

Censored and Truncated Data

- An observation is right censored at y :
Unit is in our data, we know $T > y$.
Contribution to L : $P(T > y) = R(y)$.
- An observation is left censored at y :
Unit is in our data, we know $T < y$.
Contribution to L : $P(T < y) = F(y)$.
- An observation is right truncated at y :
Unit is in our data only if $T \leq y$. We do not know about the units with $T > y$.
Contribution to L of observed failure at t :
 $\Delta^{-1}P(t \leq T \leq t + \Delta | T \leq y) \approx f(t)/F(y)$.
- An observation is left truncated at y :
Unit is in our data only if $T \geq y$. We do not know about the units with $T < y$.
Contribution to L of observed failure at t :
 $\Delta^{-1}P(t \leq T \leq t + \Delta | T \geq y) \approx f(t)/R(y)$.

Examples of left truncation:

- Ultrasonic inspection of material. Signal amplitude only trusted when above limit τ . Condition for being in the data set is $T > \tau$.
- Life data with pretest screening. Electronic component is burn-in tested for 1000 hours. Only the ones that passed this test are observed later. The number of components failing at burn-in is unknown. Condition for being in the data set is $T > 1000$.

Example of right truncation:

- Casting for automobile engine mounts. Pore size distribution below 10 microns only are recorded (other units are immediately discarded). Condition for being in the data set is $T < 10$ microns.
- Study group of individuals with AIDS diagnosis before July 1, 1986, and known date of HIV-infection (due to blood-transfusion). Let $T_i =$ time from HIV-infection to AIDS diagnosis for i th individual. Then condition for being in the data set is that $T_i \leq v_i$ where v_i is time from HIV-infection of the i th individual to July 1, 1986. (Kalbfleisch and Lawless, 1989)

COMPUTER PROGRAM EXECUTION TIME vs SYSTEM LOAD

Data: 17 observations of (T,x)

- Time to complete a computationally intensive task.
- Information from the Unix uptime command
- Predictions needed for scheduling subsequent steps in a multi-step computational process.

Seconds (T)	Load (x)	Seconds (T)	Load (x)
123	2,74	110	,60
704	5,47	213	2,10
184	2,13	284	3,10
113	1,00	317	5,86
94	,32	142	1,18
76	,31	127	,57
78	,51	96	1,10
98	,29	111	1,89
240	,96		

Covariates (explanatory variables) for failure times

Useful covariates explain/predict why some units fail quickly and some units survive a long time:

- Continuous variables like stress, temperature, voltage, and pressure.
- Discrete variables like number of hardening treatments or number of simultaneous users of a system.
- Categorical variables like manufacturer, design, and location.

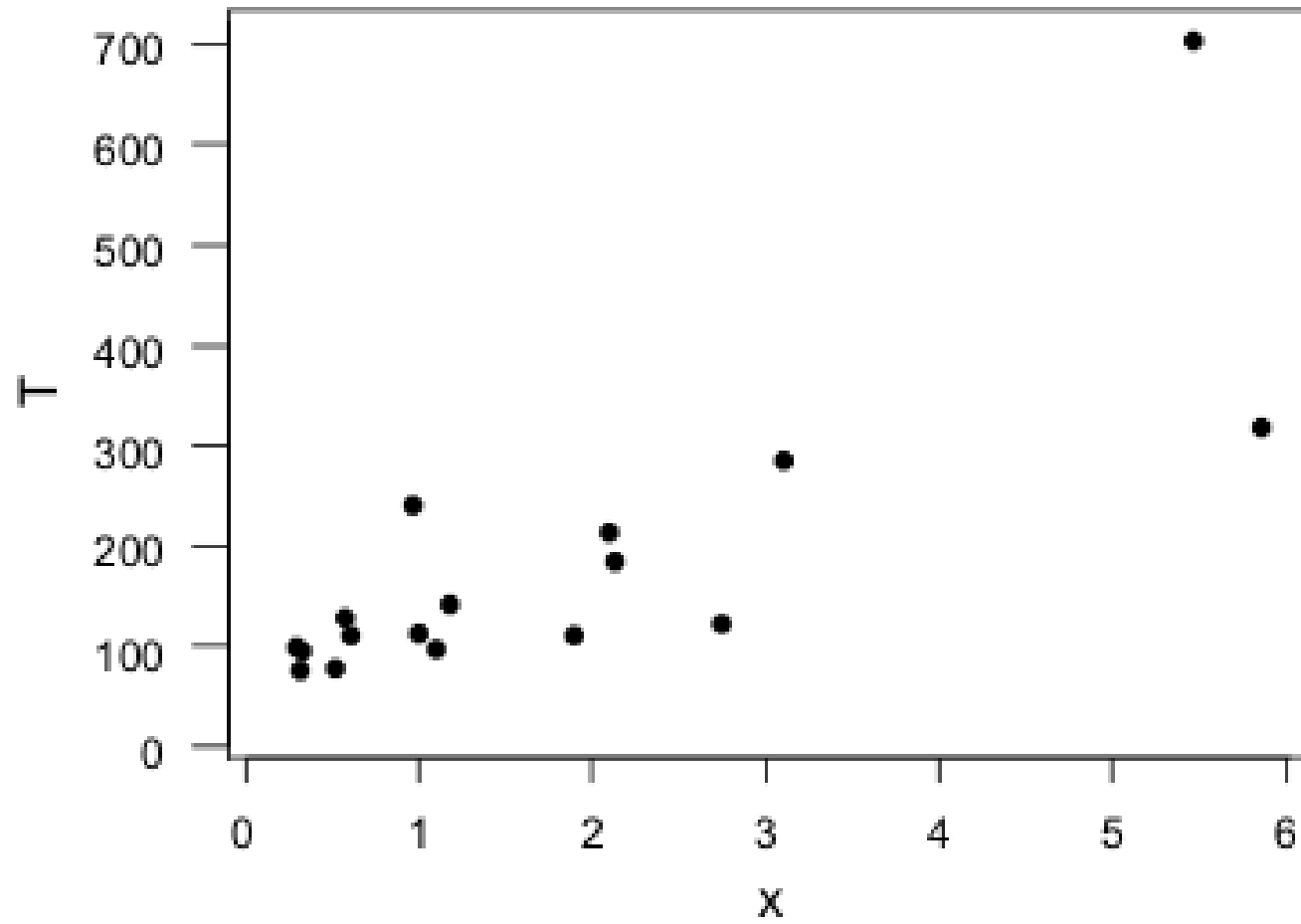
Regression model relates failure time distribution to covariates $x = (x_1, \dots, x_k)$:

$$P(T \leq t) = F(t) = F(t; x)$$

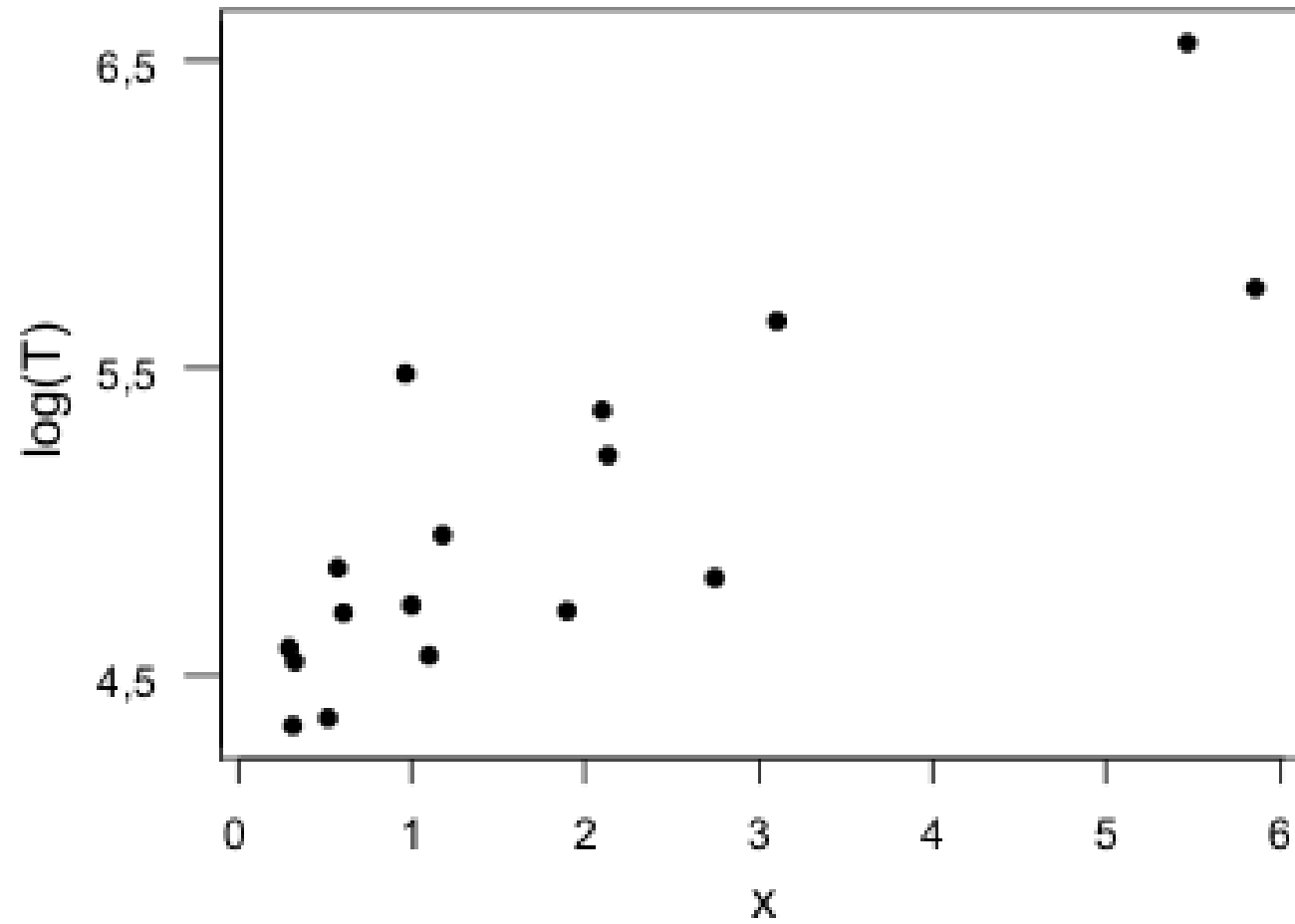
Why regression models?

- Want to find factors which explain the reliability of an item
- Want to exclude factors which do not influence the reliability
- Obtain new knowledge about failure mechanisms
- Make better predictions for reliability of an item

Computer data



Computer data



The screenshot shows the MINITAB software interface. The 'Stat' menu is open, and the path 'Reliability/Survival' > 'Regression with Life Data...' is selected. The main window displays the following text:

```

Session
----- 01.03
Welcome to Minitab
Saving file as: C:\Levetidsanalyse\Minitabplot\C11.MTW

Results for: C11.MTW

Plot C1 * C2

Plot T * x

MTB > let c3=log(c1)
MTB > Plot c3*c2;
SUBC> Symbol;
SUBC> ScFrame;
SUBC> ScAnnotation.

Plot log(T) * x

MTB >

```

The data table at the bottom of the window is as follows:

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15	C16	C17
	T	x	log(T)														
1	123	2,74	4,81218														
2	704	5,47	6,55678														
3	184	2,13	5,21494														
4	113	1,00	4,72739														
5	94	0,32	4,54329														
6	76	0,31	4,33073														
7	78	0,51	4,35671														
8	98	0,29	4,58497														
9	240	0,96	5,48064														
10	110	0,60	4,70048														
11	213	2,10	5,36129														
12	284	3,10	5,64897														

The Windows taskbar at the bottom shows the Start button, several open applications including Internet Explorer, Folder, WinEdt 5.3, Yap 0.99a, Corel PHOTO-PAIN, and MINITAB - Untit..., and the system clock showing 20:35.

MINITAB - Untitled

File Edit Manip Calc Stat Graph Editor Window Help

Session

01.03.2003 20:16:11

Welcome to Minitab, press F1 for help.
Saving file as: C:\Documents and Settings\Bo Lindqvist\My Documents\Jobb\Fag\Levetidsanalyse\Minitabplot\C11.MTW

Results for: C11.MTW

Plot C1 * C2

Plot T * x

```

MTB > let c3=log(c1)
MTB > Plot c3*c2;
SUBC> Symbol;
SUBC> ScFrame;
SUBC> ScAnnotation.

```

Plot log(T) * x

```

MTB >

```

Regression with Life Data

C1 T
C2 x
C3 log(T)

Responses are uncens/right censored data
 Responses are uncens/arbitrarily censored data

Variables/
Start variables: c1
End variables:

Freq. columns:
(optional)

Model:
c2

Factors (optional):

Assumed distribution: Lognormal base e

Select Help Censor... Estimate... Graphs... Results... Options... Storage... OK Cancel

	C1	C2	C3
	T	x	log(T)
1	123	2,74	4,81218
2	704	5,47	6,55678
3	184	2,13	5,21494
4	113	1,00	4,72739
5	94	0,32	4,54329
6	76	0,31	4,33073
7	78	0,51	4,35671
8	98	0,29	4,58497
9	240	0,96	5,48064
10	110	0,60	4,70048
11	213	2,10	5,36129
12	284	3,10	5,64897

Projec... 20:39

Welcome to Minitab, press F1 for help.

Start 3 Internet Expl... Foler WinEdt 5.3 - [C:\... Yap 0.99a - [Forel... Corel PHOTO-PAIN... MINITAB - Untit... 20:39

Regression with Life Data: T versus x

Response Variable: T

Censoring Information Count
Uncensored value 17

Estimation Method: Maximum Likelihood
Distribution: Lognormal base e

Regression Table

Predictor	Coef	Standard Error	Z	P	95,0% Normal CI	
					Lower	Upper
Intercept	4,4936	0,1112	40,39	0,000	4,2756	4,7116
x	0,29075	0,04595	6,33	0,000	0,20069	0,38080
Scale	0,31247	0,05359			0,22327	0,43730

Log-Likelihood = -89,498

Anderson-Darling (adjusted) Goodness-of-Fit

Standardized Residuals = 0,8356; Cox-Snell Residuals = 0,8170

Regression with Life Data: C1 versus C2

Response Variable: C1

Censoring Information	Count
Uncensored value	17

Estimation Method: Maximum Likelihood
Distribution: Weibull

Regression Table

Predictor	Coef	Standard Error	Z	P	95,0% Normal CI	
					Lower	Upper
Intercept	4,6182	0,1219	37,88	0,000	4,3792	4,8572
C2	0,31118	0,04939	6,30	0,000	0,21437	0,40799
Shape	3,0604	0,5245			2,1873	4,2820

Log-Likelihood = -91,504

Anderson-Darling (adjusted) Goodness-of-Fit

Likelihood for Lognormal Distribution Simple Regression Model with Right Censored Data

The likelihood for n independent observations has the form

$$\begin{aligned} L(\beta_0, \beta_1, \sigma) &= \prod_{i=1}^n L_i(\beta_0, \beta_1, \sigma; \text{data}_i) \\ &= \prod_{i=1}^n \left\{ \frac{1}{\sigma t_i} \phi_{\text{nor}} \left[\frac{\log(t_i) - \mu_i}{\sigma} \right] \right\}^{\delta_i} \left\{ 1 - \Phi_{\text{nor}} \left[\frac{\log(t_i) - \mu_i}{\sigma} \right] \right\}^{1-\delta_i} \end{aligned}$$

where $\text{data}_i = (x_i, t_i, \delta_i)$, $\mu_i = \beta_0 + \beta_1 x_i$,

$$\delta_i = \begin{cases} 1 & \text{exact observation} \\ 0 & \text{right censored observation} \end{cases}$$

$\phi_{\text{nor}}(z)$ is the standardized normal pdf and $\Phi_{\text{nor}}(z)$ is the corresponding normal cdf.

The parameters are $\theta = (\beta_0, \beta_1, \sigma)$.

Estimated Parameter Variance-Covariance Matrix

Local (observed information) estimate

$$\begin{aligned}\hat{\Sigma}_{\hat{\theta}} &= \begin{bmatrix} \widehat{\text{Var}}(\hat{\beta}_0) & \widehat{\text{Cov}}(\hat{\beta}_0, \hat{\beta}_1) & \widehat{\text{Cov}}(\hat{\beta}_0, \hat{\sigma}) \\ \widehat{\text{Cov}}(\hat{\beta}_1, \hat{\beta}_0) & \widehat{\text{Var}}(\hat{\beta}_1) & \widehat{\text{Cov}}(\hat{\beta}_1, \hat{\sigma}) \\ \widehat{\text{Cov}}(\hat{\sigma}, \hat{\beta}_0) & \widehat{\text{Cov}}(\hat{\sigma}, \hat{\beta}_1) & \widehat{\text{Var}}(\hat{\sigma}) \end{bmatrix} \\ &= \begin{bmatrix} -\frac{\partial^2 \mathcal{L}(\beta_0, \beta_1, \sigma)}{\partial \beta_0^2} & -\frac{\partial^2 \mathcal{L}(\beta_0, \beta_1, \sigma)}{\partial \beta_0 \partial \beta_1} & -\frac{\partial^2 \mathcal{L}(\beta_0, \beta_1, \sigma)}{\partial \beta_0 \partial \sigma} \\ -\frac{\partial^2 \mathcal{L}(\beta_0, \beta_1, \sigma)}{\partial \beta_1 \partial \beta_0} & -\frac{\partial^2 \mathcal{L}(\beta_0, \beta_1, \sigma)}{\partial \beta_1^2} & -\frac{\partial^2 \mathcal{L}(\beta_0, \beta_1, \sigma)}{\partial \beta_1 \partial \sigma} \\ -\frac{\partial^2 \mathcal{L}(\beta_0, \beta_1, \sigma)}{\partial \sigma \partial \beta_0} & -\frac{\partial^2 \mathcal{L}(\beta_0, \beta_1, \sigma)}{\partial \sigma \partial \beta_1} & -\frac{\partial^2 \mathcal{L}(\beta_0, \beta_1, \sigma)}{\partial \sigma^2} \end{bmatrix}^{-1}\end{aligned}$$

Partial derivatives are evaluated at $\hat{\beta}_0, \hat{\beta}_1, \hat{\sigma}$.

Standard Errors and Confidence Intervals for Parameters

- Lognormal ML estimates for the computer time experiment were $\hat{\theta} = (\hat{\beta}_0, \hat{\beta}_1, \hat{\sigma}) = (4.49, .290, .312)$ and an estimate of the variance-covariance matrix for $\hat{\theta}$ is

$$\hat{\Sigma}_{\hat{\theta}} = \begin{bmatrix} .012 & -.0037 & 0 \\ -.0037 & .0021 & 0 \\ 0 & 0 & .0029 \end{bmatrix}.$$

- Normal-approximation confidence interval for the computer execution time regression slope is

$$[\tilde{\beta}_1, \tilde{\beta}_1] = \hat{\beta}_1 \pm z_{(.975)} \widehat{se}_{\hat{\beta}_1} = .290 \pm 1.96(.046) = [.20, .38]$$

where $\widehat{se}_{\hat{\beta}_1} = \sqrt{.0021} = .046$.

Probability Plot for SResids Comp Data Weibull

Smallest Extreme Value - 95% CI

Complete Data - ML Estimates

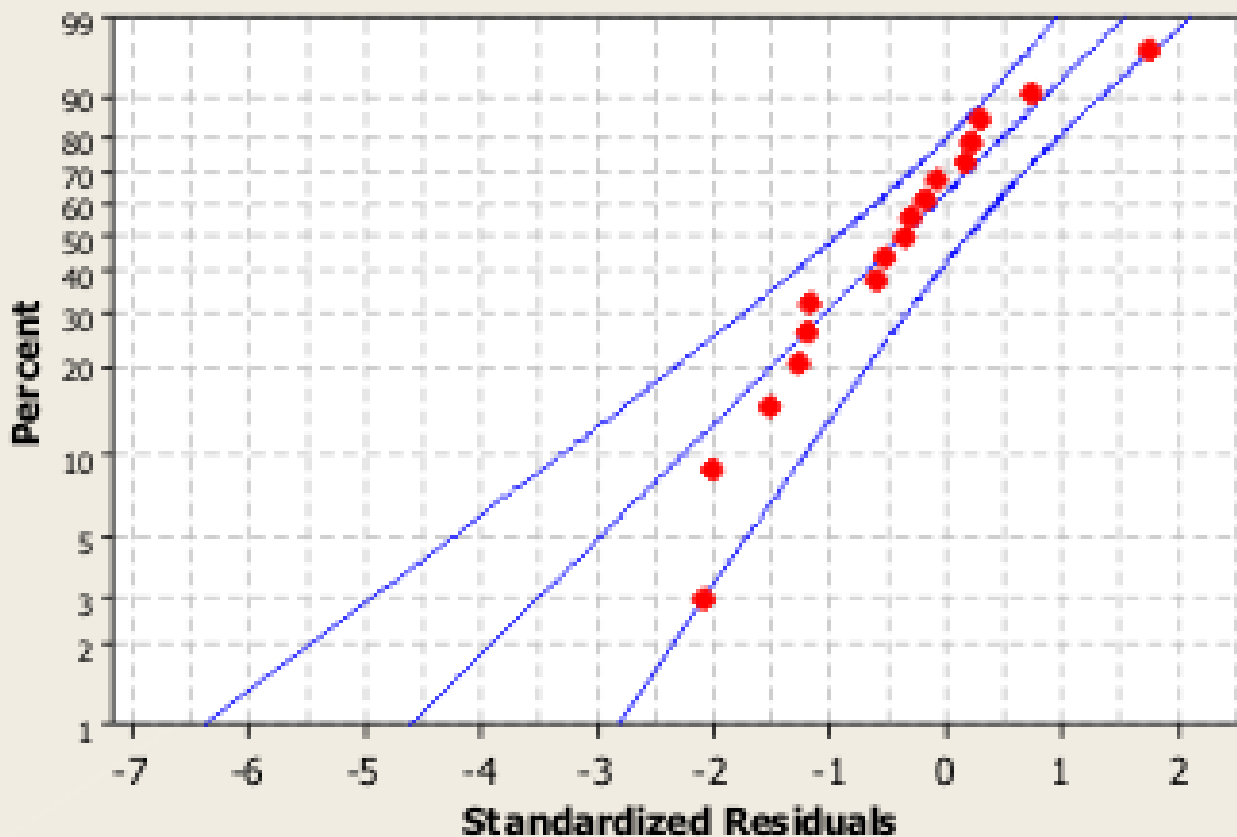


Table of Statistics	
Loc	0,000000
Scale	1,00000
Mean	-0,577216
StDev	1,28255
Median	-0,366513
IQR	1,57253
Failure	17
Censor	0
AD*	0,894

Probability Plot for CSResids Comp Data Weibull

Exponential - 95% CI

Complete Data - ML Estimates

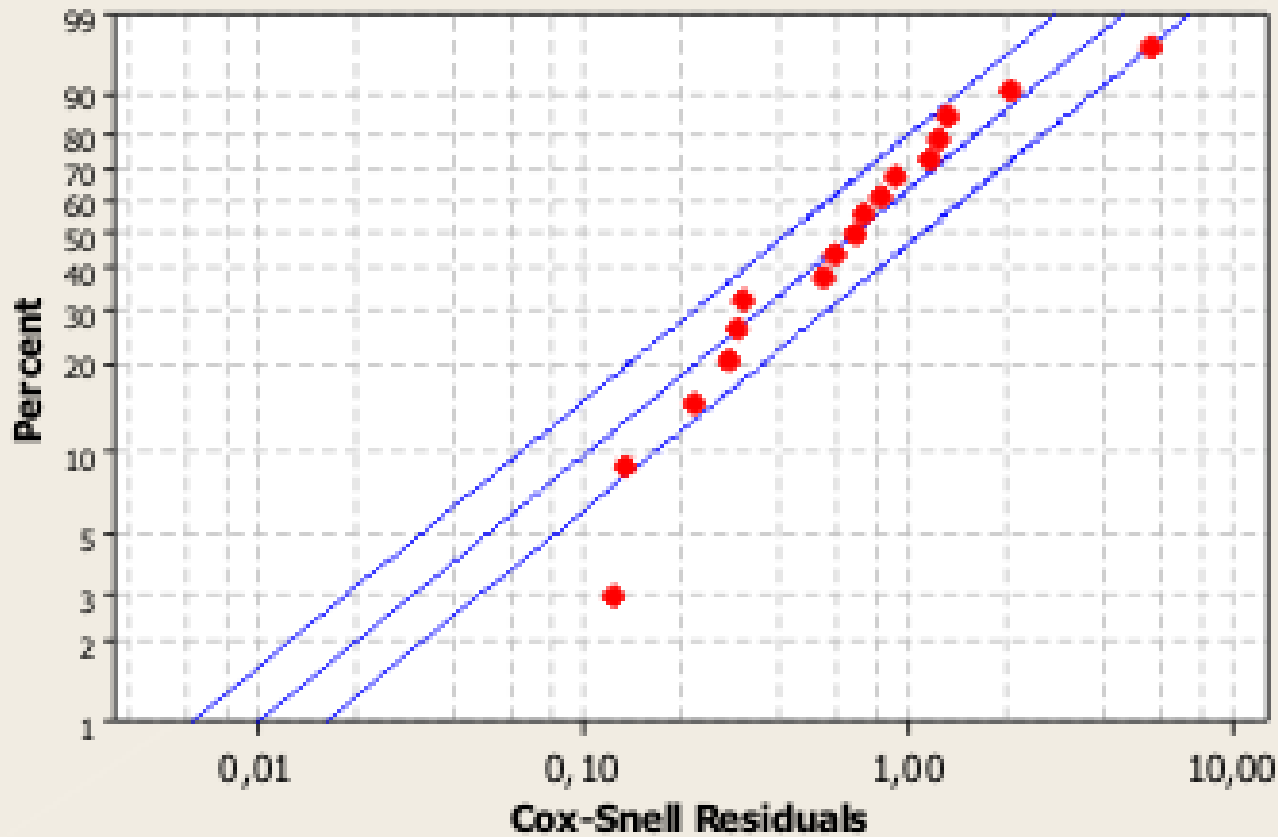


Table of Statistics	
Mean	1,00000
Std ev	1,00000
Median	0,693147
IQR	1,09861
Failure	17
Censor	0
AD*	0,894

Ordinary residuals

$$y_i - x'_i \hat{\beta}$$

where

y_i is the i th response value

x'_i is the vector of predictor values associated with the i th response value

$\hat{\beta}$ represents the estimated regression coefficients

Standardized residuals

$$\frac{y_i - x'_i \hat{\beta}}{\hat{\sigma}}$$

where $\hat{\sigma}$ is the estimated scale parameter.

Cox-Snell residuals

$$-\ln(\hat{R}(y_i))$$

where

$\hat{R}(y_i)$ is the estimated survival (reliability) probability for the response value y_i

$\ln(x)$ is the natural log of x

Probability Plot for SResids Comp Data Lognormal

Normal - 95% CI

Complete Data - ML Estimates

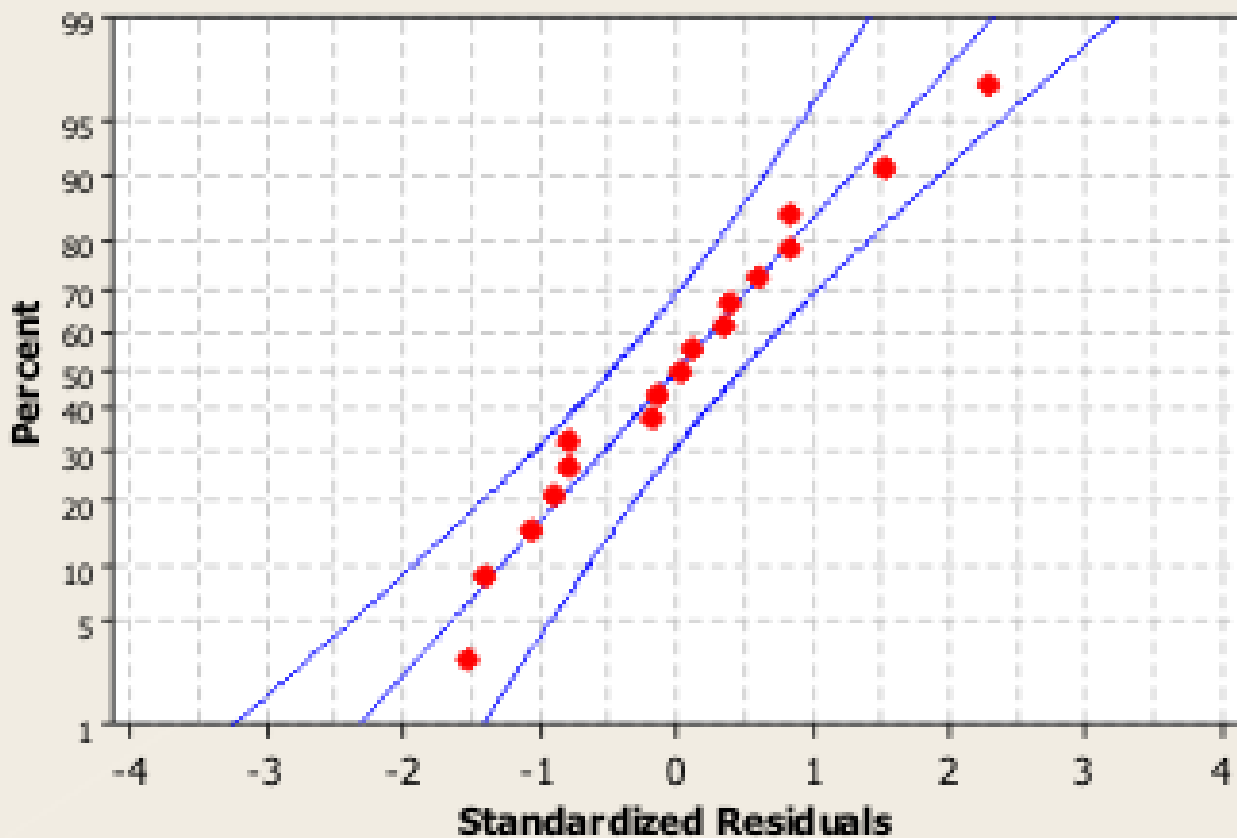


Table of Statistics	
Mean	-0,000000
StDev	1,00000
Median	-0,000000
IQR	1,34898
Failure	17
Censor	0
AD*	0,637

Probability Plot for CSR esids Comp Data Lognormal

Exponential - 95% CI

Complete Data - ML Estimates

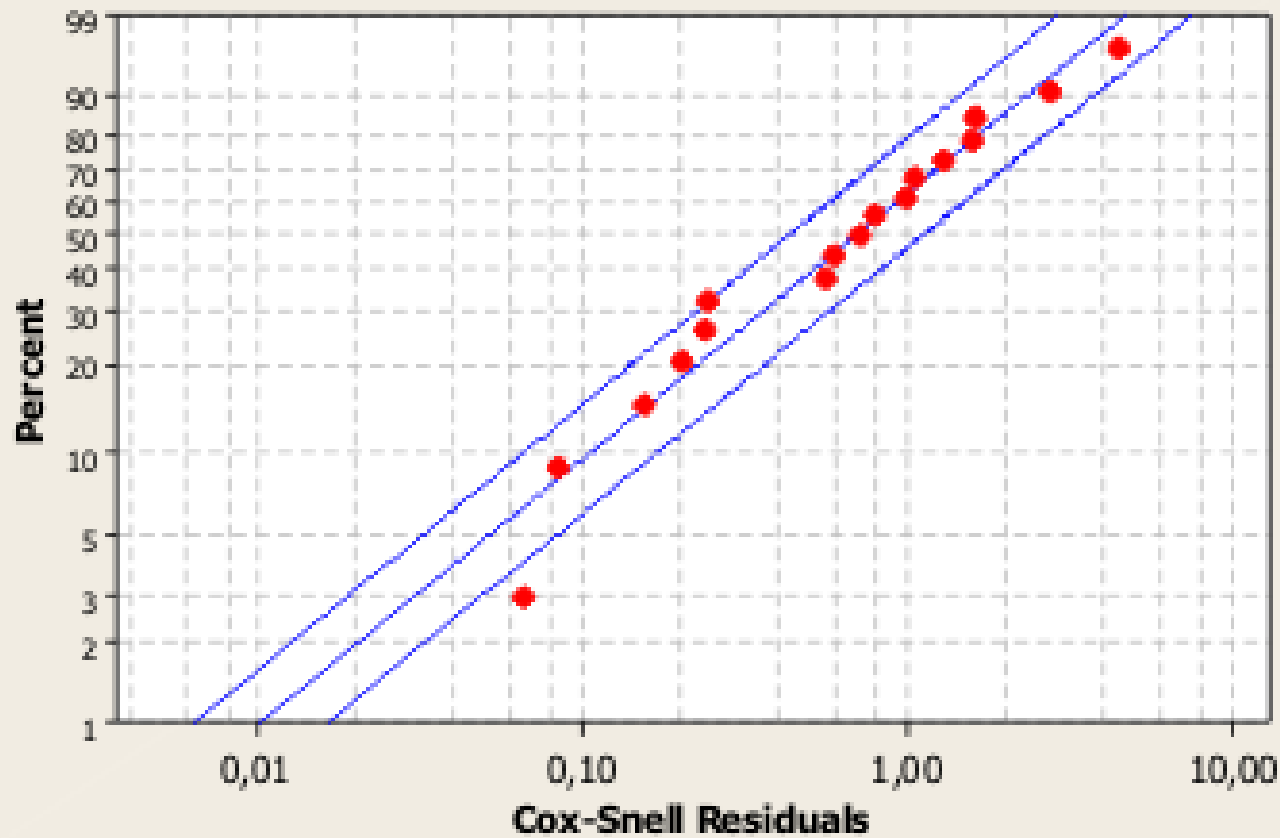
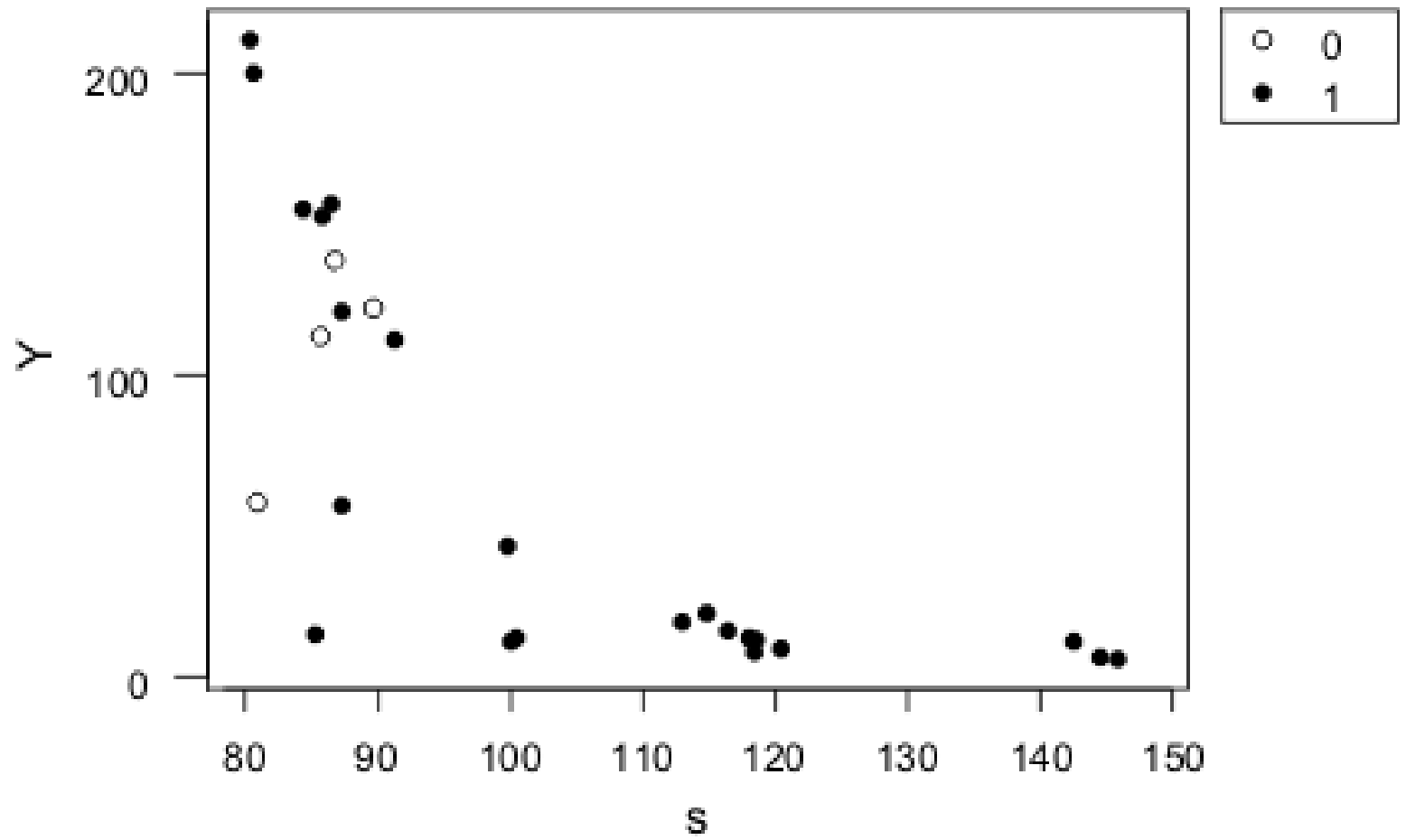


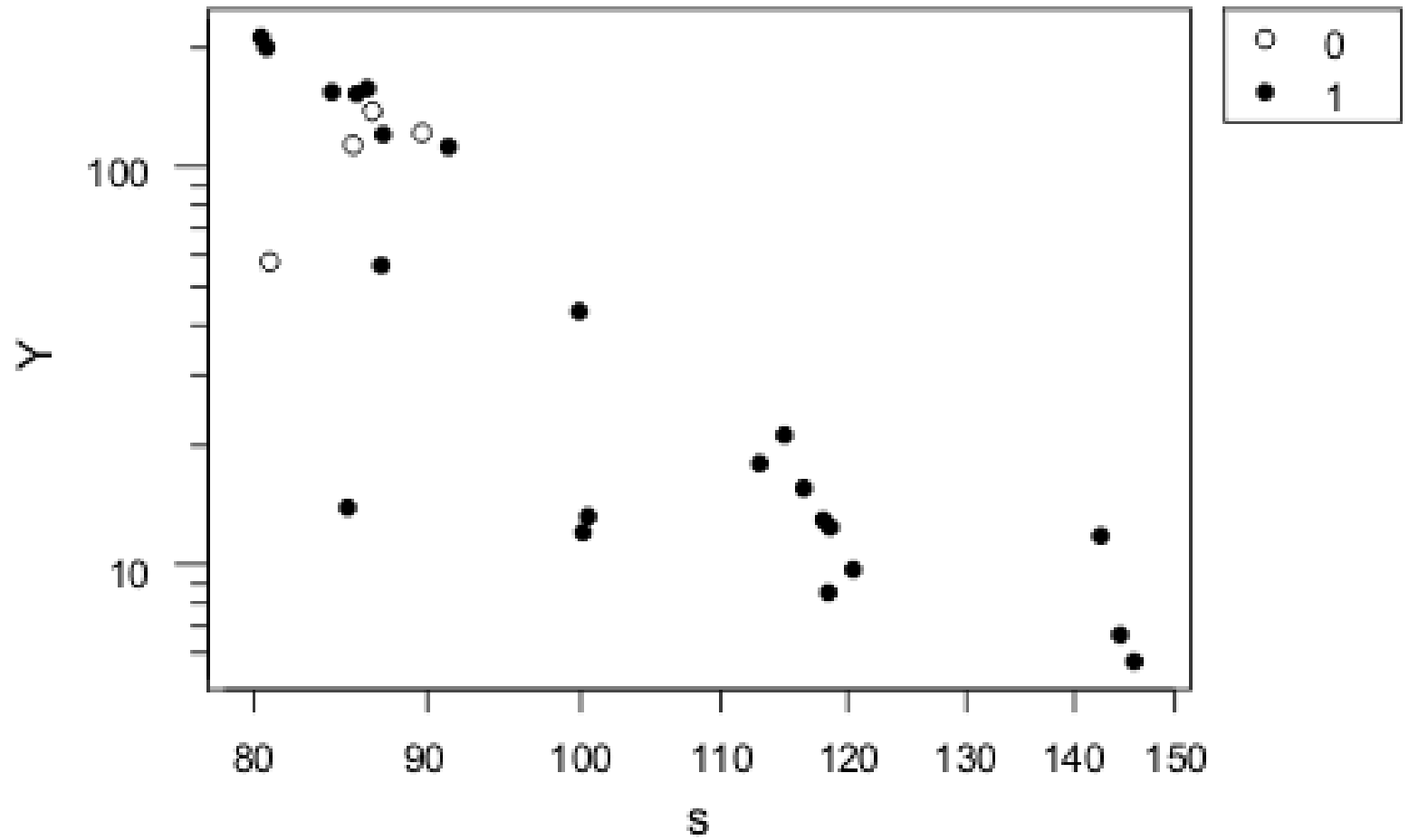
Table of Statistics	
Mean	1,01537
Std ev	1,01537
Median	0,703802
IQR	1,11550
Failure	17
Censor	0
AD*	0,619

Row	Pseudo- stress	k-Cycles	Status (1=failed, 0=censored)	
i	s	Y	C	
1	80,3	211,629	1	DATA DESCRIPTION: Low-Cycle Fatigue Life of Nickel-Base Superalloy Specimens (in units of thousands of cycles to failure).
2	80,6	200,027	1	
3	80,8	57,923	0	
4	84,3	155,000	1	
5	85,2	13,949	1	Data from Nelson (1990):
6	85,6	112,968	0	
7	85,8	152,680	1	
8	86,4	156,725	1	SUPER ALLOY DATA
9	86,7	138,114	0	
10	87,2	56,723	1	
11	87,3	121,075	1	
12	89,7	122,372	0	
13	91,3	112,002	1	
14	99,8	43,331	1	
15	100,1	12,076	1	
16	100,5	13,181	1	
17	113,0	18,067	1	
18	114,8	21,300	1	
19	116,4	15,616	1	
20	118,0	13,030	1	
21	118,4	8,489	1	
22	118,6	12,434	1	
23	120,4	9,750	1	
24	142,5	11,865	1	
25	144,5	6,705	1	
26	145,9	5,733	1	

Plot of Y vs s



Plot of $\log(Y)$ vs $\log(s)$



Regression with Life Data: Y versus x

Response Variable: Y

Censoring Information	Count
Uncensored value	22
Right censored value	4
Censoring value: C = 0	

Estimation Method: Maximum Likelihood
Distribution: Weibull

Regression Table

Predictor	Coef	Standard Error	Z	P	95,0% Normal CI	
					Lower	Upper
Intercept	31,432	2,008	15,65	0,000	27,496	35,368
x	-5,9600	0,4329	-13,77	0,000	-6,8085	-5,1116
Shape	2,2105	0,3894			1,5651	3,1221

Log-Likelihood = -97,155

Anderson-Darling (adjusted) Goodness-of-Fit

Standardized Residuals = 1,0768; Cox-Snell Residuals = 1,0768

Regression with Life Data: Y versus x

Response Variable: Y

Censoring Information	Count
Uncensored value	22
Right censored value	4

Censoring value: C = 0

Estimation Method: Maximum Likelihood
Distribution: Weibull

Regression Table

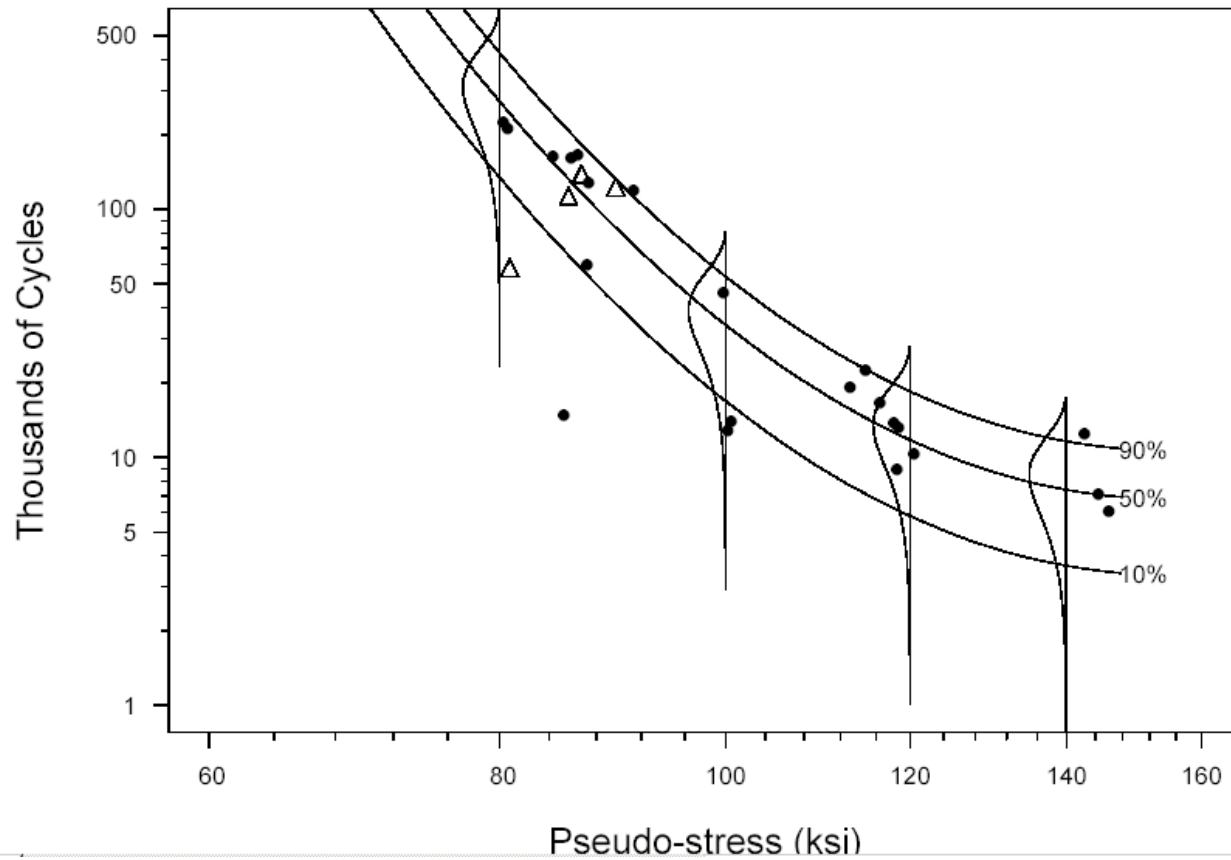
Predictor	Coef	Standard Error	Z	P	95,0% Normal CI	
					Lower	Upper
Intercept	217,61	62,13	3,50	0,000	95,83	339,39
x	-85,52	26,55	-3,22	0,001	-137,55	-33,49
x*x	8,483	2,831	3,00	0,003	2,934	14,032
Shape	2,6685	0,4777			1,8789	3,7900

Log-Likelihood = -93,382

Anderson-Darling (adjusted) Goodness-of-Fit

Standardized Residuals = 0,9283; Cox-Snell Residuals = 0,9283

**Log-Quadratic Weibull Regression Model with
Constant ($\beta = 1/\sigma$) Fit to the Fatigue Data**
 $\log[\hat{t}_p(x)] = \hat{\mu}(x) + \Phi_{sev}^{-1}(p)\hat{\sigma}, x = \log(\text{pseudo-stress})$



Regression with Life Data: Y versus x

Response Variable: Y

Table of Percentiles

Percent	s	x	Percentile	Standard Error	95,0% Normal CI	
					Lower	Upper
10	80	4,3820	133,3747	34,0579	80,8565	220,0048
10	100	4,6052	16,7928	3,4263	11,2577	25,0494
10	120	4,7875	5,7830	1,2364	3,8034	8,7929
10	140	4,9416	3,6458	0,8760	2,2766	5,8386
50	80	4,3820	270,1879	56,0580	179,9121	405,7621
50	100	4,6052	34,0186	4,3027	26,5494	43,5891
50	120	4,7875	11,7151	1,5950	8,9713	15,2980
50	140	4,9416	7,3856	1,2828	5,2547	10,3807
90	80	4,3820	423,6933	90,4646	278,8097	643,8659
90	100	4,6052	53,3461	6,8162	41,5281	68,5272
90	120	4,7875	18,3709	2,4567	14,1351	23,8760
90	140	4,9416	11,5817	1,9813	8,2824	16,1952

ESTIMERT KOVARIANSMATRISSE FOR $(\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2, \hat{\sigma})$

3860,37	-1649,17	175,82	-0,80
-1649,17	704,70	-75,15	0,33
175,82	-75,15	8,02	-0,03
-0,80	0,33	-0,03	0,23

Probability Plot for SResids of Y
 Smallest Extreme Value - 95% CI
 Censoring Column in C - ML Estimates

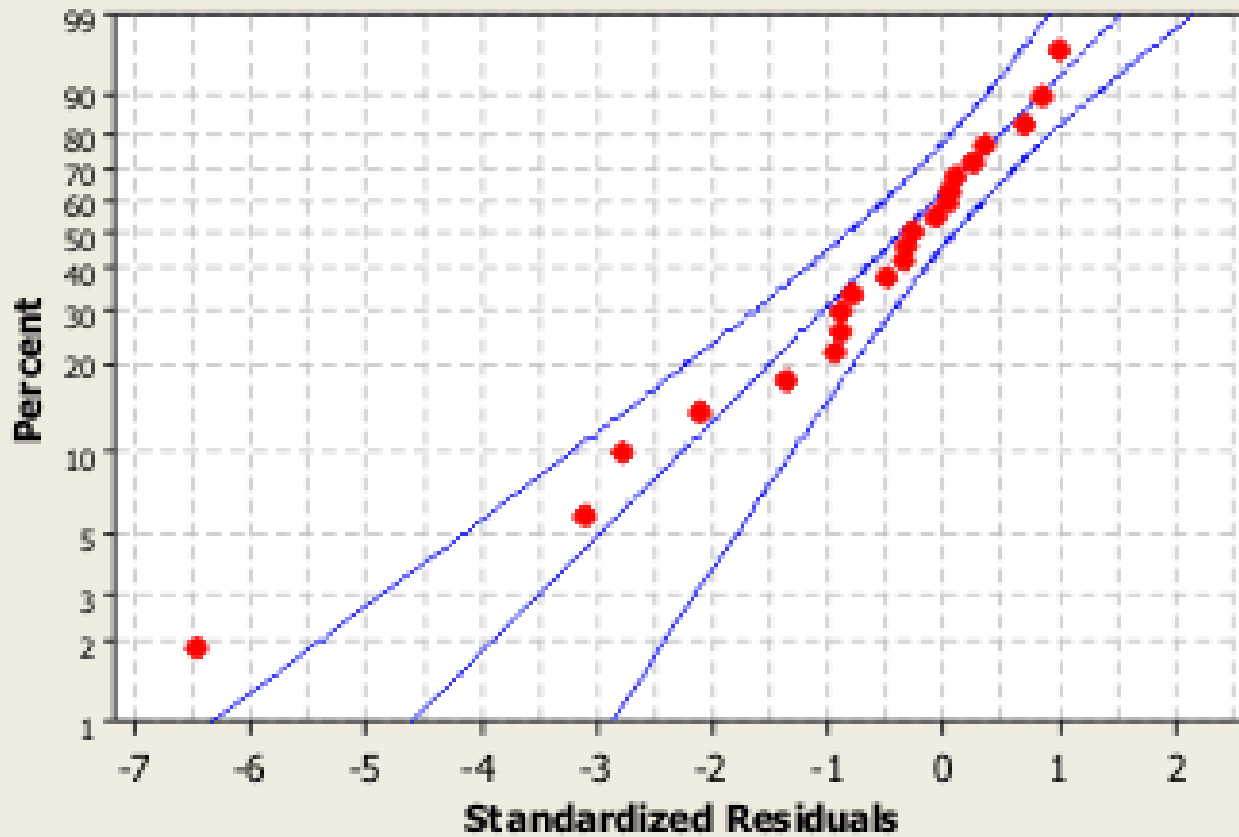


Table of Statistics	
Loc	0,0000000
Scale	1
Mean	-0,577216
StDev	1,28255
Median	-0,366513
IQR	1,57253
Failure	22
Censor	4
AD*	0,928

Probability Plot for CSResids of Y

Exponential - 95% CI

Censoring Column in C - ML Estimates

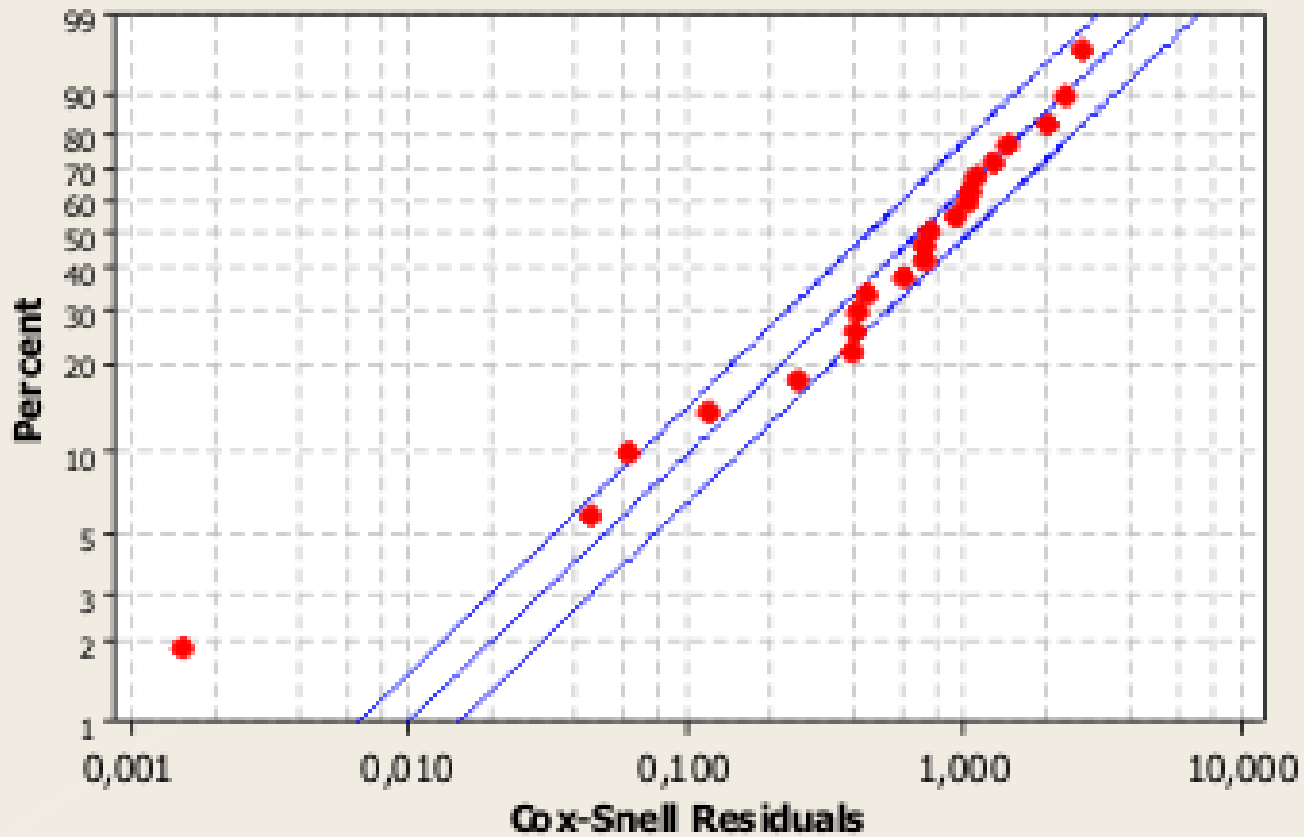
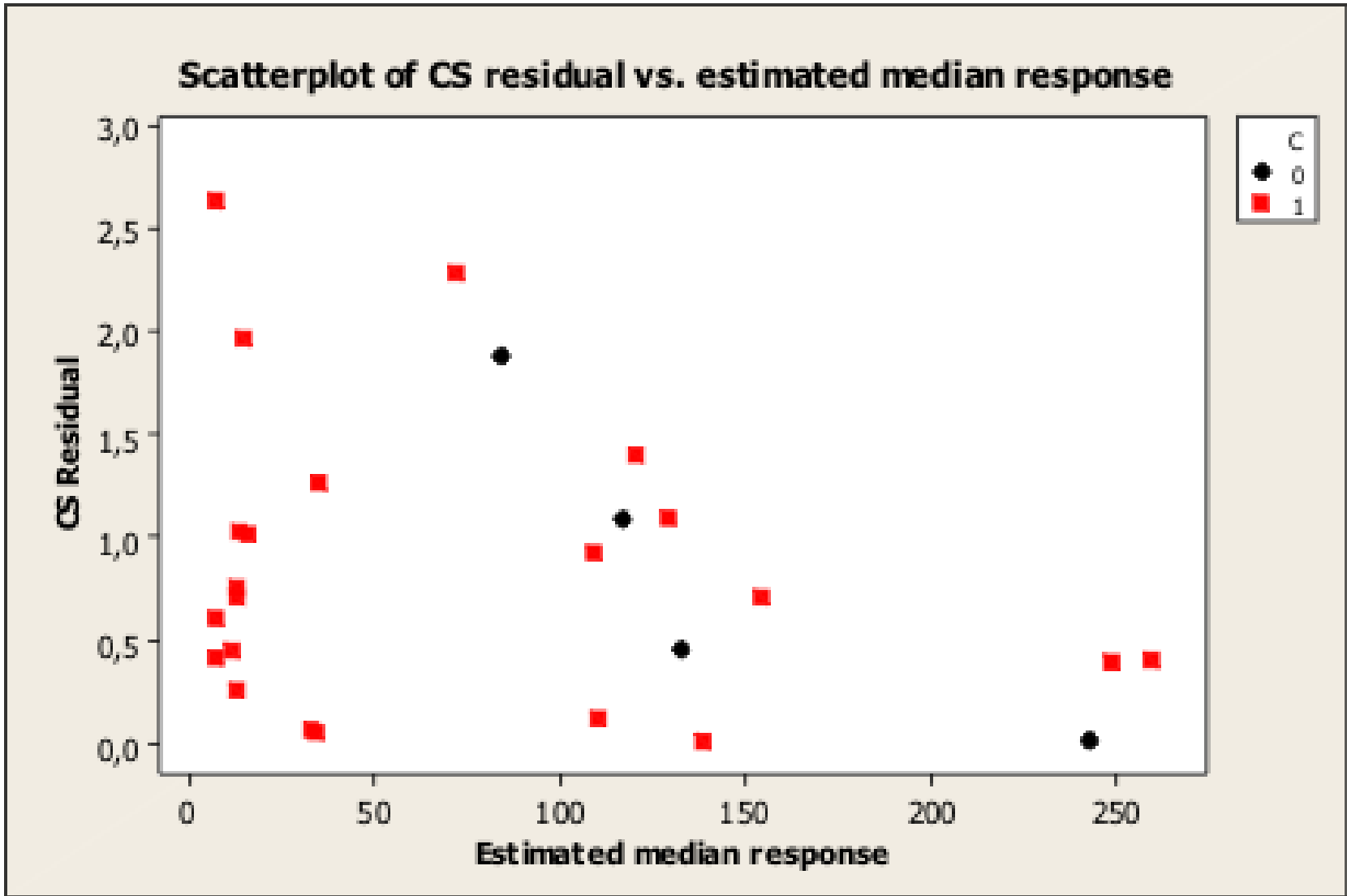


Table of Statistics	
Mean	1
Std ev	1
Median	0,693147
IQR	1,09861
Failure	22
Censor	4
AD*	0,928



SIMPLE EXAMPLE COX-REGRESSION

j	Y_j	x_j	δ_j
1	5	12	0
2	10	10	1
3	40	3	0
4	80	5	0
5	120	3	1
6	400	4	1
7	600	1	0

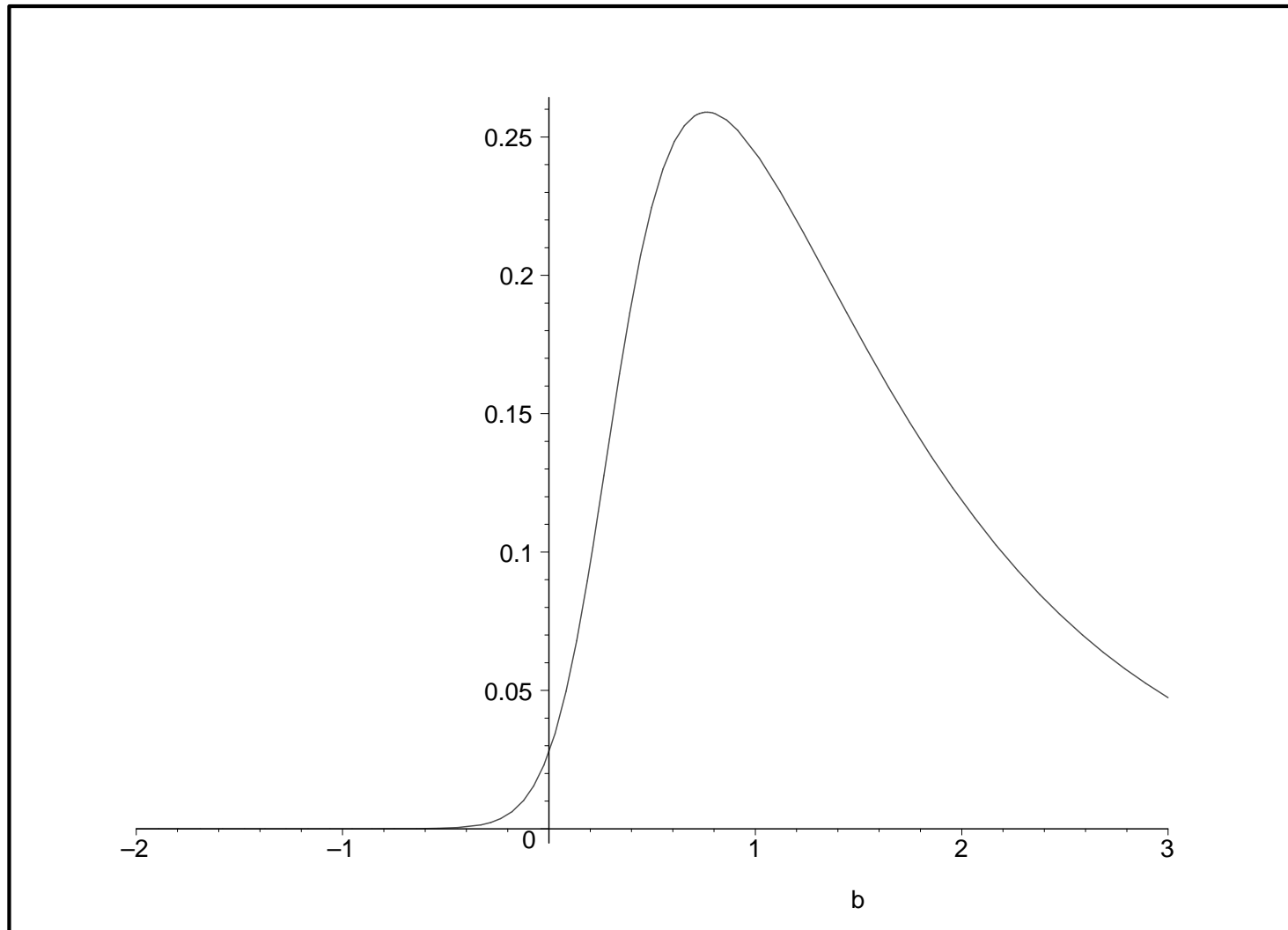
Model:

- $z(t|x) = z_0(t) \exp\{\beta x\}$

Partial likelihood:

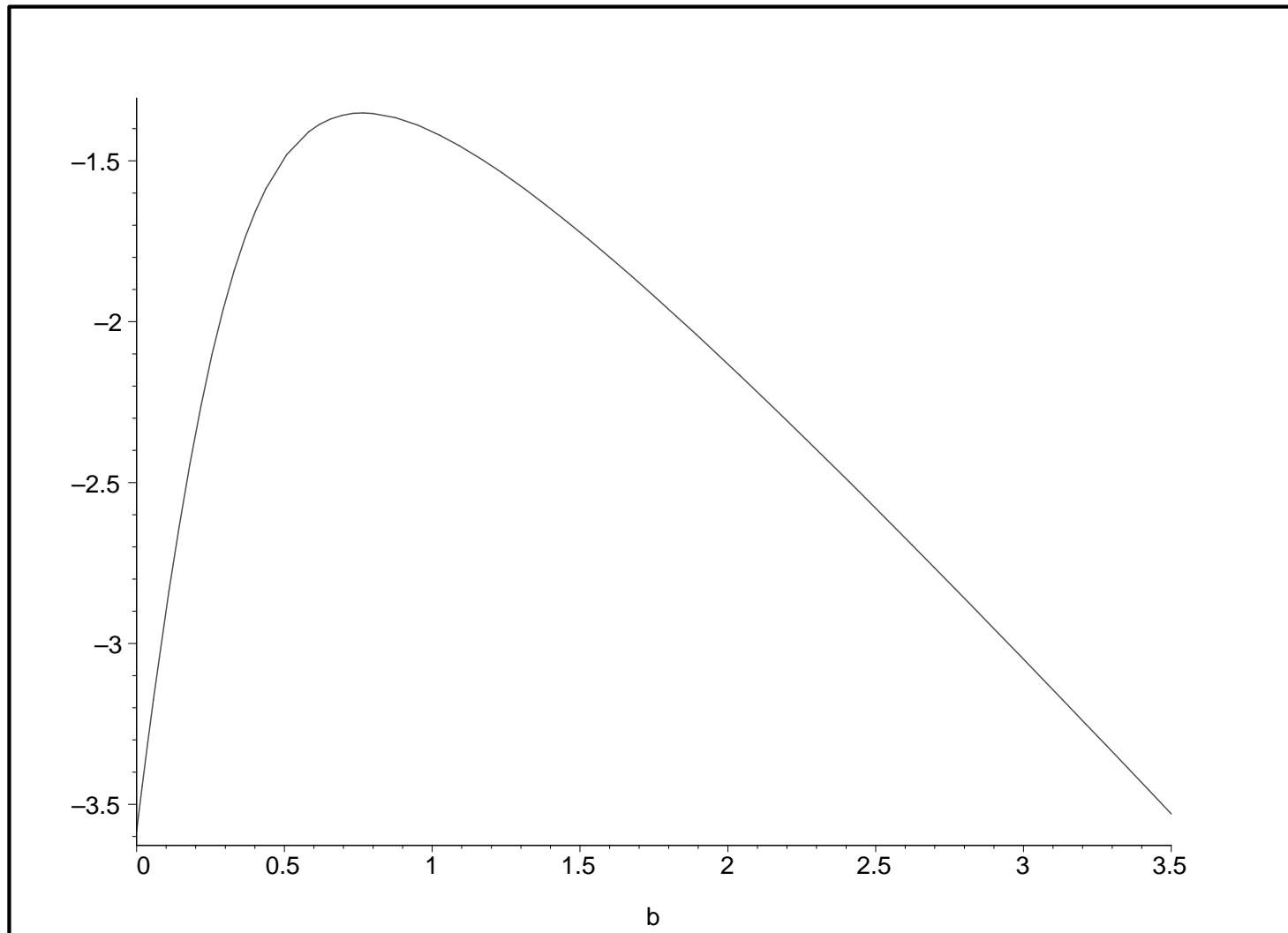
$$L(\beta) = \frac{e^{10\beta}}{e^{10\beta} + e^{3\beta} + e^{5\beta} + e^{3\beta} + e^{4\beta} + e^{\beta}} \cdot \frac{e^{3\beta}}{e^{3\beta} + e^{4\beta} + e^{\beta}} \cdot \frac{e^{4\beta}}{e^{4\beta} + e^{\beta}}$$

Cox' partial likelihood $L(\beta)$ in the example:



Maximum likelihood estimate: $\hat{\beta} = 0.765$.

Cox' partial log-likelihood $l(\beta)$ in the example:



Maximum likelihood estimate: $\hat{\beta} = 0.765$.

95% likelihood confidence interval: (0.1, 3.2).

Weibull regression (Cox-example)

Estimation Method: Maximum Likelihood

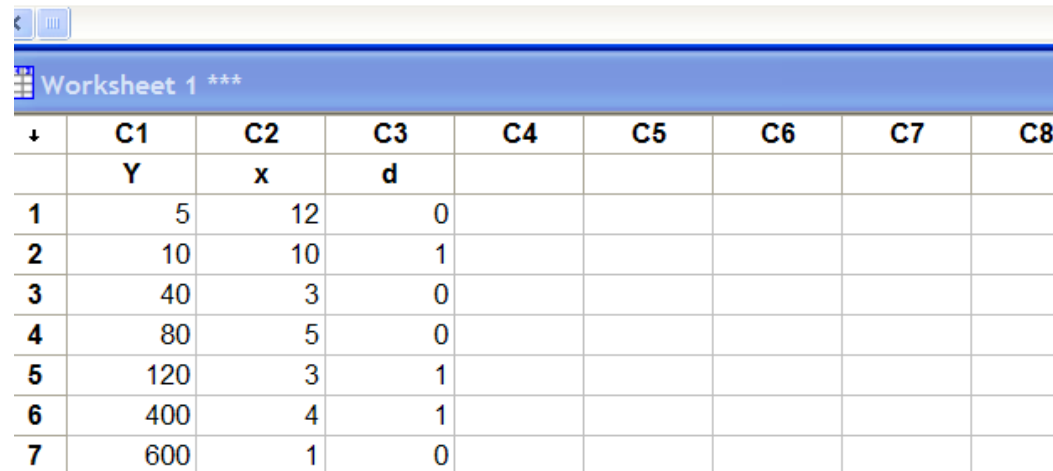
Distribution: Weibull

Relationship with accelerating variable(s): Linear

Regression Table

Predictor	Coef	Standard Error	Z	P	95,0% Normal CI	
					Lower	Upper
Intercept	7,58636	0,548229	13,84	0,000	6,51185	8,66087
x	-0,468235	0,0842830	-5,56	0,000	-0,633427	-0,303044
Shape	2,05563	0,872169			0,894943	4,72167

Log-Likelihood = -17,450



Worksheet 1 ***

	C1	C2	C3	C4	C5	C6	C7	C8
	Y	x	d					
1	5	12	0					
2	10	10	1					
3	40	3	0					
4	80	5	0					
5	120	3	1					
6	400	4	1					
7	600	1	0					

Data from Ansell & Phillips (s. 63)

Table 3.2. Lifetimes (in cycles) of sodium sulphur batteries

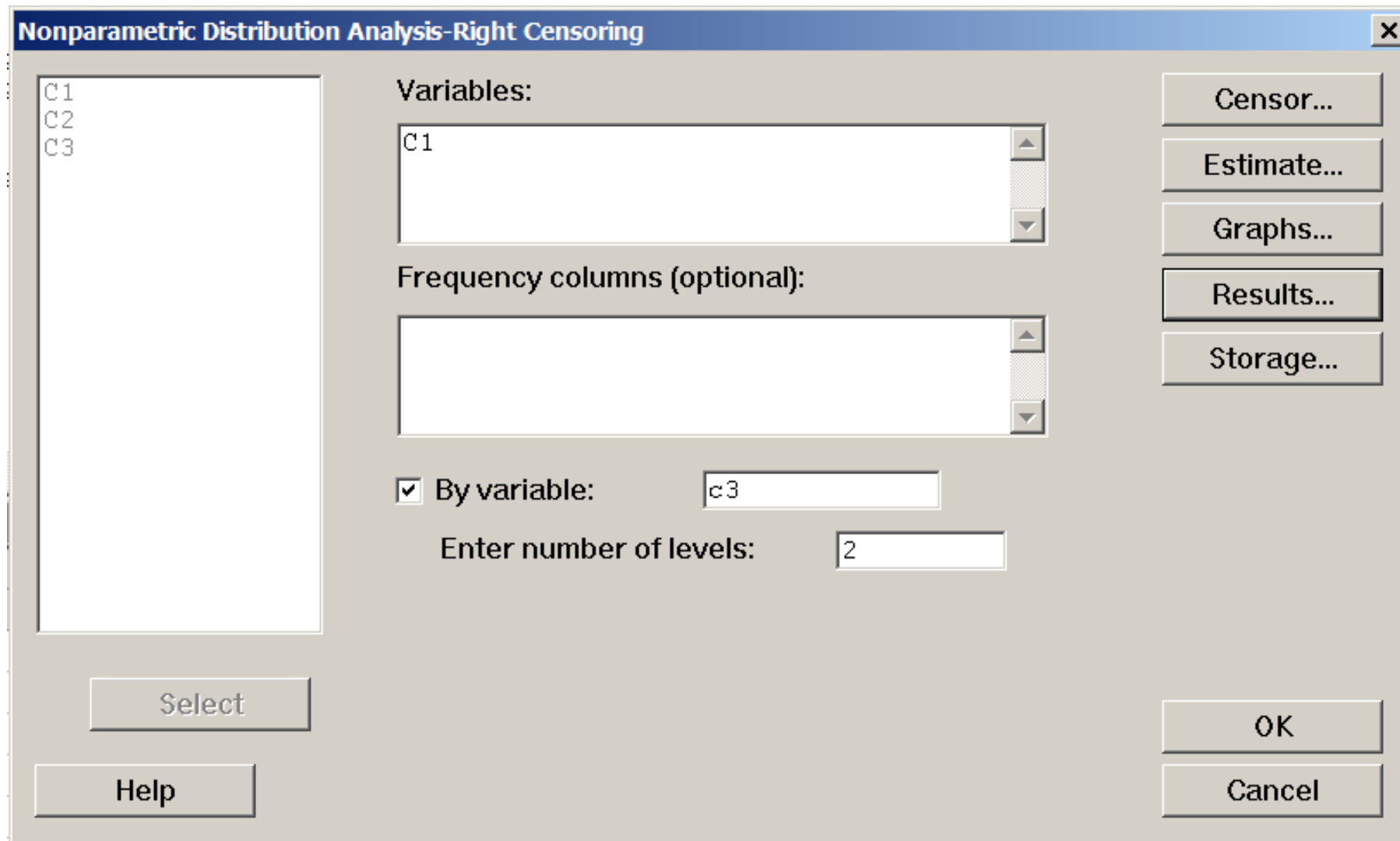
Batch 1	164	164	218	230	263	467	538	639	669
	917	1148	1678+	1678+	1678+	1678+			
Batch 2	76	82	210	315	385	412	491	504	522
	646+	678	775	884	1131	1446	1824	1827	2248
	2385	3077							

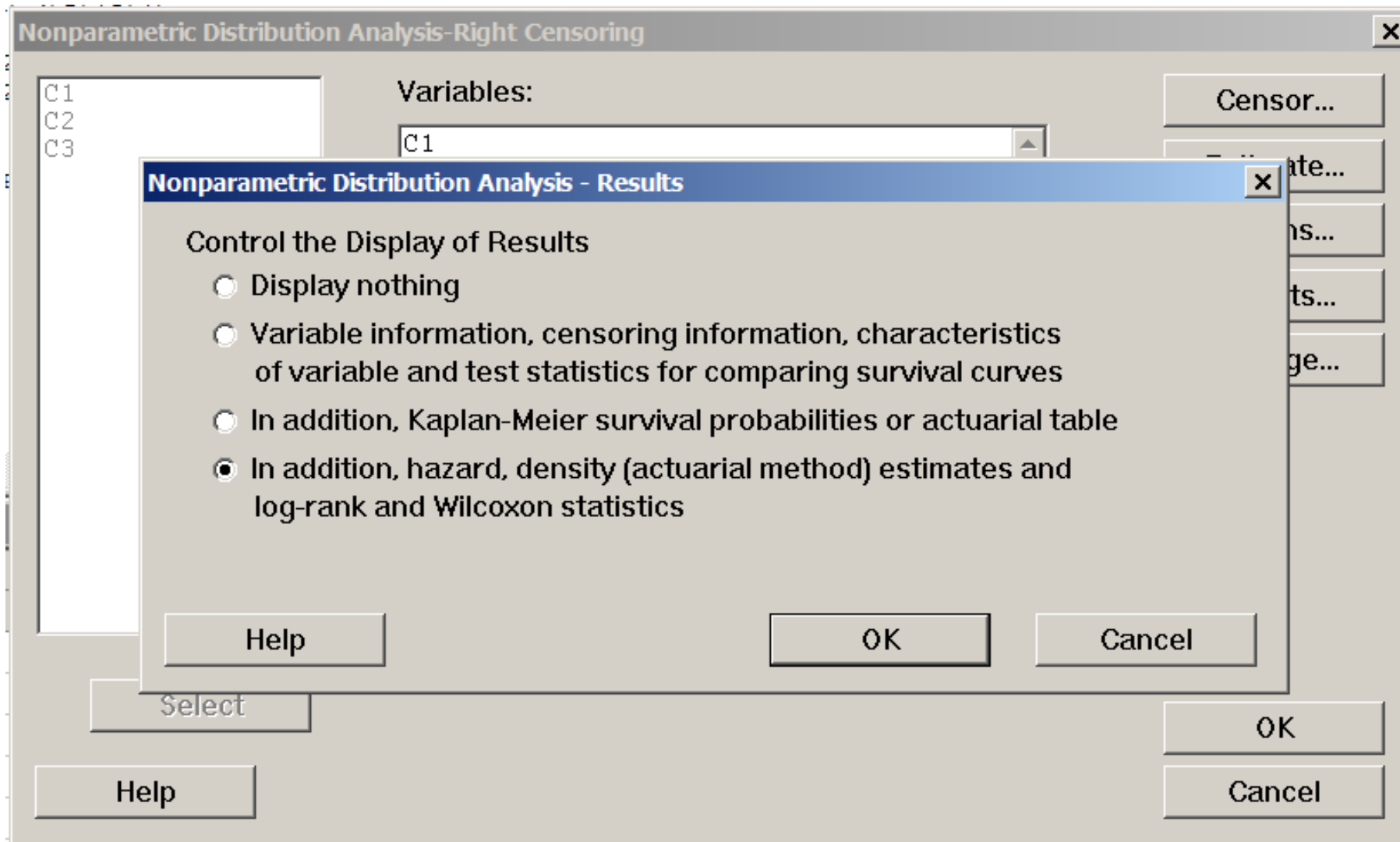
Note: Lifetimes with + are right censored observations, not failures.

C1=Obs. times, C2=censoring, C3="batch" no.

Ansell32.MTW ***												
↓	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12
1	164	1	1									
2	164	1	1									
3	218	1	1									
4	230	1	1									
5	263	1	1									
6	467	1	1									
7	538	1	1									
8	639	1	1									
9	669	1	1									
10	917	1	1									
11	1148	1	1									
12	1678	0	1									
13	1678	0	1									
14	1678	0	1									
15	1678	0	1									
16	76	1	2									
17	82	1	2									
18	210	1	2									
19	315	1	2									
20	385	1	2									
21	412	1	2									
22	491	1	2									
23	504	1	2									
24	522	1	2									
25	646	0	2									

MINITAB-analysis of data from Ansell & Phillips:

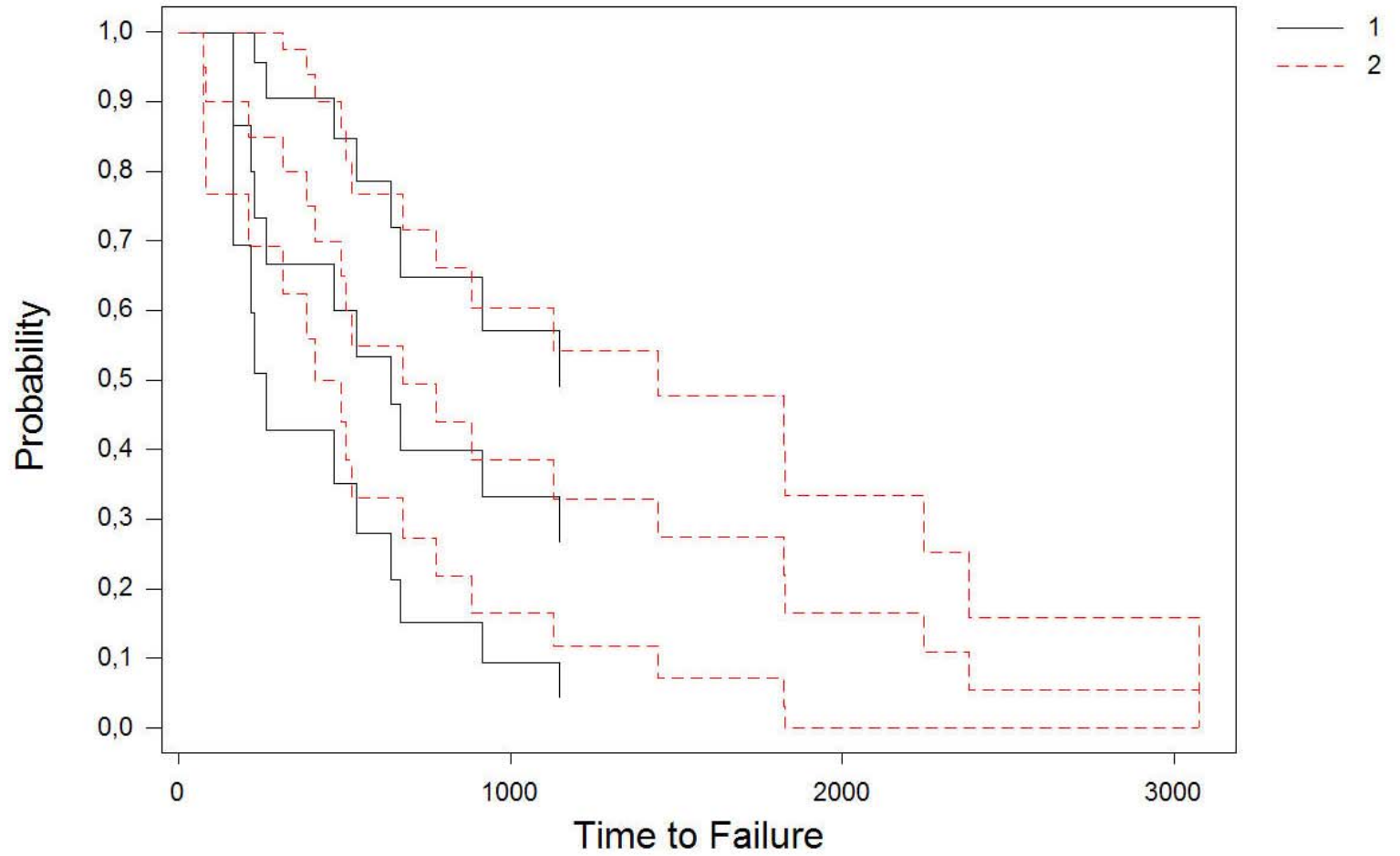




Nonparametric Survival Plot for C1

Kaplan-Meier Method - 95,0% CI

Censoring Column in C2



MINITAB-result (among other output):

Test Statistics Method	Chi-Square	DF	P-Value	Log-Rank
	0,04855	1	0,8256	

This is in accordance with the result of p. 72 (Example 3.5.2) in Ansell & Phillips.

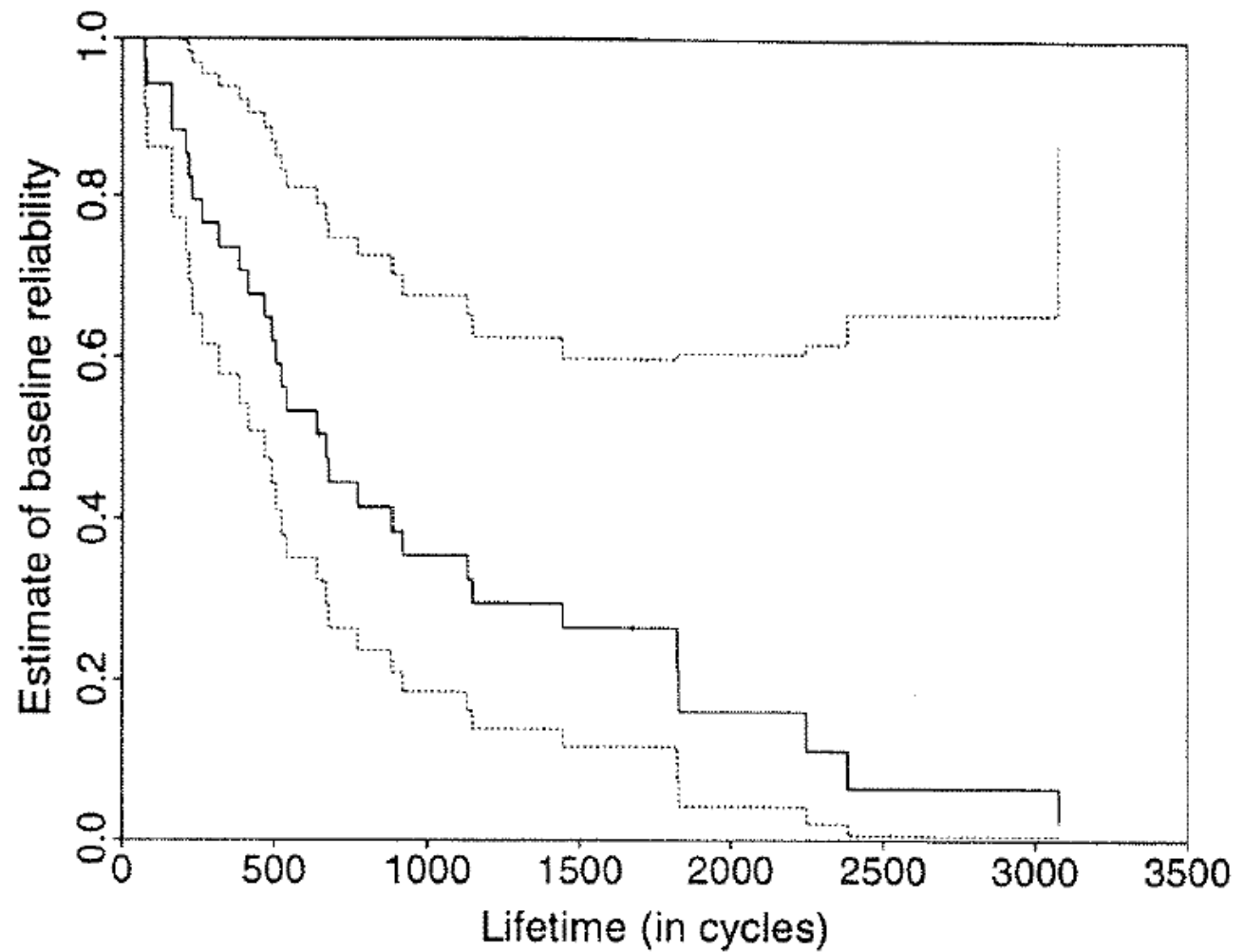


Fig. 3.3. Plot of the baseline reliability function for the proportional hazards model for the sodium sulphur battery data with 95% confidence limits

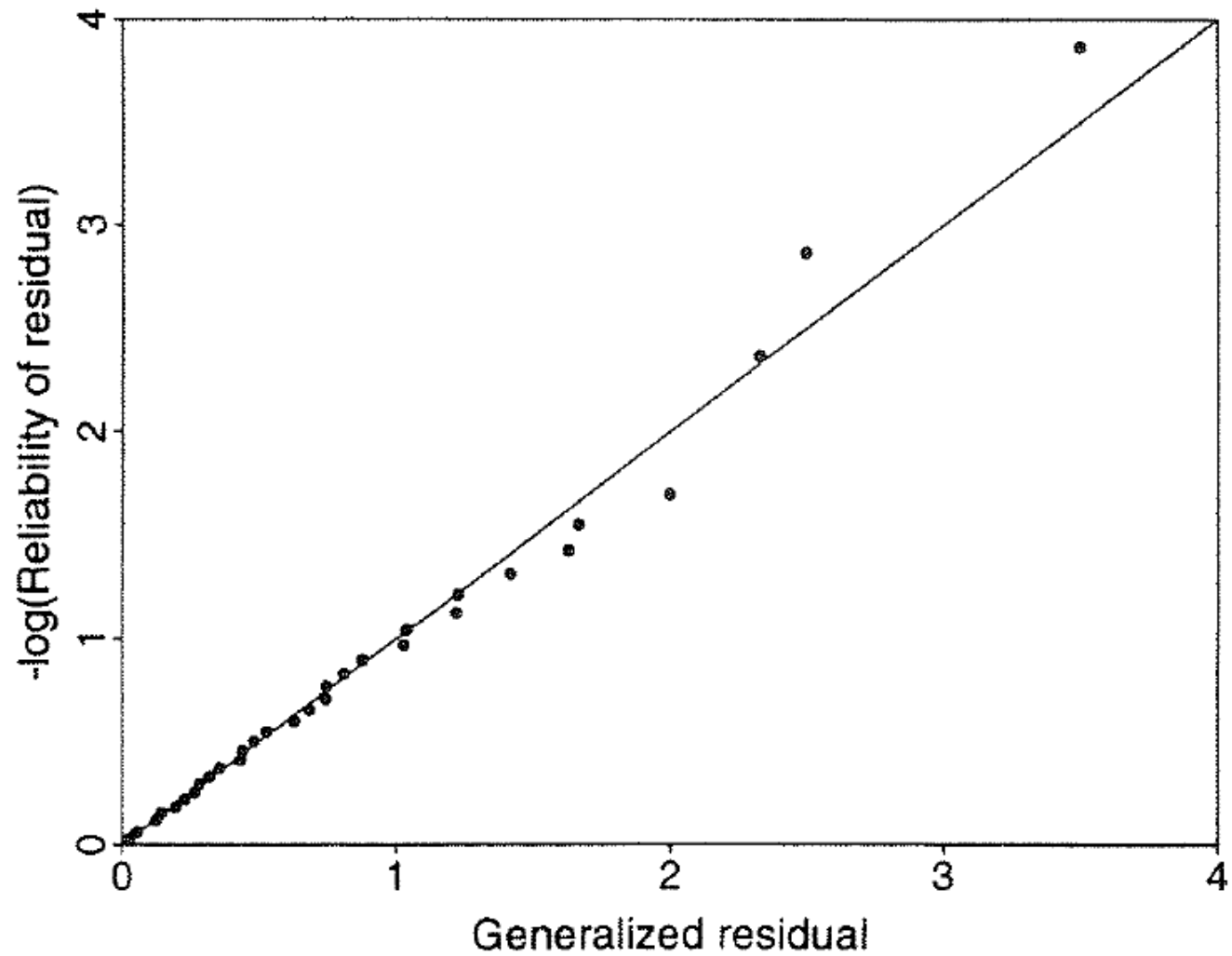


Fig. 3.5. Plot of the generalized residuals of the proportional hazards model for the sodium sulphur battery data

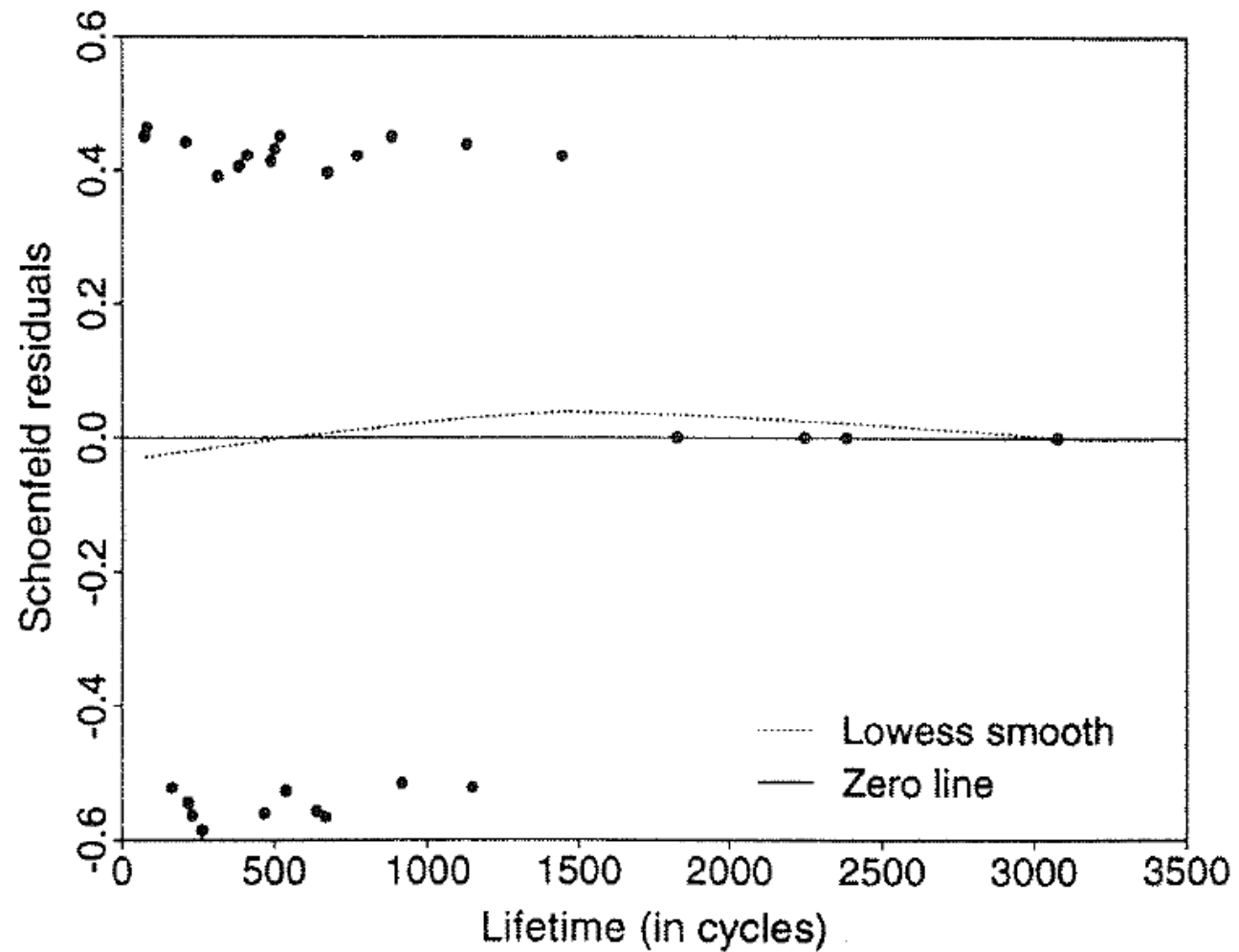
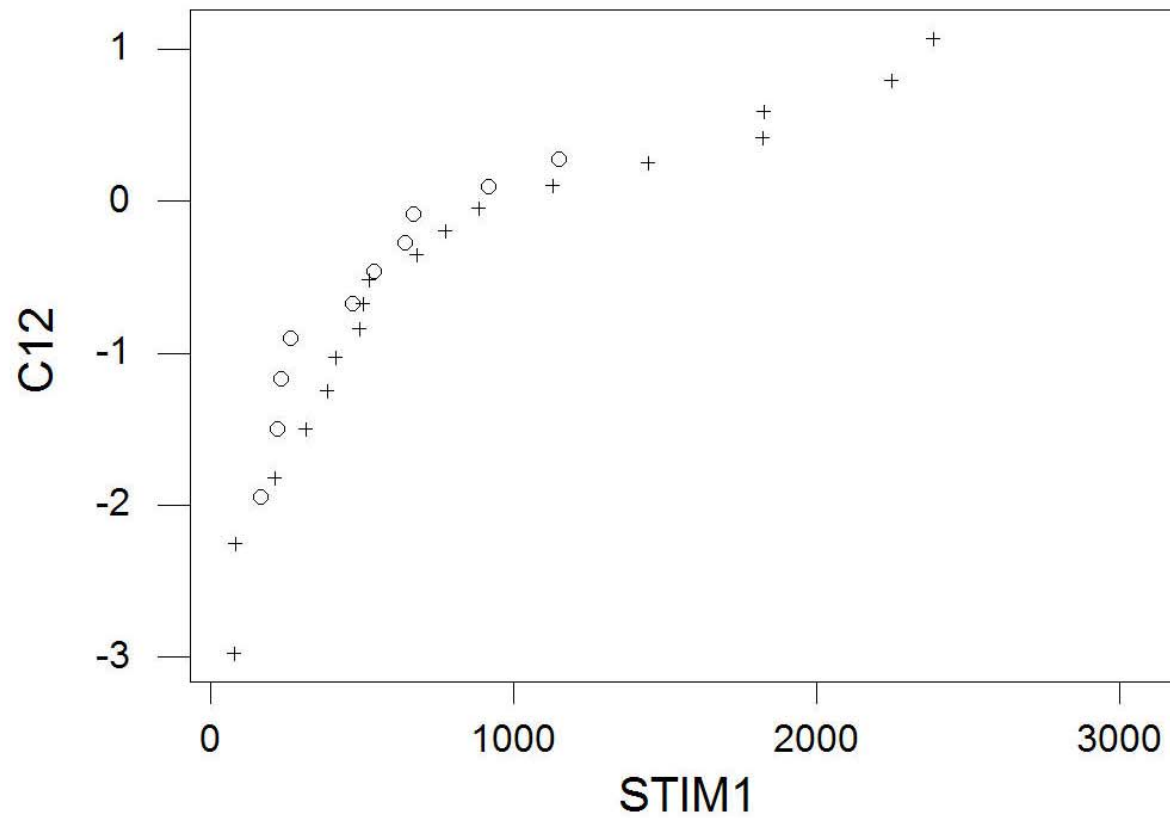


Fig. 3.7. Plot of the Schoenfeld residuals for batch of the proportional hazards model for the sodium sulphur battery data

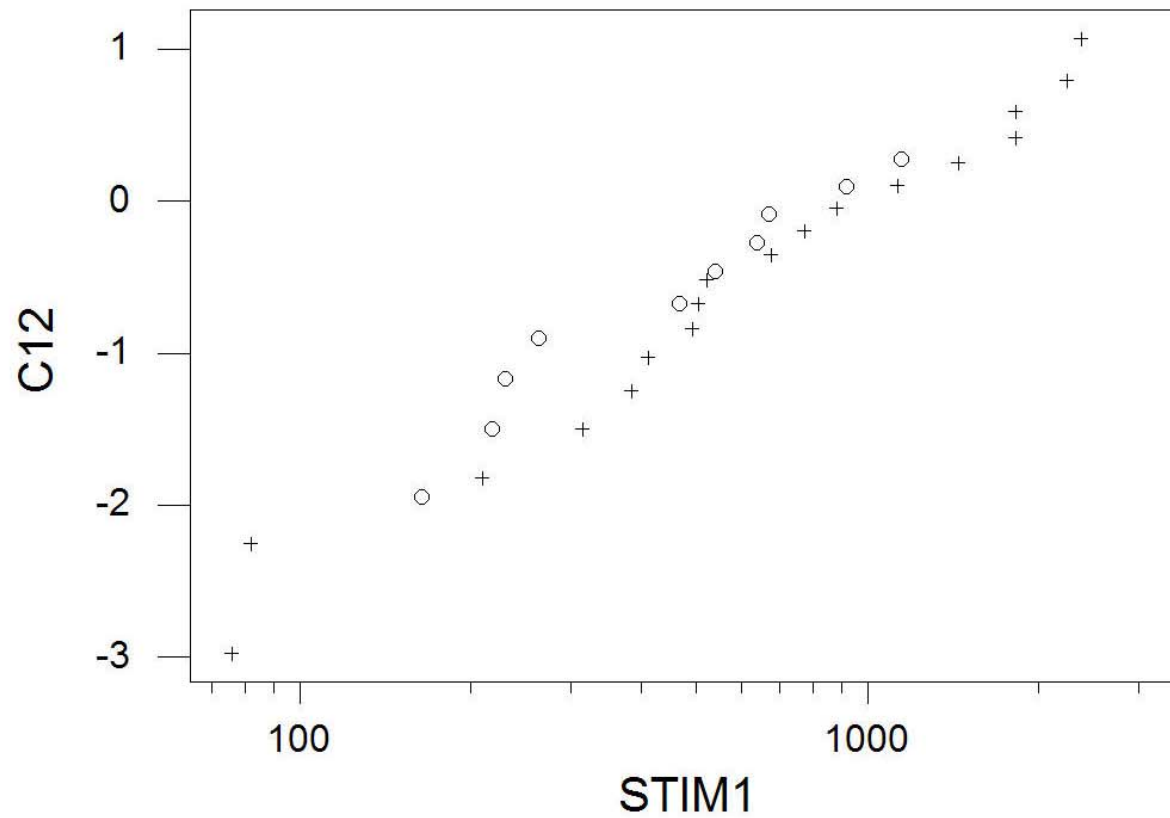
LOG MINUS LOG PLOT vs. "t" FOR A&P DATA (s.63)

Parallel plots indicate Proportional Hazard



LOG MINUS LOG PLOT vs. "log t" FOR A&P DATA (s.63)

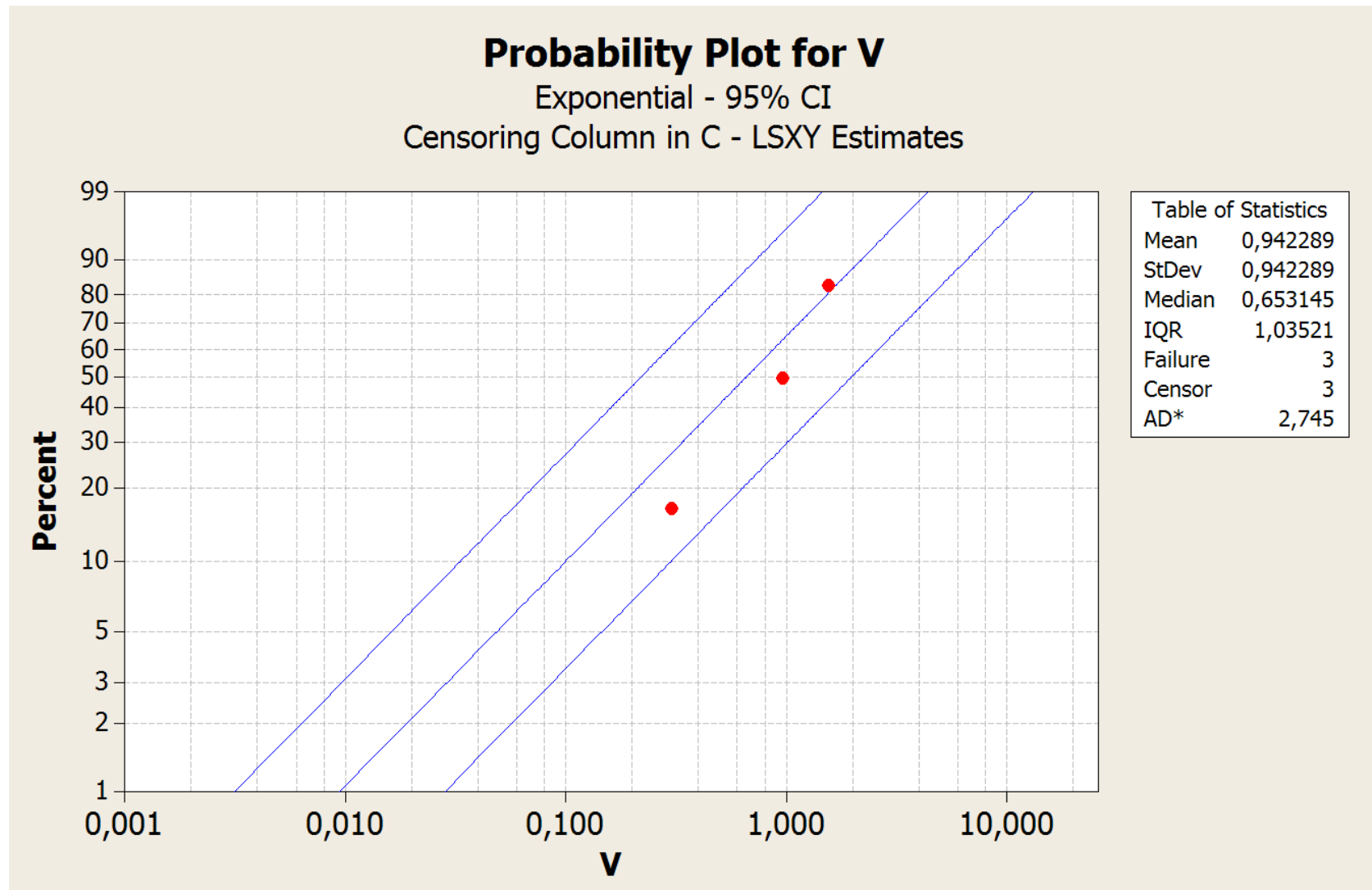
Straight lines indicate Weibull distribution



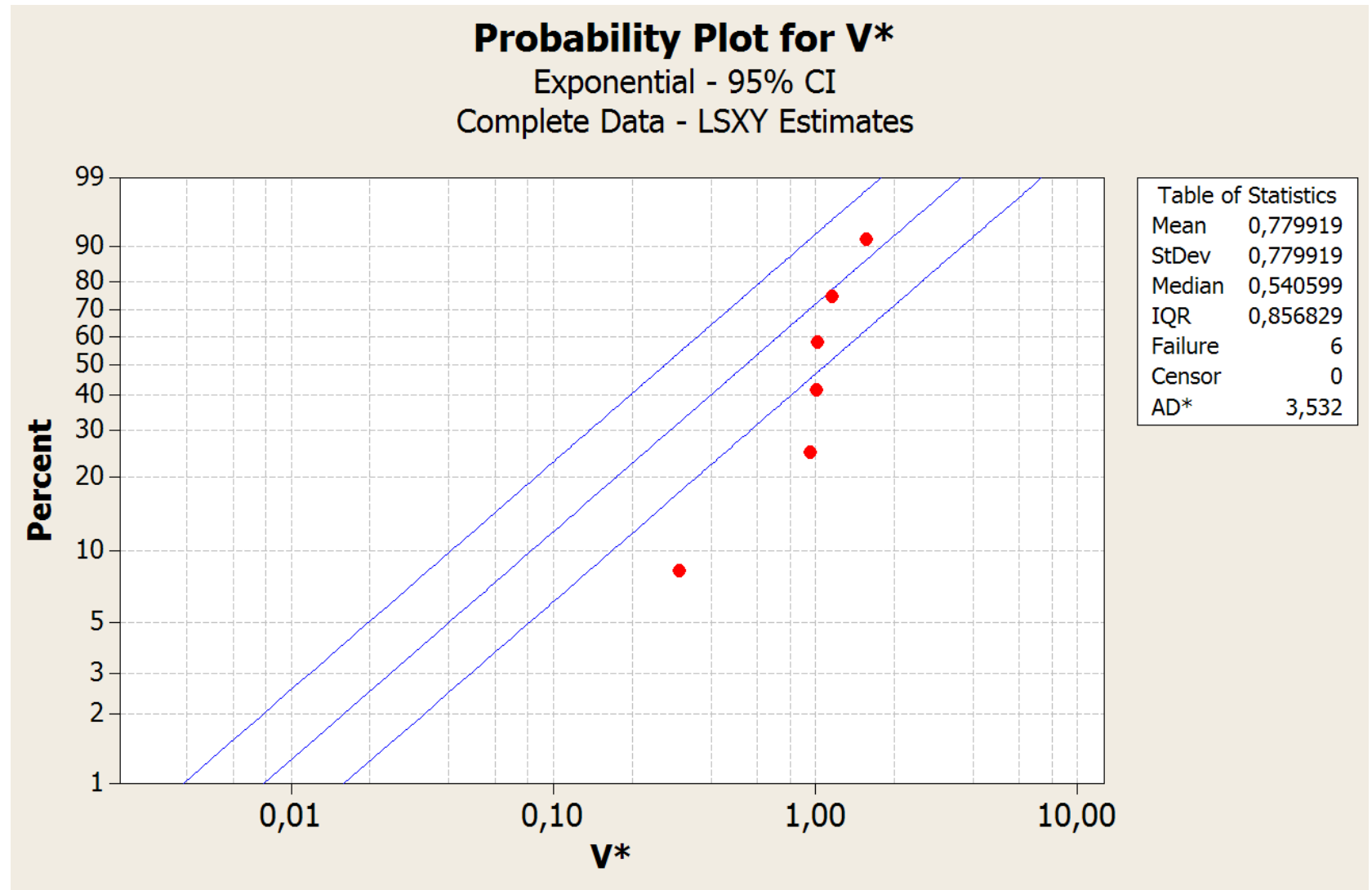
COX-SNELL RESIDUALS SIMPLE COX-EXAMPLE

Worksheet 1 ***				
+	C1	C2	C3	C
	V	C	V*	
1	0,9593	1	0,9593	
2	0,0045	0	1,0045	
3	0,0209	0	1,0209	
4	0,3017	1	0,3017	
5	1,5567	1	1,5567	
6	0,1569	0	1,1569	
7				
-				

COX-SNELL RESIDUALS SIMPLE COX-EXAMPLE



MODIFIED COX-SNELL RESIDUALS SIMPLE COX-EXAMPLE





Methods and Formulas – Accelerated Life Testing

Equation	Models	Residuals
Lifetime regression	Linear	Ordinary
Response variable	Arrhenius	Standardized
Error term	Inverse temp	Cox-Snell
	Loge (Power)	

Equation

Lifetime regression

The regression model estimates the percentiles of the failure time distribution:

$$Y = \beta_0 + \beta_1 X + \sigma \varepsilon$$

where:

Y = either failure time or $\log(\text{failure time})$

β_0 = y-intercept (constant)

β_1 = regression coefficient

X = predictor values (may be transformed)

σ = 1/shape (Weibull distribution) or scale (other distributions)

ε = random error term

Response variable

Depending on the distribution, Y = failure time or $\log(\text{failure time})$:

- For the Weibull, exponential, lognormal, and loglogistic distributions, $Y = \log(\text{failure time})$
- For the normal, extreme value, and logistic distributions, $Y = \text{failure time}$

When $Y = \log(\text{failure time})$, Minitab takes the antilog to display the percentiles on the original scale.

Error term

The value of the error distribution also depends on the distribution chosen.

- For the normal distribution, the error distribution is the standard normal distribution – normal (0,1). For the lognormal distribution, Minitab takes the \log base e of the data and uses a normal distribution.
- For the logistic distribution, the error distribution is the standard logistic distribution – logistic (0, 1). For the loglogistic distribution, Minitab takes the \log of the data and uses a logistic distribution.
- For the extreme value distribution, the error distribution is the standard extreme value distribution – extreme value (0, 1). For the Weibull distribution and the exponential distribution (a type of Weibull distribution), Minitab takes the \log of the data and uses the extreme value distribution.

[Back to top](#)

Models

Linear

$$Y = \beta_0 + \beta_1 * \text{accelerating variable} + \sigma \varepsilon$$

where:

- Y is the failure time or log failure time
- σ is the reciprocal of the shape parameter (Weibull distribution) or the scale parameter (other distributions)
- ε is the random error term

Arrhenius

$$Y = \beta_0 + \beta_1 * [11604.83/\text{Degrees Celsius} + 273.16]] + \sigma \varepsilon$$

where:

- Y is the failure time or log failure time
- σ is the reciprocal of the shape parameter (Weibull distribution) or the scale parameter (other distributions)
- ε is the random error term

Inverse temp

$$Y = \beta_0 + \beta_1 * [1/(\text{Degrees Celsius} + 273.16)] + \sigma \varepsilon$$

where:

- Y is the failure time or log failure time
- σ is the reciprocal of the shape parameter (Weibull distribution) or the scale parameter (other distributions)
- ε is the random error term

Loge (Power)

$$Y = \beta_0 + \beta_1 * \log(\text{accelerating variable}) + \sigma \varepsilon$$

where:

- Y is the failure time or log failure time
- σ is the reciprocal of the shape parameter (Weibull distribution) or the scale parameter (other distributions)
- ε is the random error term

[Back to top](#)

+	C1	C2	C3	C4	C5-T	C6	C7	C8	C9	C10	C11
	Temp	ArrTemp	Plant	FailureT	Censor	Design	NewTemp	ArrNewT	NewPlant		
1	170	26,1865	1	343	F	80	80	32,8600	1		
2	170	26,1865	1	869	F	100	80	32,8600	2		
3	170	26,1865	1	244	C		100	31,0988	1		
4	170	26,1865	1	716	F		100	31,0988	2		
5	170	26,1865	1	531	F						
6	170	26,1865	1	738	F						
7	170	26,1865	1	461	F						
8	170	26,1865	1	221	F						
9	170	26,1865	1	665	F						
10	170	26,1865	1	384	C						
11	170	26,1865	2	394	C						
12	170	26,1865	2	369	F						
13	170	26,1865	2	366	F						
14	170	26,1865	2	507	F						
15	170	26,1865	2	461	F						
16	170	26,1865	2	431	F						
17	170	26,1865	2	479	F						
18	170	26,1865	2	106	F						
19	170	26,1865	2	545	F						
20	170	26,1865	2	536	F						
21	150	27,4242	1	2134	C						
22	150	27,4242	1	2746	F						
23	150	27,4242	1	2859	F						
24	150	27,4242	1	1826	C						

MINITAB Help

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Example of Accelerated Life Testing

[main topic](#) [interpreting results](#) [session command](#) [see also](#)

Suppose you want to investigate the deterioration of an insulation used for electric motors. The motors normally run between 80 and 100° C. To save time and money, you decide to use accelerated life testing.

First you gather failure times for the insulation at abnormally high temperatures – 110, 130, 150, and 170° C – to speed up the deterioration. With failure time information at these temperatures, you can then extrapolate to 80 and 100° C. It is known that an Arrhenius relationship exists between temperature and failure time. To see how well the model fits, you will draw a probability plot based on the standardized residuals.

- 1 Open the worksheet INSULATE.MTW.
- 2 Choose **Stat > Reliability/Survival > Accelerated Life Testing**.
- 3 In **Variables/Start variables**, enter **FailureT**. In **Accelerating variable**, enter **Temp**.
- 4 From **Relationship**, choose **Arrhenius**.
- 5 Click **Censor**. In **Use censoring columns**, enter **Censor**, then click **OK**.
- 6 Click **Graphs**. In **Enter design value to include on plot**, enter **80**. Click **OK**.
- 7 Click **Estimate**. In **Enter new predictor values**, enter **Design**, then click **OK** in each dialog box.

Session window output

Regression with Life Data: FailureT versus Temp

Response Variable: FailureT

Censoring Information	Count
Uncensored value	66
Right censored value	14

Censoring value: Censor = C

Estimation Method: Maximum Likelihood
 Distribution: Weibull
 Transformation on accelerating variable: Arrhenius

Regression Table

Predictor	Coef	Standard Error	Z	P	95.0% Normal CI	
					Lower	Upper
Intercept	-15.1874	0.9862	-15.40	0.000	-17.1203	-13.2546
Temp	0.83072	0.03504	23.71	0.000	0.76204	0.89940
Shape	2.8246	0.2570			2.3633	3.3760

Log-Likelihood = -564.693

Anderson-Darling (adjusted) Goodness-of-Fit

At each accelerating level

Level	Fitted Model
110	*
130	*
150	*
170	*

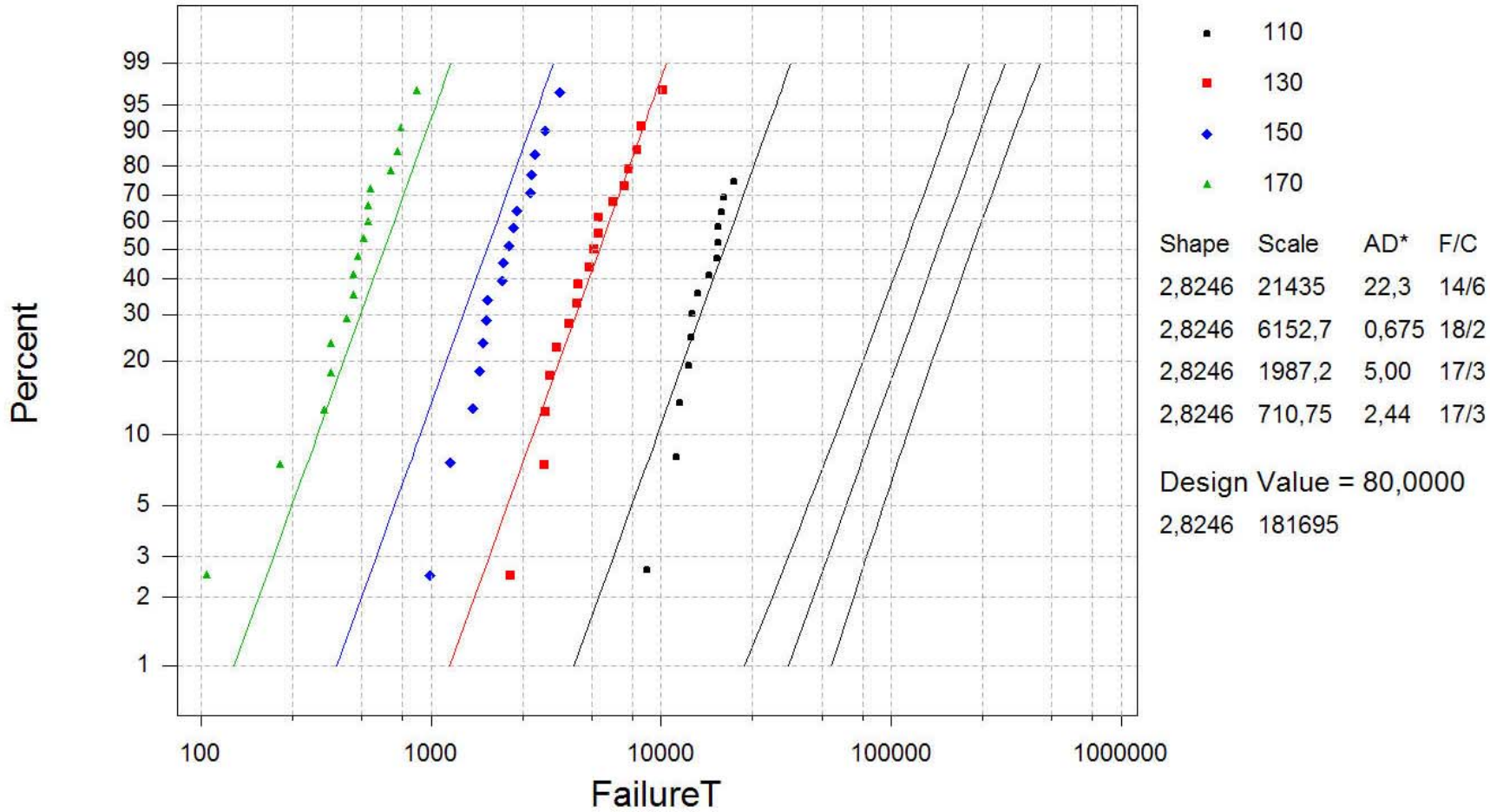
Table of Percentiles

Percent	Temp	Percentile	Standard Error	95.0% Normal CI	
				Lower	Upper
50	80.0000	159584.5	27446.85	113918.2	223557.0
50	100.0000	36948.57	4216.511	29543.36	46209.94

Probability Plot (Fitted Arrhenius) for FailureT

Weibull Distribution - ML Estimates - 95,0% CI

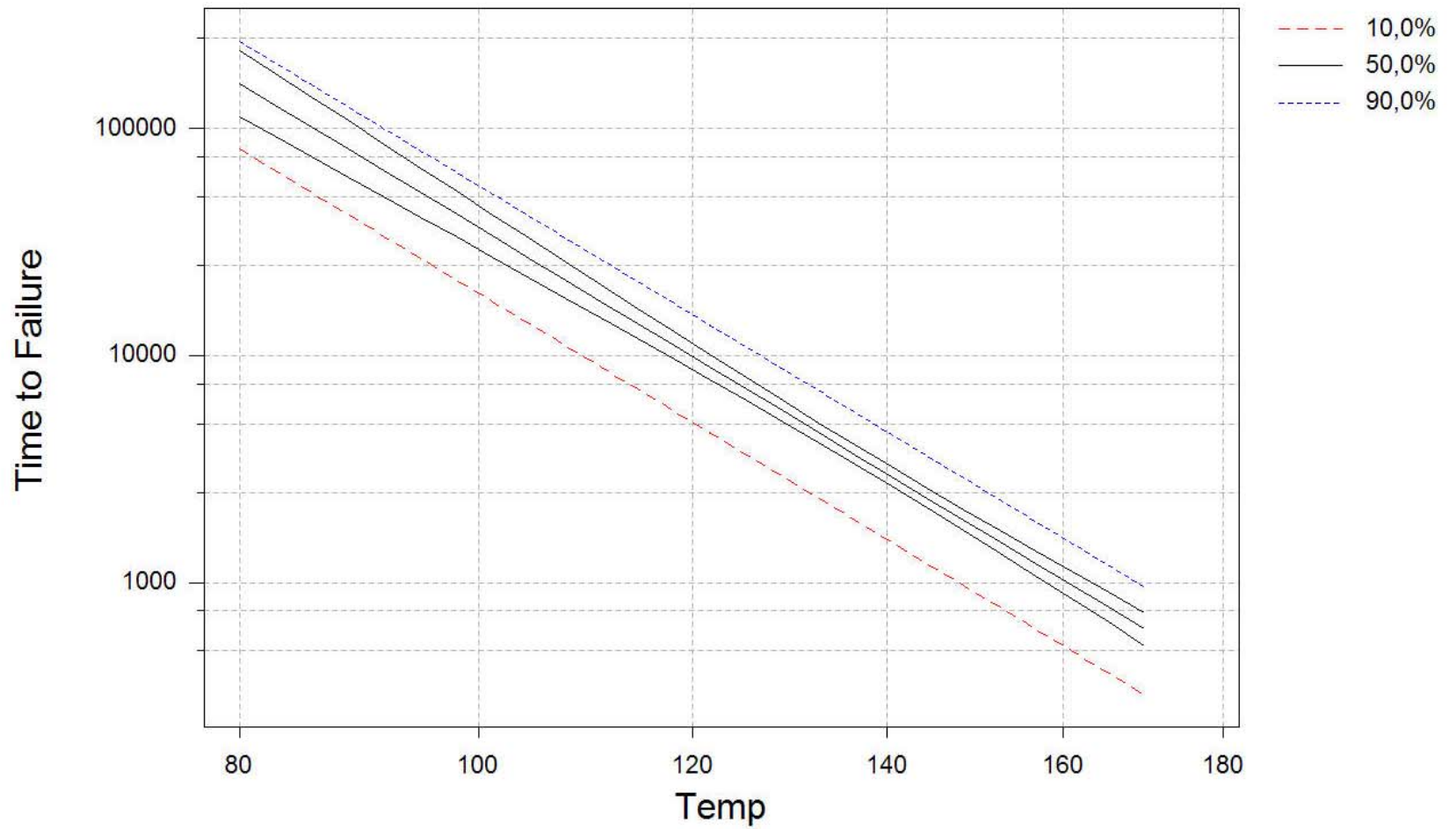
Censoring Column in Censor



Relation Plot (Fitted Arrhenius) for FailureT

Weibull Distribution - ML Estimates - 95,0% CI

Censoring Column in Censor



ADDING THE FACTOR "PLANT":

Accelerated Life Testing

Responses are uncens/right censored data
 Responses are uncens/arbitrarily censored data

Variables/ Start variables: FailureT
End variables:
Freq. columns: (optional)

Accelerating var: Temp Relationship: Arrhenius

Second Variable
 Accelerating: Relationship: Linear
 Factor: Plant
 Include interaction term between variables

Assumed distribution: Weibull

Select
Help

Censor...
Estimate...
Graphs...
Results...
Options...
Storage...
OK
Cancel