

WEEK 12, 2006  
March 23 and 24

## TMA4275 LIFETIME ANALYSIS

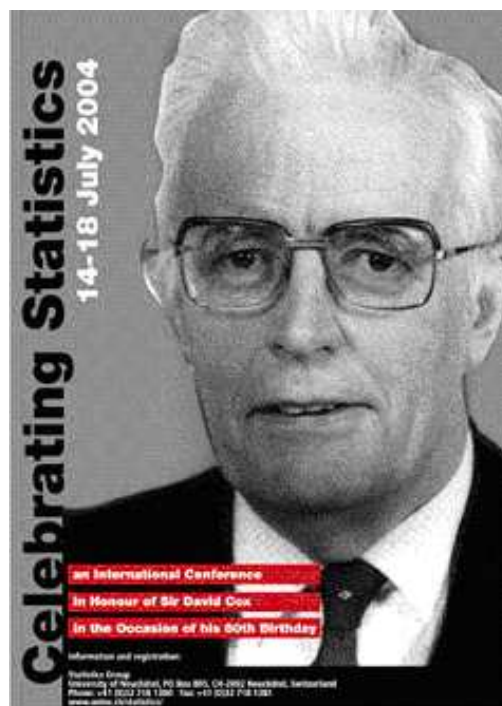
Bo Lindqvist

*Department of Mathematical Sciences*  
*NTNU*

bo@math.ntnu.no  
<http://www.math.ntnu.no/~bo/>

1

Sir David Cox (1924–)



2

## SIMPLE EXAMPLE COX-REGRESSION

$j$	$Y_j$	$x_j$	$\delta_j$
1	5	12	0
2	10	10	1
3	40	3	0
4	80	5	0
5	120	3	1
6	400	4	1
7	600	1	0

Model:

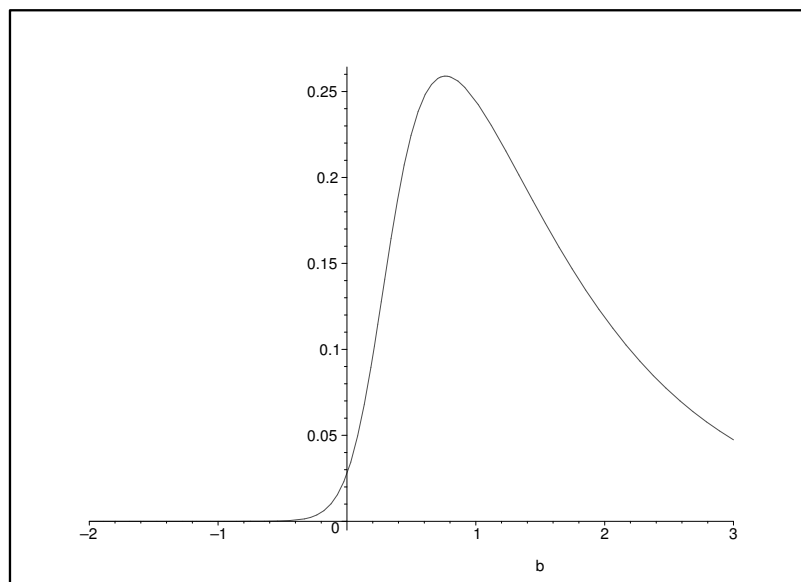
- $z(t|x) = z_0(t) \exp\{\beta x\}$

Partial likelihood:

$$L(\beta) = \frac{e^{10\beta}}{e^{10\beta} + e^{3\beta} + e^{5\beta} + e^{3\beta} + e^{4\beta} + e^{\beta}} \cdot \frac{e^{3\beta}}{e^{3\beta} + e^{4\beta} + e^{\beta}} \cdot \frac{e^{4\beta}}{e^{4\beta} + e^{\beta}}$$

3

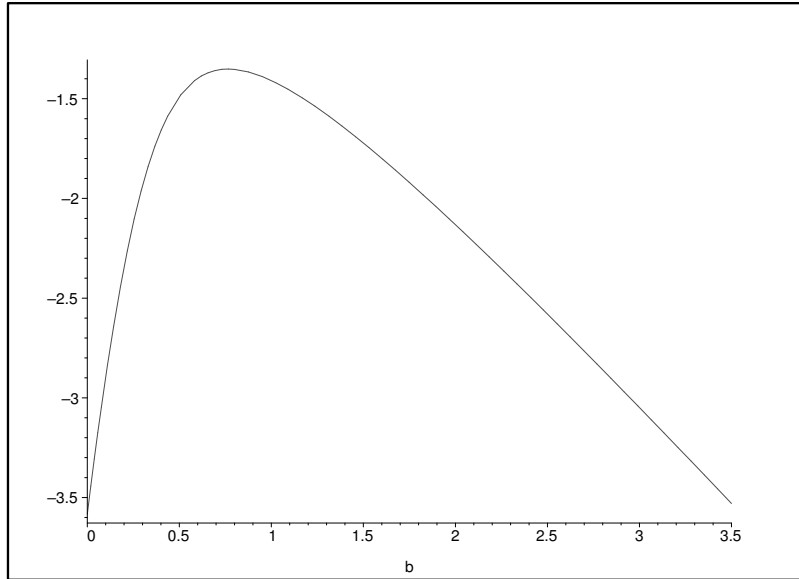
Cox' partial likelihood  $L(\beta)$  in the example:



Maximum likelihood estimate:  $\hat{\beta} = 0.765$ .

4

Cox' partial log-likelihood  $l(\beta)$  in the example:



Maximum likelihood estimate:  $\hat{\beta} = 0.765$ .

95% likelihood confidence interval: (0.1, 3.2).

5

## Weibull regression (Cox-example)

Estimation Method: Maximum Likelihood

Distribution: Weibull

Relationship with accelerating variable(s): Linear

Regression Table

Predictor	Coef	Standard Error	Z	P	95,0% Normal CI	
					Lower	Upper
Intercept	7,58636	0,548229	13,84	0,000	6,51185	8,66087
x	-0,468235	0,0842830	-5,56	0,000	-0,633427	-0,303044
Shape	2,05563	0,872169			0,894943	4,72167

Log-Likelihood = -17,450

	C1	C2	C3	C4	C5	C6	C7	C8
	Y	x	d					
1	5	12	0					
2	10	10	1					
3	40	3	0					
4	80	5	0					
5	120	3	1					
6	400	4	1					
7	600	1	0					

6

Data from Ansell & Phillips (s. 63)

**Table 3.2.** Lifetimes (in cycles) of sodium sulphur batteries

Batch 1	164	164	218	230	263	467	538	639	669
	917	1148	1678+	1678+	1678+	1678+			
Batch 2	76	82	210	315	385	412	491	504	522
	646+	678	775	884	1131	1446	1824	1827	2248
	2385	3077							

Note: Lifetimes with + are right censored observations, not failures.

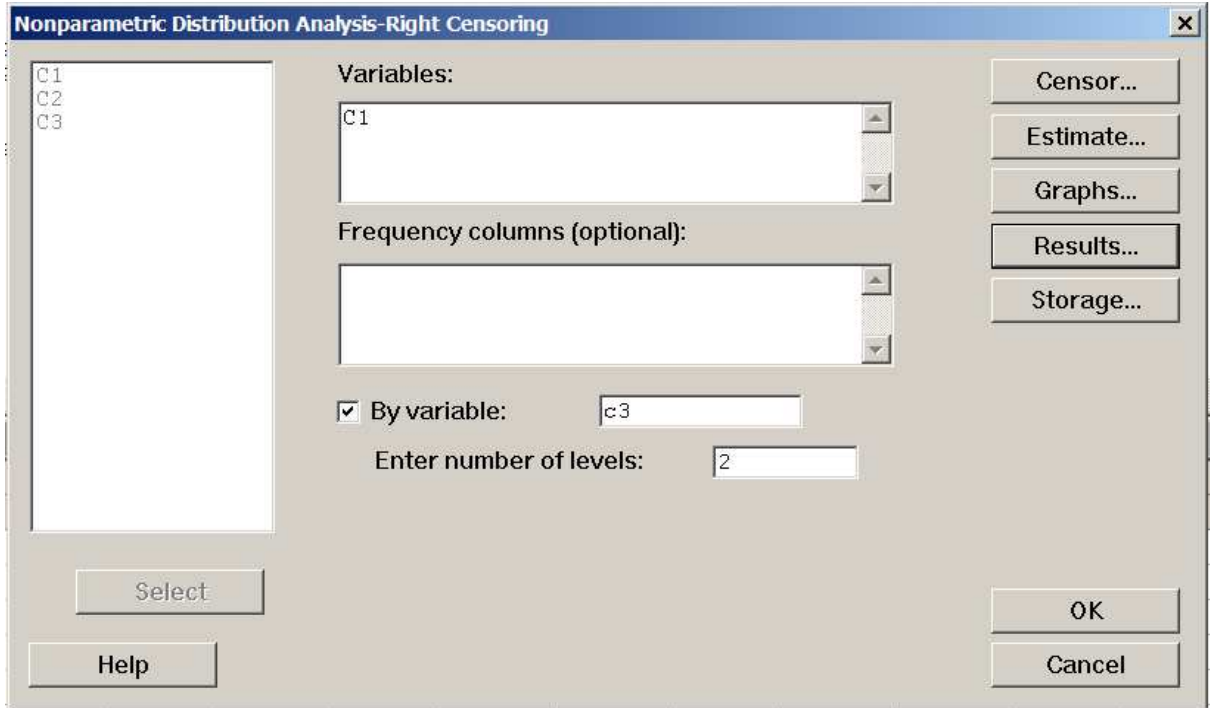
7

C1=Obs. times, C2=censoring, C3="batch" no.

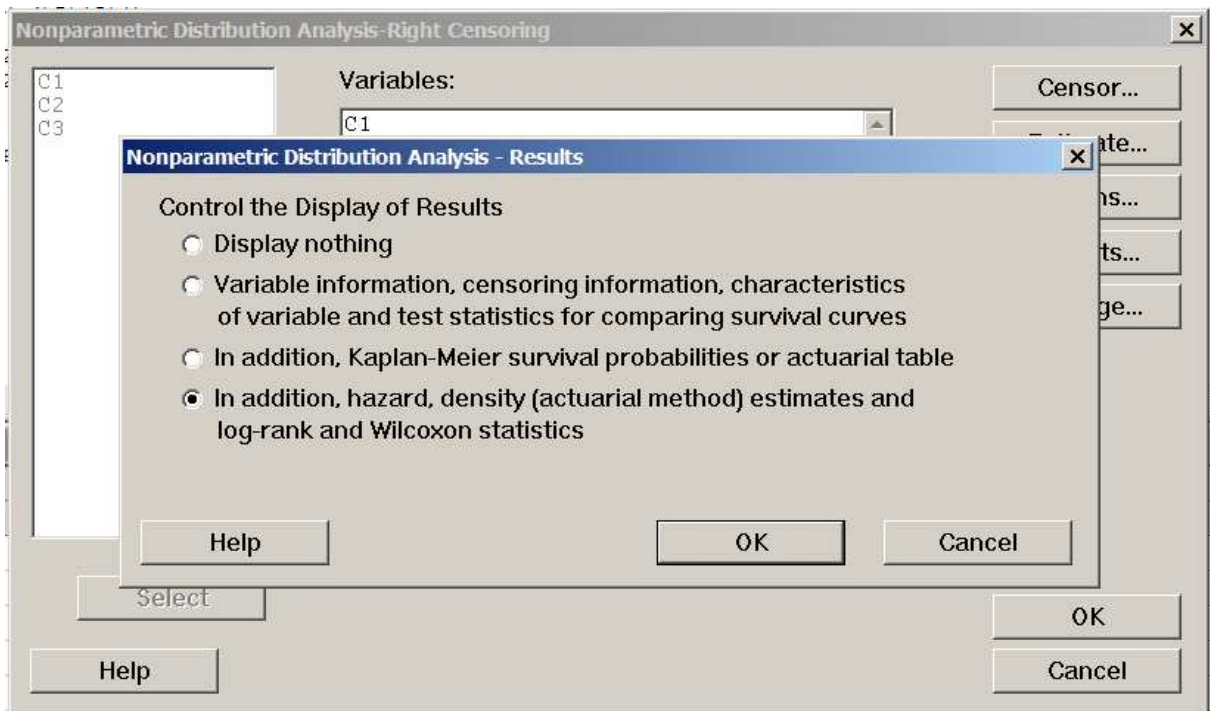
Ansell32.MTW ***												
+	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12
1	164	1	1									
2	164	1	1									
3	218	1	1									
4	230	1	1									
5	263	1	1									
6	467	1	1									
7	538	1	1									
8	639	1	1									
9	669	1	1									
10	917	1	1									
11	1148	1	1									
12	1678	0	1									
13	1678	0	1									
14	1678	0	1									
15	1678	0	1									
16	76	1	2									
17	82	1	2									
18	210	1	2									
19	315	1	2									
20	385	1	2									
21	412	1	2									
22	491	1	2									
23	504	1	2									
24	522	1	2									
25	646	0	2									

8

MINITAB-analysis of data from Ansell & Phillips:



9

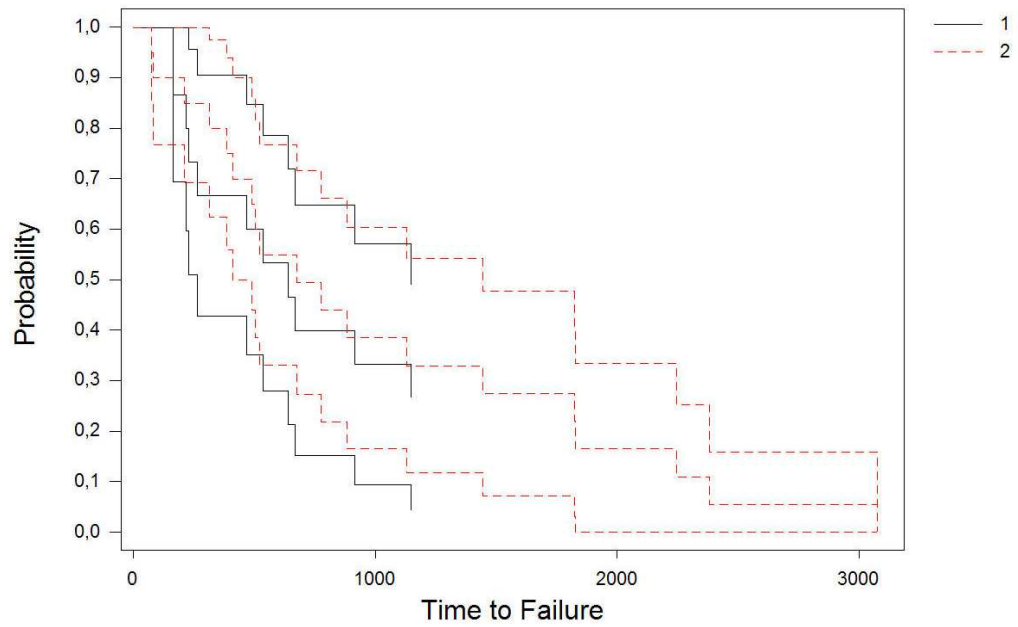


10

### Nonparametric Survival Plot for C1

Kaplan-Meier Method - 95,0% CI

Censoring Column in C2



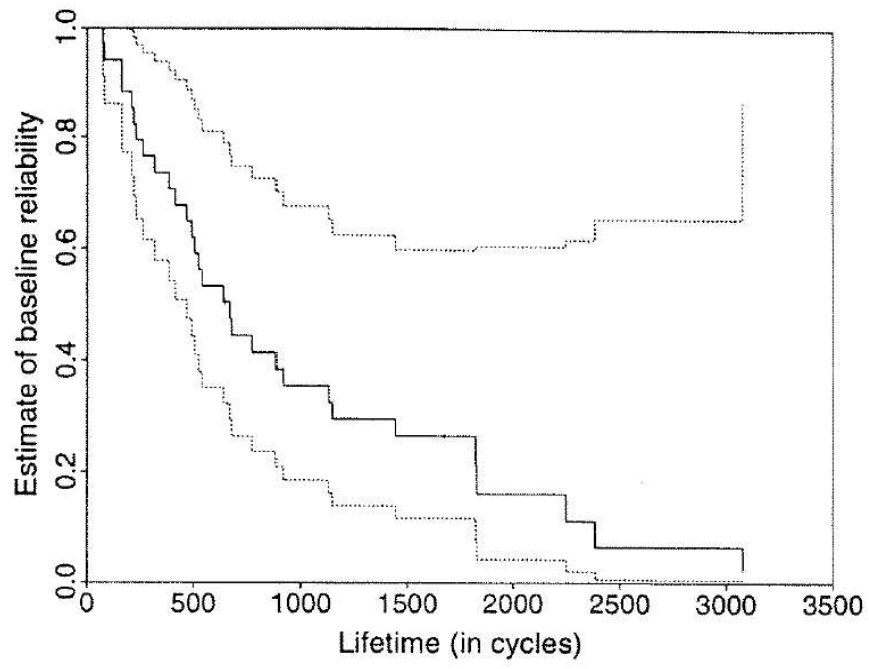
11

MINITAB-result (among other output):

Test Statistics Method	Chi-Square	DF	P-Value	Log-Rank
	0,04855	1	0,8256	

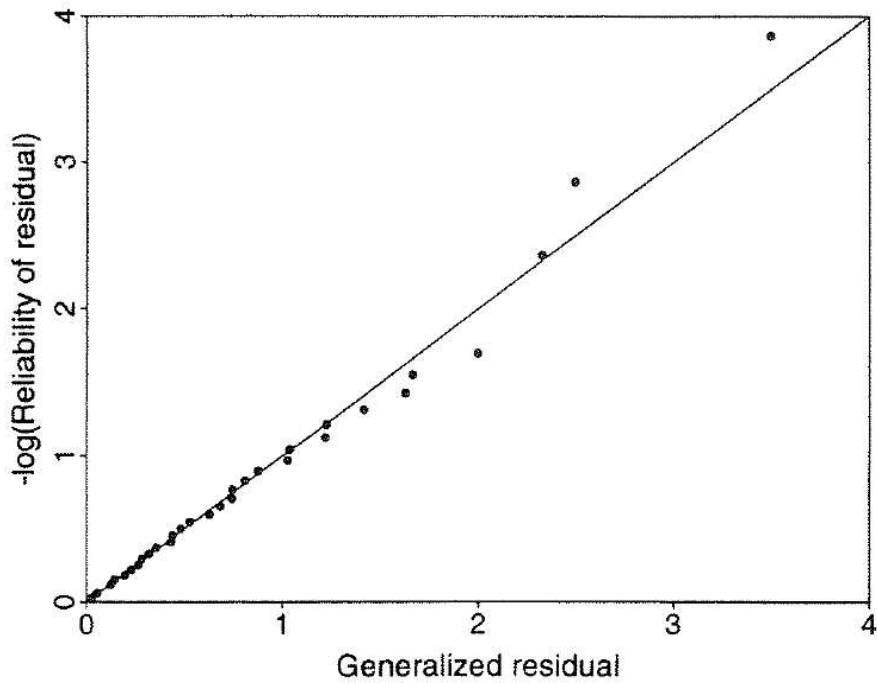
This is in accordance with the result of p. 72 (Example 3.5.2) in Ansell & Phillips.

12



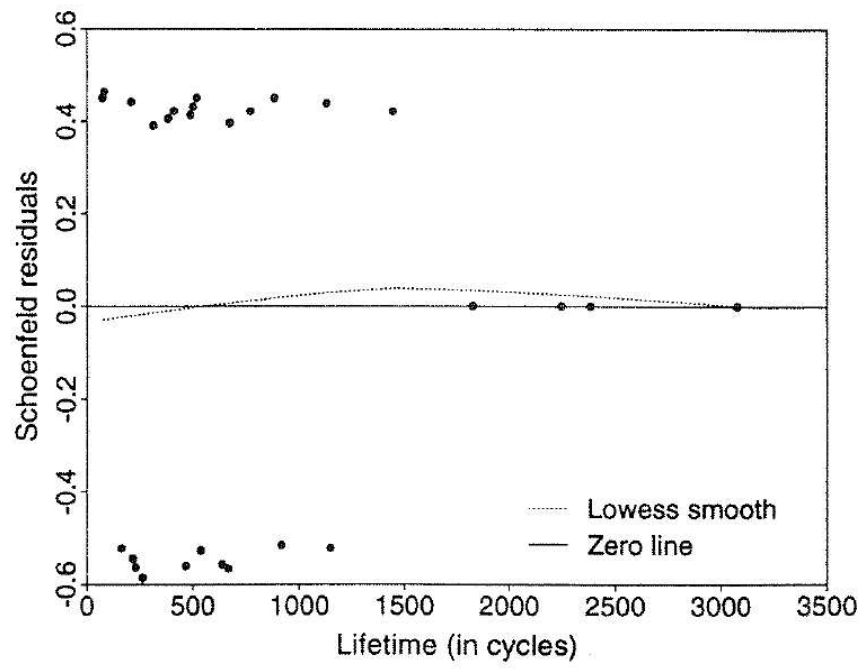
**Fig. 3.3.** Plot of the baseline reliability function for the proportional hazards model for the sodium sulphur battery data with 95% confidence limits

13



**Fig. 3.5.** Plot of the generalized residuals of the proportional hazards model for the sodium sulphur battery data

14

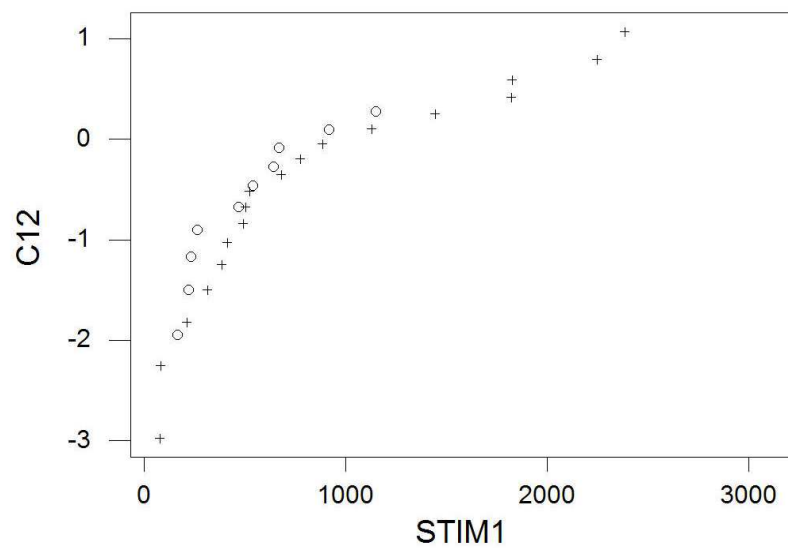


**Fig. 3.7.** Plot of the Schoenfeld residuals for batch of the proportional hazards model for the sodium sulphur battery data

15

LOG MINUS LOG PLOT vs. "t" FOR A&P DATA (s.63)

Parallel plots indicate Proportional Hazard

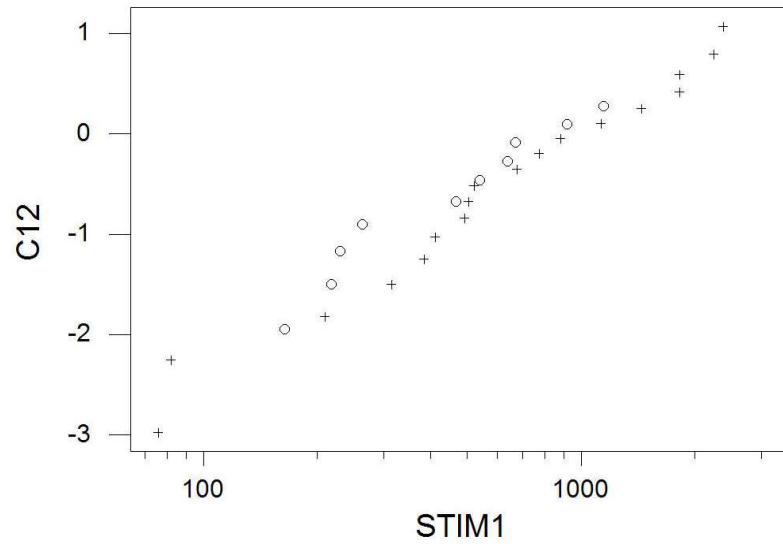


16



LOG MINUS LOG PLOT vs. "log t" FOR A&P DATA (s.63)

Straight lines indicate Weibull distribution



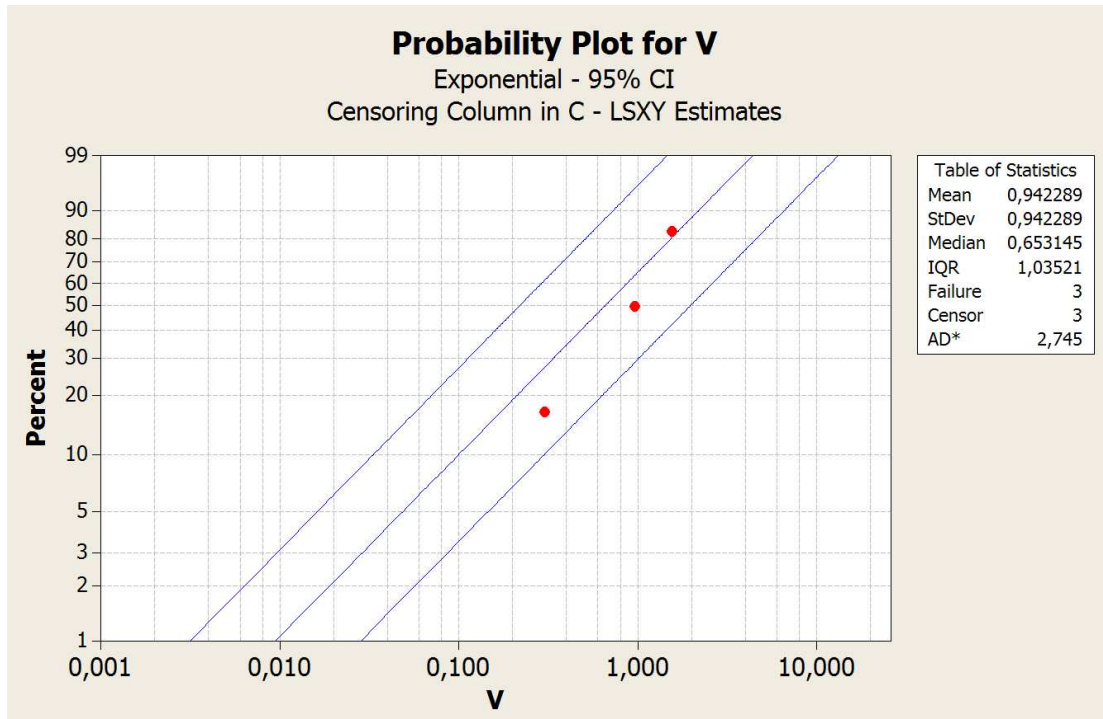
17

COX-SNELL RESIDUALS SIMPLE COX-EXAMPLE

Worksheet 1 ***				
↓	C1	C2	C3	C
	V	C	V*	
1	0,9593	1	0,9593	
2	0,0045	0	1,0045	
3	0,0209	0	1,0209	
4	0,3017	1	0,3017	
5	1,5567	1	1,5567	
6	0,1569	0	1,1569	
7				

18

COX-SNELL RESIDUALS SIMPLE COX-EXAMPLE



MODIFIED COX-SNELL RESIDUALS SIMPLE COX-EXAMPLE

