

SIMPLE EXAMPLE COX-REGRESSION

i	Y_i	x_i	δ_i
1	5	12	0
2	10	10	1
3	40	3	0
4	80	5	0
5	120	3	1
6	400	4	1
7	600	1	0

Model:

- $z(t|x) = z_0(t) \exp\{\beta x\}$

Partial likelihood:

$$L(\beta) = \frac{e^{10\beta}}{e^{10\beta} + e^{3\beta} + e^{5\beta} + e^{3\beta} + e^{4\beta} + e^{\beta}} \cdot \frac{e^{3\beta}}{e^{3\beta} + e^{4\beta} + e^{\beta}} \cdot \frac{e^{4\beta}}{e^{4\beta} + e^{\beta}}$$

$$\hat{\beta} = 0.765$$

BRESLOW-ESTIMATOR

$$\hat{Z}_0(t) = \sum_{T_j \leq t} \frac{1}{\sum_{i \in R_j} e^{\hat{\beta}' \mathbf{x}_i}}$$

SO

$$\begin{aligned}\hat{Z}_0(10) &= \frac{1}{e^{10\hat{\beta}} + e^{3\hat{\beta}} + e^{5\hat{\beta}} + e^{3\hat{\beta}} + e^{4\hat{\beta}} + e^{\hat{\beta}}} = 4.57 \cdot 10^{-4} \\ \hat{Z}_0(120) &= 4.57 \cdot 10^{-4} + \frac{1}{e^{3\hat{\beta}} + e^{4\hat{\beta}} + e^{\hat{\beta}}} = 0.0304 \\ \hat{Z}_0(400) &= 0.0304 + \frac{1}{e^{4\hat{\beta}} + e^{\hat{\beta}}} = 0.0730\end{aligned}$$

ESTIMATED SURVIVAL FUNCTION $P(T > t) = R(t; \mathbf{x})$

$$\hat{R}(t; \mathbf{x}) = \exp\{-\hat{Z}_0(t)e^{\hat{\beta}'\mathbf{x}}\}$$

so

$$\begin{aligned}\hat{R}(10; x) &= \exp\{-4.57 \cdot 10^{-4}e^{0.765x}\} \\ \hat{R}(120; x) &= \exp\{-0.0304e^{0.765x}\} \\ \hat{R}(400; x) &= \exp\{-0.0730e^{0.765x}\}\end{aligned}$$

x	$\hat{R}(10; x)$	$\hat{R}(120; x)$	$\hat{R}(400; x)$
0*	0.9995	0.9701	0.9296
1	0.9990	0.9368	0.8548
3	0.9955	0.7396	0.4846
5	0.9793	0.2483	0.0352
10	0.3829	0.0000	0.0000
KM**	0.8333	0.5556	0.2778

* Baseline survival function

**KM-estimator does not use the value of x

COX-SNELL RESIDUALS

$$\hat{V}_i = \hat{Z}_0(Y_i)e^{\hat{\beta}'x_i}$$

which should behave like *expon*(1)

i	Y_i	x_i	δ_i	\hat{V}_i	
1	5	12	0	$\hat{Z}_0(0)e^{0.765 \cdot 12}$	= 0.0000
2	10	10	1	$\hat{Z}_0(10)e^{0.765 \cdot 10}$	= 0.9593
3	40	3	0	$\hat{Z}_0(10)e^{0.765 \cdot 3}$	= 0.0045
4	80	5	0	$\hat{Z}_0(10)e^{0.765 \cdot 5}$	= 0.0209
5	120	3	1	$\hat{Z}_0(120)e^{0.765 \cdot 3}$	= 0.3017
6	400	4	1	$\hat{Z}_0(400)e^{0.765 \cdot 4}$	= 1.5567
7	600	1	0	$\hat{Z}_0(400)e^{0.765 \cdot 1}$	= 0.1569