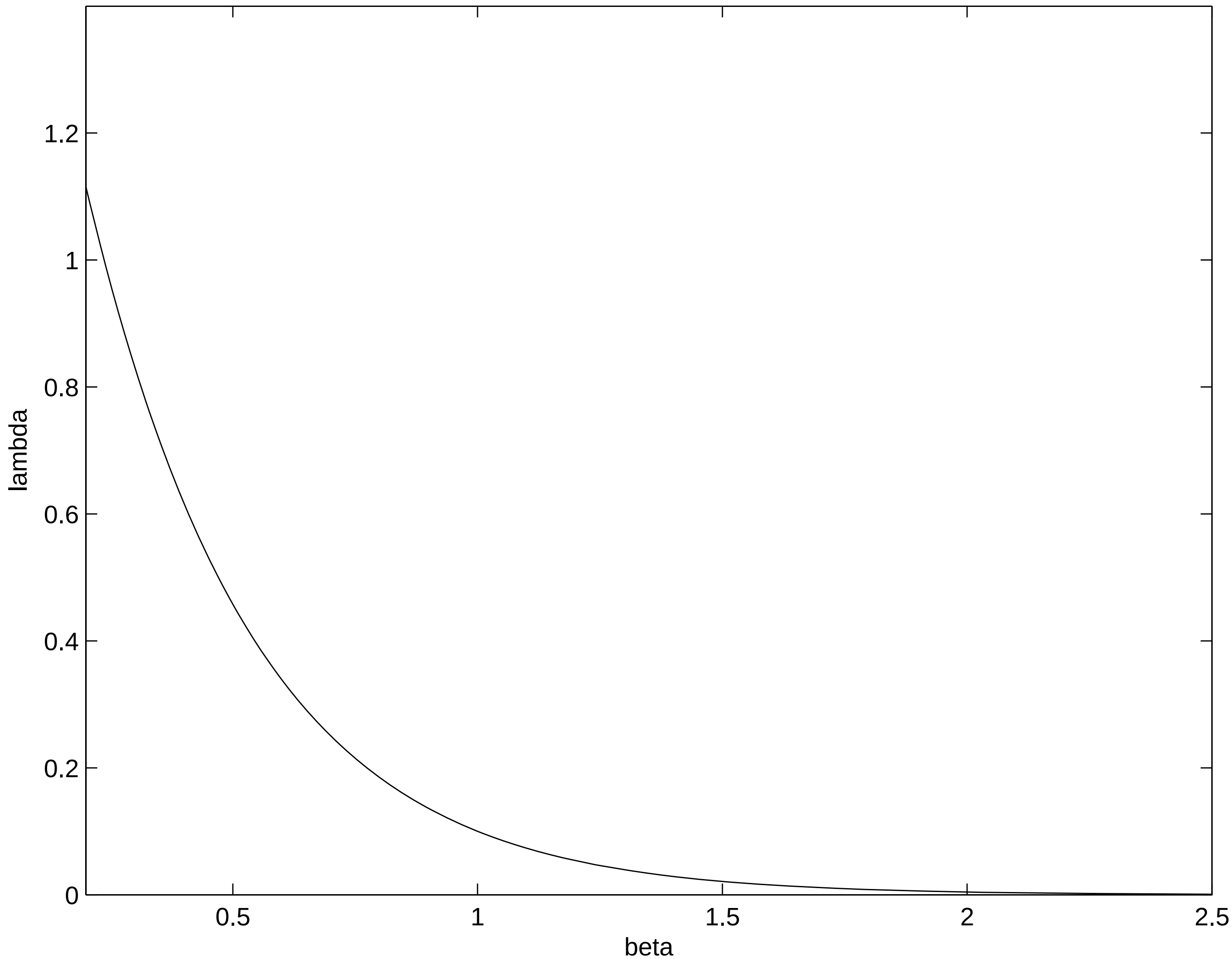
- For simplicity, above we assumed that the intervals are contiguous. Obvious changes to the formula above give the likelihood when there are gaps among the intervals.

- In both cases (the interval data or exact recurrences data) the same methods used in Chapters 7, 8 can be used to obtain the ML estimate $\widehat{\boldsymbol{\theta}}$ and confidence regions for $\boldsymbol{\theta}$ or functions of $\boldsymbol{\theta}$.

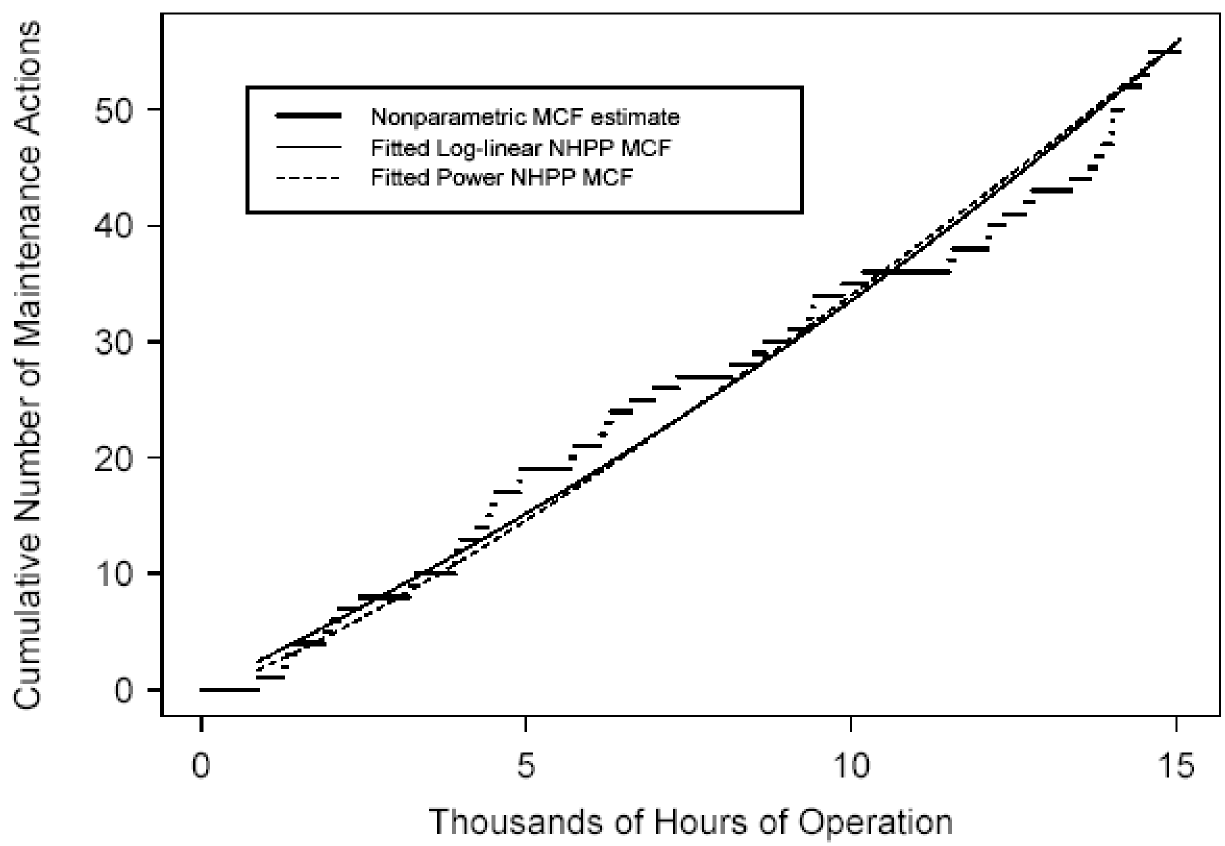
PROFILE LIKELIHOOD FOR BETA
("SIMPLE EXAMPLE")
$\widehat{\beta} = 1.20$, $\widehat{\lambda} = 0.0538$.



CONNECTION BETWEEN LAMBDA OG BETA
("SIMPLE EXAMPLE")
$\widehat{\beta} = 1.20,\ \widehat{\lambda} = 0.0538.$



Cumulative Number of Unscheduled Maintenance Actions Versus Operating Hours with Power and Loglinear NHPP Models for a USS Grampus Diesel Engine



Results of Fitting NHPP Models to the USS Grampus Diesel Engine Data

- Both models seem to fit the data very well.
- For the power recurrence rate model, $\widehat{\beta}$=1.22 and $\widehat{\eta}$ =0.553.
- For the loglinear recurrence rate model, $\widehat{\gamma}_0$=1.01 and $\widehat{\gamma}_1$ =.0377.
- Times between recurrences are consistent with a HPP:
  - ▶ the Lewis-Robinson test gave $Z_{\text{LR}} = 1.02$ with p-value $p = .21$.
  - ▶ the MIL-HDBk-189 test gave $X^2_{\text{MHB}} = 92$ with p-value $p = .08$.

Comparison of trend tests

main topic

Minitab provides five trend tests for data with multiple systems: MIL-hdbk-189 (TTT-based), MIL-hdbk-189 (Pooled), Laplace's (TTT-based), Laplace's (Pooled), and Anderson-Darling. The pooled Laplace and military handbook tests reduce to their respective TTT-based tests when there is only one system. These tests behave differently under the following two circumstances:

1 the data follow a non-monotonic trend

2 the data are from heterogeneous systems

Monotonic and non-monotonic trends

There is a trend in the pattern of times between failure if the times change in a systematic way. Trends can be:

- monotonic – times between failures are getting either consistently longer (decreasing trend) or consistently shorter (increasing trend)
- non-monotonic – times between failures alternate between increasing and decreasing trend (cyclic) or have a decreasing trend, no trend, and then increasing trend (bathtub)

The Anderson-Darling test will reject the null hypothesis in the presence of both monotonic and non-monotonic trends. The other tests will generally only detect monotonic trends. While the Anderson-Darling test is useful if you suspect the existence of a cyclic or other non-monotonic trend, the other tests are more powerful in the case of a monotonic trend.

Homogeneous and heterogeneous systems

The null hypothesis of no trend differs slightly for the different tests:

- The null hypothesis for the pooled tests (MIL-hdbk-189 and Laplace's) is that the data come from a homogeneous Poisson processes (HPP) with a possibly different MTBF for each system. Thus, rejecting the null hypothesis means that you can definitely conclude there is a trend in your data.
- The null hypothesis for the TTT-based tests (MIL-hdbk-189, Laplace's, and Anderson-Darling) is that the data come from a homogeneous Posson process (HPP) with the same MTBF for each system. Thus, rejecting the null hypothesis could mean that either there is a trend in your data or your data come from heterogeneous systems. Therefore, you should use TTT-based tests only when you are confident that your systems are homogeneous.

The table below summarizes the different null hypotheses associated with the trend tests.

	MIL-hdbk-189 (Pooled)	**MIL-hdbk-189 (TTT-based)**	**Laplace's (Pooled)**	**Laplace's (TTT-based)**	**Anderson-Darling**
Null Hypothesis	HPP (possibly different MTBFs)	HPP (equal MTBFs)	HPP (possibly different MTBFs)	HPP (equal MTBFs)	HPP (possibly different MTBFs)
Rejecting H_0 means...	monotonic trend	monotonic trend or systems are heterogeneous	monotonic trend	monotonic trend or systems are heterogeneous	monotonic trend or non-monotonic trend or systems are heterogeneous

See [12] for more information concerning these tests.

PII: S0951-8320(97)00099-9

Reliability Engineering and System Safety **60** (1998) 13–28

0951–8320/98/$19.00

TTT-based tests for trend in repairable systems data

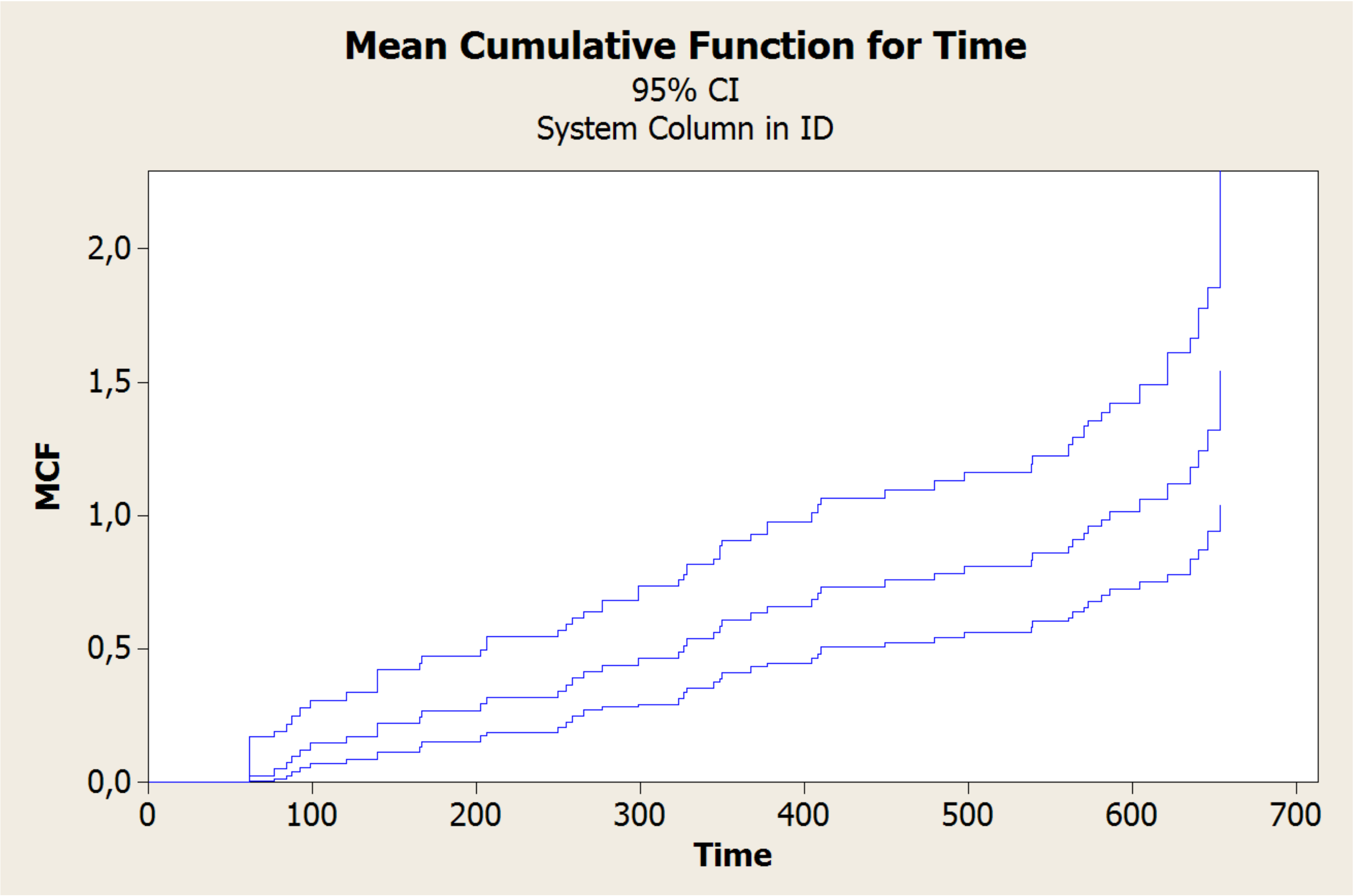
Jan Terje Kvaløy & Bo Henry Lindqvist

Department of Mathematical Sciences, Norwegian University of Science and Technology, N-7034 Trondheim, Norway

(Received 25 September 1996; revised 24 January 1997; accepted 15 July 1997)

A major aspect of analysis of failure data for repairable systems is the testing for a possible trend in interfailure times. This paper reviews some important and popular graphical methods and tests for the nonhomogeneous Poisson process model. In particular, the total time on test (TTT) plot is considered, and trend tests based on the TTT-statistic are motivated and derived. In particular, a test based on the Anderson–Darling statistic is suggested. The tests are evaluated and compared in a simulation study, both with respect to the achievement of correct significance level and rejection power. The considered alternatives to 'no trend' are the log-linear, power law and a class of bathtub-shaped intensity functions. The simulation study involves single systems, as well as the case where several independent systems of the same kind are observed. © 1998 Elsevier Science Limited.

Valveseat Data



Valveseat Data

Trend Tests

	MIL-Hdbk-189		Laplace's		
	TTT-based	Pooled	TTT-based	Pooled	Anderson-Darling
Test Statistic	80,28	66,15	0,46	2,38	0,80
P-Value	0,249	0,017	0,645	0,017	0,478
DF	96	96			

TTT-analysis Simple Example

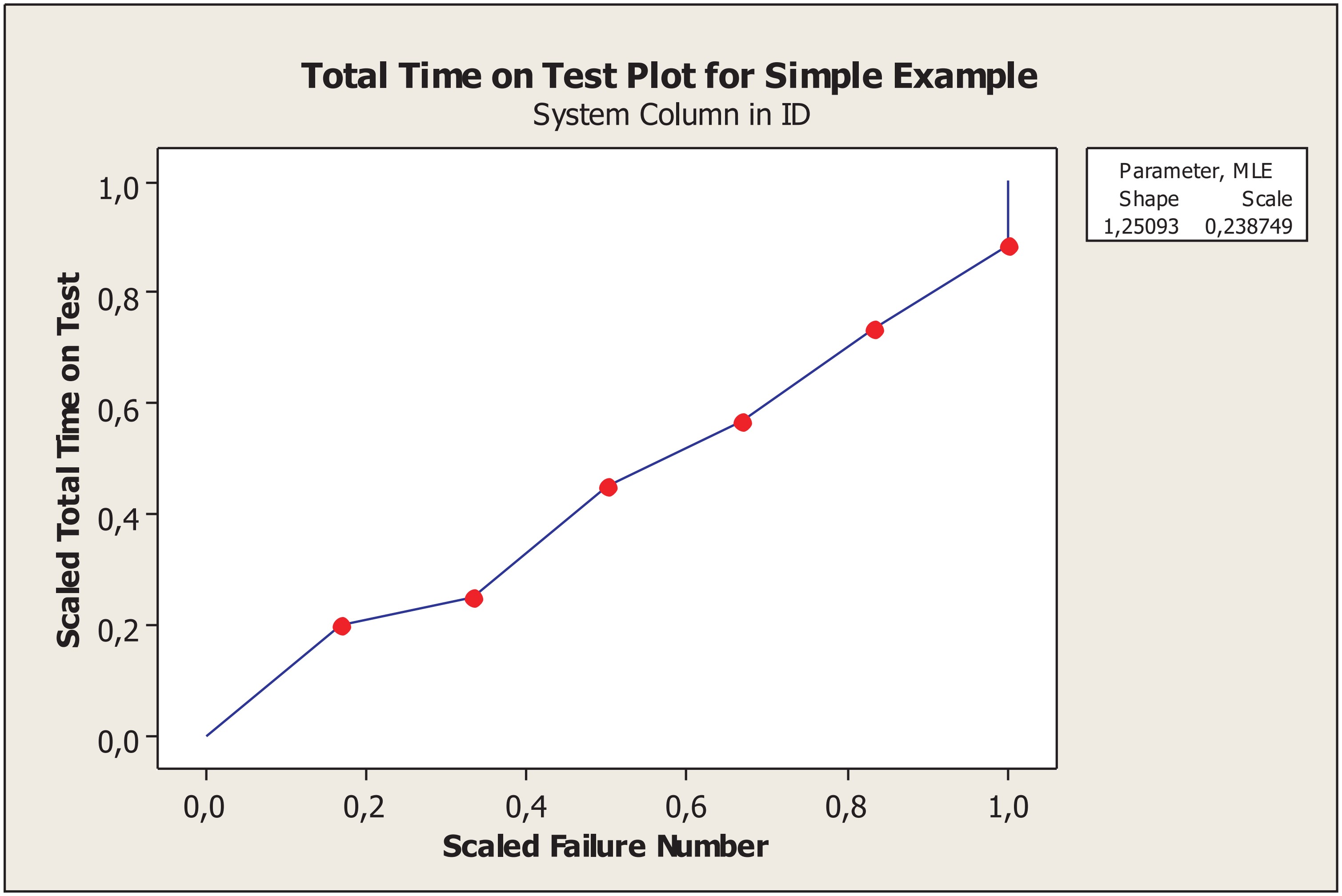
Row	STTT	ID	Scaled
1	12	1	0,20000
2	15	1	0,25000
3	27	1	0,45000
4	34	1	0,56667
5	44	1	0,73333
6	53	1	0,88333
7	60	1	1,00000

Parameter Estimates

		Standard	95% Normal CI	
Parameter	Estimate	Error	Lower	Upper
Shape	1,25093	0,511	0,249996	2,25186
Scale	0,238749	0,160	-0,0746105	0,552109

Trend Tests

	MIL-Hdbk-189	Laplace's	Anderson-Darling
Test Statistic	9,59	0,12	0,24
P-Value	0,697	0,906	0,977
DF	12		


Total Time on Test Plot for Simple Example
System Column in ID
Parameter, MLE
Shape Scale
1,25093 0,238749
Scaled Total Time on Test
1,0
0,8
0,6
0,4
0,2
0,0
0,0
0,2
0,4
0,6
0,8
1,0
Scaled Failure Number

TTT-analysis of Valve Seat Data

Parametric Growth Curve: C1

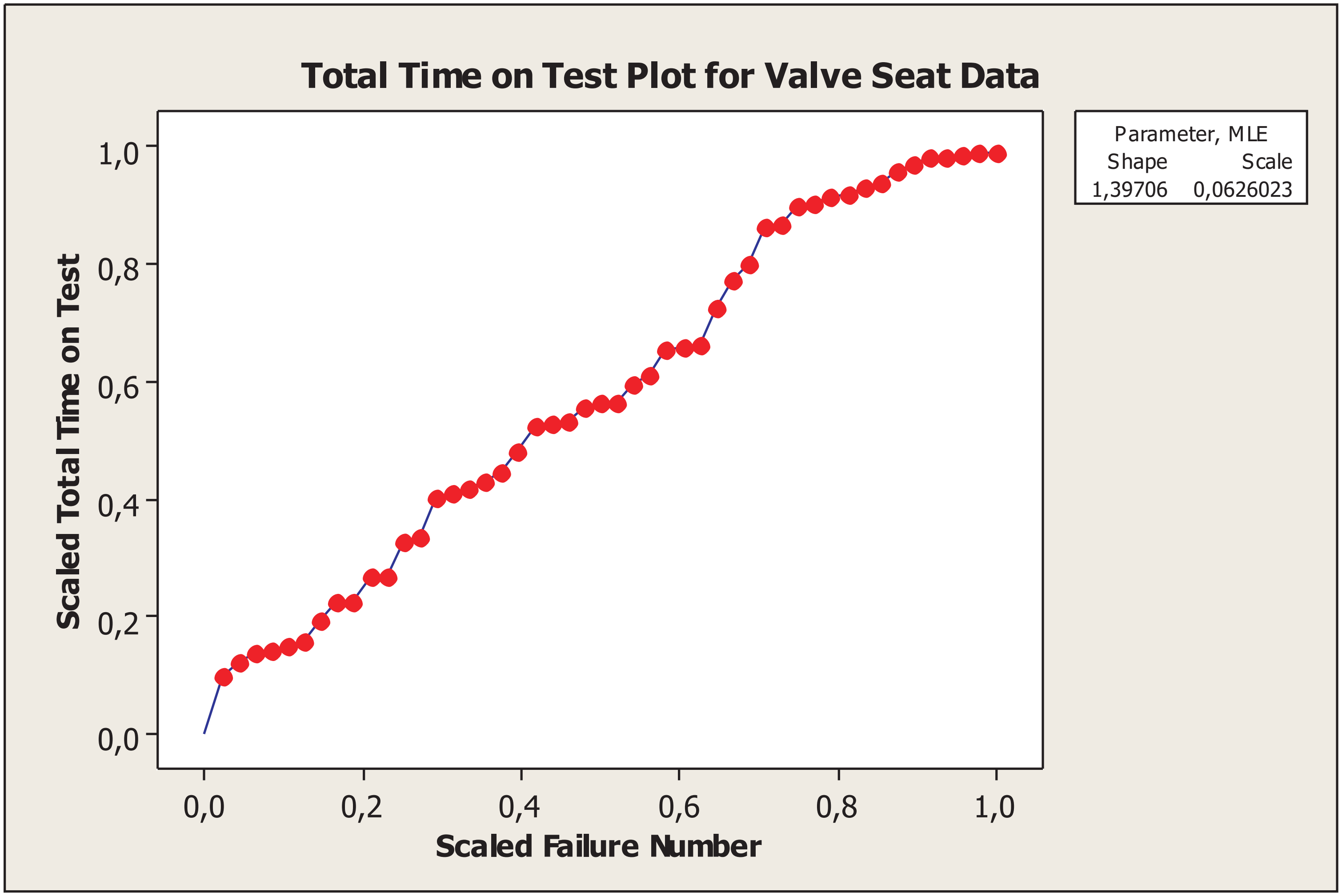
Model: Power-Law Process
Estimation Method: Maximum Likelihood

Parameter Estimates

		Standard	95% Normal CI	
Parameter	Estimate	Error	Lower	Upper
Shape	1,39706	0,202	1,00184	1,79229
Scale	0,0626023	0,026	0,0119179	0,113287

Trend Tests

	MIL-Hdbk-189	Laplace's	Anderson-Darling
Test Statistic	68,72	2,03	3,17
P-Value	0,032	0,043	0,022
DF	96		


Total Time on Test Plot for Valve Seat Data
Parameter, MLE
Shape Scale
1,39706 0,0626023
Scaled Total Time on Test
1,0
0,8
0,6
0,4
0,2
0,0
0,0
0,2
0,4
0,6
0,8
1,0
Scaled Failure Number