WEEK 11, 2006 March 16 and 17

TMA4275 LIFETIME ANALYSIS

Bo Lindqvist

Department of Mathematical Sciences NTNU

bo@math.ntnu.no http://www.math.ntnu.no/~bo/

Censored and Truncated Data

- An observation is right censored at y: Unit is in our data, we know T > y. Contribution to L: P(T > y) = R(y).
- An observation is left censored at y: Unit is in our data, we know T < y. Contribution to L: P(T < y) = F(y).
- An observation is right truncated at y: Unit is in our data only if $T \leq y$. We do not know about the units with T > y. Contribution to L of observed failure at t: $\Delta^{-1}P(t \leq T \leq t + \Delta | T \leq y) \approx f(t)/F(y)$.
- An observation is left truncated at y: Unit is in our data only if $T \ge y$. We do not know about the units with T < y. Contribution to L of observed failure at t: $\Delta^{-1}P(t \le T \le t + \Delta | T \ge y) \approx f(t)/R(y)$.

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Examples of left truncation:

- Ultrasonic inspection of material. Signal amplitude only trusted when above limit τ . Condition for being in the data set is $T > \tau$.
- Life data with pretest screening. Electronic component is burn-in tested for 1000 hours. Only the ones that passed this test are observed later. The number of components failing at burn-in is unknown. Condition for being in the data set is T > 1000.

Example of right truncation:

- Casting for automobile engine mounts. Pore size distribution below 10 microns only are recorded (other units are immediately discarded). Condition for being in the data set is T < 10 microns.
- Study group of individuals with AIDS diagnosis before July 1, 1986, and known date of HIV-infection (due to blood-transfusion). Let $T_i =$ time from HIV-infection to AIDS diagnosis for *i*th individual. Then condition for being in the data set is that $T_i \leq v_i$ where v_i is time from HIV-infection of the *i*th individual to July 1, 1986. (Kalbfleisch and Lawless, 1989)

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COMPUTER PROGRAM EXECUTION TIME vs SYSTEM LOAD

Data: 17 observations of (T,x)

- Time to complete a computationally intensive task.
- Information from the Unix uptime command
- Predictions needed for scheduling subsequent steps in a multistep computational process.

Seconds (T)	Load (x)	Seconds (T)	Load (x)
123	2,74	110	,60
704	5,47	213	2,10
184	2,13	284	3,10
113	1,00	317	5,86
94	,32	142	1,18
76	,31	127	,57
78	,51	96	1,10
98	,29	111	1,89
240	,96		

Covariates (explanatory variables) for failure times

Useful covariates explain/predict why some units fail quickly and some units survive a long time:

- Continuous variables like stress, temperature, voltage, and pressure.
- Discrete variables like number of hardening treatments or number of simultaneous users of a system.
- Categorical variables like manufacturer, design, and location.

Regression model relates failure time distribution to covariates $x = (x_1, \ldots, x_k)$:

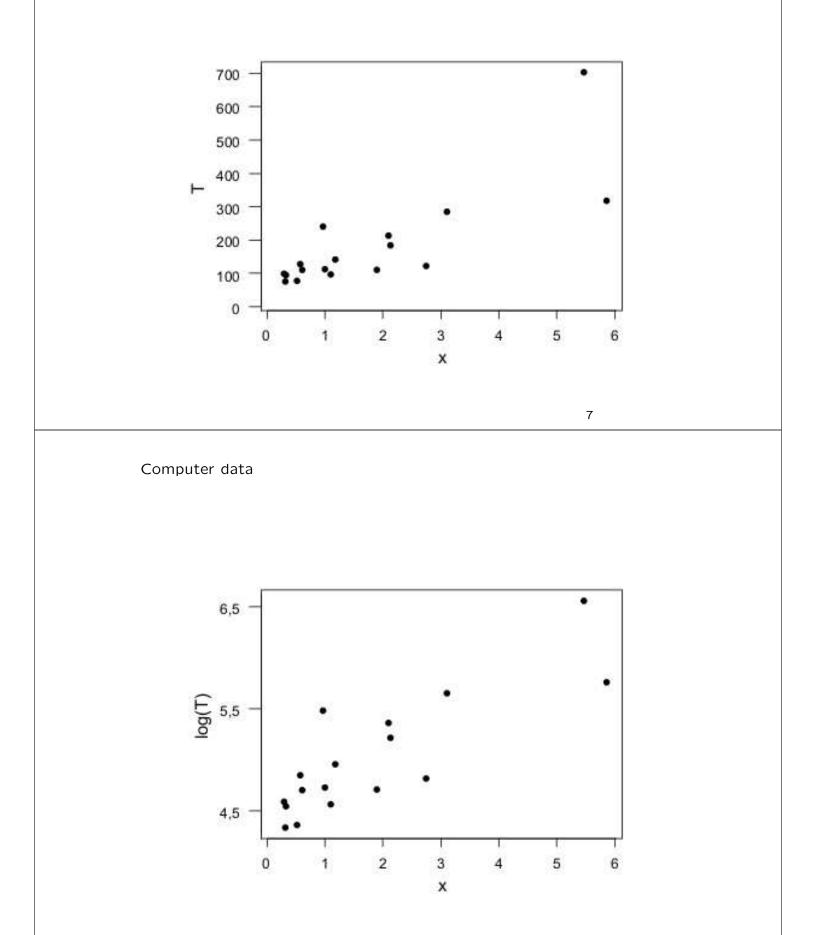
 $P(T \le t) = F(t) = F(t;x)$

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Why regression models?

- Want to find factors which explain the reliability of an item
- Want to exclude factors which do not influence the reliability
- Obtain new knowledge about failure mechanisms
- Make better predictions for reliability of an item

Computer data



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•	C1 T	x	log(T)	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15	C16	Antonio	
C11	C1 T 123	x 2,74	log(T) 4,81218	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15	C16	Antonio	
C11	C1 T 123 704	x 2,74 5,47	log(T) 4,81218 6,55678	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15	C16	Antonio	
C11	C1 T 123 704 184	x 2,74 5,47 2,13	log(T) 4,81218 6,55678 5,21494	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15	C16	Antonio	
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C11	C1 T 123 704 184 113 94 76	x 2,74 5,47 2,13 1,00 0,32 0,31	log(T) 4,81218 6,55678 5,21494 4,72739 4,54329 4,33073	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15	C16	Antonio	
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MTB > Let c3=log(c1) ** C Responses are uncens/arbitrarily censored data Estimate MTB > Plot c3*c2; Start variables: C1 Graphs SUBC> Schnotation. Freq. columns: Options Plot log(T) * x Model: Options * C1 C2 C3 Model: C1 * C1 C2 C3 T Keults Options * C1 C2 C3 C1 C1 C2 C3 * C1 C2 C3 C3 C4 C3 C4 C4 * C1 C2 C3 C3 C4 C1 C4 C4 * C1 C2 C3 C3 C4 C4 C4 C4 1 123 2,74 4,81218 Select Assumed distribution: Cognormal base e OK Cancel C4 C4 4 113 1,00 4,72739 Help Cancel C3 C3 C3 C3 C3 C4 C4 C4 C4 C4 C4 C4 C4 <				C1 T	Responses	are uncens/ric	ht censored dat	ta	Censor	1			
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Regression with Life Data: T versus x Response Variable: T Censoring Information Count Uncensored value 17 Estimation Method: Maximum Likelihood Distribution: Lognormal base e Regression Table Standard 95,0% Normal CI Z P Lower Upper 4,2756 4,7116 Predictor Coef Error Intercept 4,4936 0,1112 40,39 0,000 4,2756 0,20069 0,38080 x 0,29075 0,04595 6,33 0,000 Scale 0,31247 0,05359 0,22327 0,43730 Log-Likelihood = -89,498Anderson-Darling (adjusted) Goodness-of-Fit

Standardized Residuals = 0,8356; Cox-Snell Residuals = 0,8170

Regression with Life Data: C1 versus C2

Response Variable: C1

Censoring Information Count Uncensored value 17

Estimation Method: Maximum Likelihood Distribution: Weibull

Regression Table

		Standard			95,0%	Normal CI
Predictor	Coef	Error	Z	P	Lower	Upper
Intercept	4,6182	0,1219	37,88	0,000	4,3792	4,8572
C2	0,31118	0,04939	6,30	0,000	0,21437	0,40799
Shape	3,0604	0,5245			2,1873	4,2820

Log-Likelihood = -91,504

Anderson-Darling (adjusted) Goodness-of-Fit

Likelihood for Lognormal Distribution Simple Regression Model with Right Censored Data

The likelihood for n independent observations has the form

$$L(\beta_0, \beta_1, \sigma) = \prod_{i=1}^n L_i(\beta_0, \beta_1, \sigma; \mathsf{data}_i)$$

=
$$\prod_{i=1}^n \left\{ \frac{1}{\sigma t_i} \phi_{\mathsf{nor}} \left[\frac{\mathsf{log}(t_i) - \mu_i}{\sigma} \right] \right\}^{\delta_i} \left\{ 1 - \Phi_{\mathsf{nor}} \left[\frac{\mathsf{log}(t_i) - \mu_i}{\sigma} \right] \right\}^{1 - \delta_i}$$

where $\mathsf{data}_i = (x_i, t_i, \delta_i), \ \mu_i = \beta_0 + \beta_1 x_i.$

where data_i = (x_i, t_i, δ_i) , $\mu_i = \beta_0 + \beta_1 x_i$,

$$\delta_i = \begin{cases} 1 & \text{exact observation} \\ 0 & \text{right censored observation} \end{cases}$$

 $\phi_{nor}(z)$ is the standardized normal pdf and $\Phi_{nor}(z)$ is the corresponding normal cdf.

The parameters are $\theta = (\beta_0, \beta_1, \sigma)$.

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Estimated Parameter Variance-Covariance Matrix

Local (observed information) estimate

$$\begin{split} \hat{\Sigma}_{\hat{\theta}} &= \begin{bmatrix} \widehat{\operatorname{Var}}(\hat{\beta}_{0}) & \widehat{\operatorname{Cov}}(\hat{\beta}_{0},\hat{\beta}_{1}) & \widehat{\operatorname{Cov}}(\hat{\beta}_{0},\hat{\sigma}) \\ \widehat{\operatorname{Cov}}(\hat{\beta}_{1},\hat{\beta}_{0}) & \widehat{\operatorname{Var}}(\hat{\beta}_{1}) & \widehat{\operatorname{Cov}}(\hat{\beta}_{1},\hat{\sigma}) \\ \widehat{\operatorname{Cov}}(\hat{\sigma},\hat{\beta}_{0}) & \widehat{\operatorname{Cov}}(\hat{\sigma},\hat{\beta}_{1}) & \widehat{\operatorname{Var}}(\hat{\sigma}) \end{bmatrix} \\ &= \begin{bmatrix} -\frac{\partial^{2}\mathcal{L}(\beta_{0},\beta_{1},\sigma)}{\partial\beta_{0}^{2}} & -\frac{\partial^{2}\mathcal{L}(\beta_{0},\beta_{1},\sigma)}{\partial\beta_{0}\partial\beta_{1}} & -\frac{\partial^{2}\mathcal{L}(\beta_{0},\beta_{1},\sigma)}{\partial\beta_{0}\partial\sigma} \\ -\frac{\partial^{2}\mathcal{L}(\beta_{0},\beta_{1},\sigma)}{\partial\beta_{1}\partial\beta_{0}} & -\frac{\partial^{2}\mathcal{L}(\beta_{0},\beta_{1},\sigma)}{\partial\beta_{1}^{2}} & -\frac{\partial^{2}\mathcal{L}(\beta_{0},\beta_{1},\sigma)}{\partial\beta_{1}\partial\sigma} \\ -\frac{\partial^{2}\mathcal{L}(\beta_{0},\beta_{1},\sigma)}{\partial\sigma\partial\beta_{0}} & -\frac{\partial^{2}\mathcal{L}(\beta_{0},\beta_{1},\sigma)}{\partial\sigma\partial\beta_{1}} & -\frac{\partial^{2}\mathcal{L}(\beta_{0},\beta_{1},\sigma)}{\partial\sigma\partial\beta_{1}} \end{bmatrix}^{-1} \end{split}$$

Partial derivatives are evaluated at $\hat{\beta}_0, \hat{\beta}_1, \hat{\sigma}$.

Standard Errors and Confidence Intervals for Parameters

• Lognormal ML estimates for the computer time experiment were $\hat{\theta} = (\hat{\beta}_0, \hat{\beta}_1, \hat{\sigma}) = (4.49, .290, .312)$ and an estimate of the variance-covariance matrix for $\hat{\theta}$ is

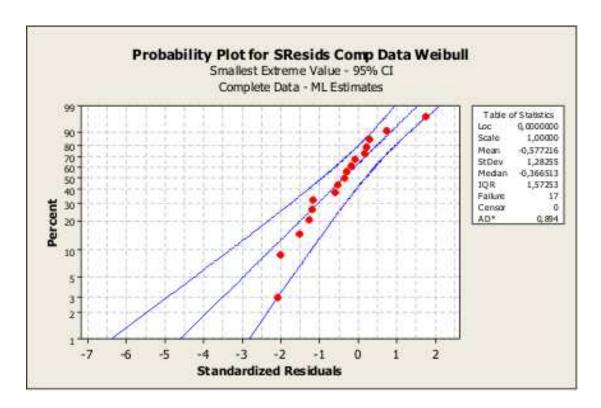
$$\hat{\Sigma}_{\hat{\theta}} = \begin{bmatrix} .012 & -.0037 & 0 \\ -.0037 & .0021 & 0 \\ 0 & 0 & .0029 \end{bmatrix}.$$

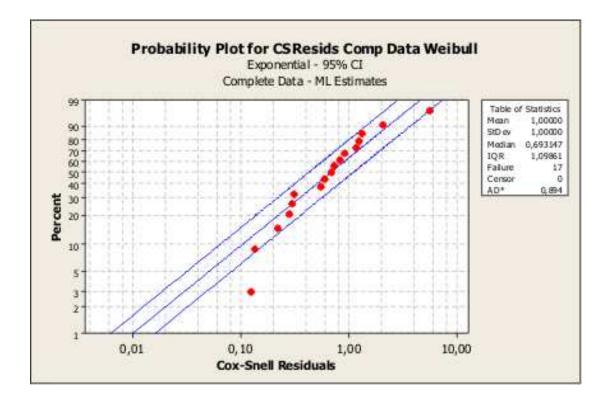
• Normal-approximation confidence interval for the computer execution time regression slope is

$$[\beta_1, \quad \tilde{\beta}_1] = \hat{\beta}_1 \pm z_{(.975)} \widehat{se}_{\hat{\beta}_1} = .290 \pm 1.96(.046) = [.20, \quad .38]$$

where $\widehat{se}_{\widehat{\beta}_1} = \sqrt{.0021} = .046$.







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Ordinary residuals

$$y_i - x'_i \hat{\beta}$$

where

yi is the ith response value

 $\mathbf{x'}_i$ is the vector of predictor values associated with the ith response value

 $\hat{\boldsymbol{\beta}}$ represents the estimated regression coefficients

Standardized residuals

 $\frac{y_i - x_i'\hat{\beta}}{\hat{\sigma}}$

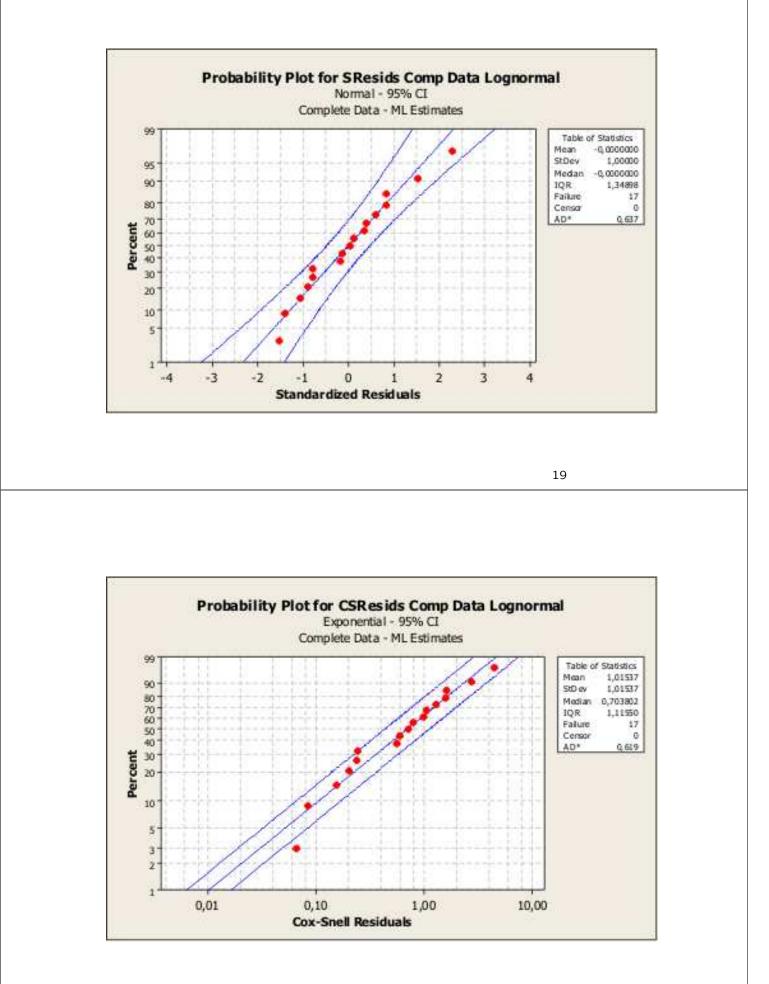
where $\hat{\sigma}$ is the estimated scale parameter.

Cox-Snell residuals

 $-\ln(\hat{R}(y_i))$

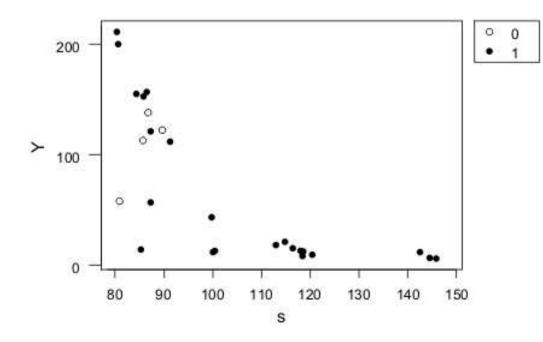
where

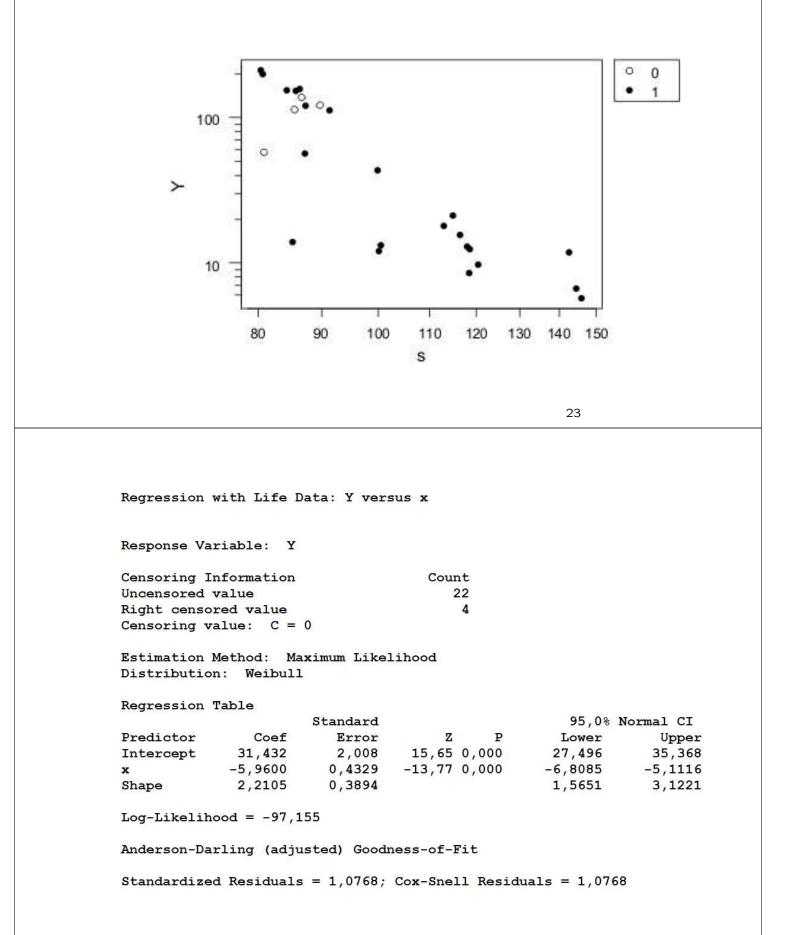
 $\hat{R}(y_i)$ is the estimated survival (reliability) probability for the response value yi ln(x) is the natural log of x



Row	Pseudo- stress	k-Cycles	Status	(1=failed, 0=censored)
i	S	Y	С	DATA DESCRIPTION:
1	80,3	211,629	1	Low-Cycle Fatigue Life of Nickel-Base
2		200,027	1	Superalloy Specimens
3	80,8	57,923	0	(in units of thousands of cycles
4	84,3	155,000	1	to failure).
5	85,2	13,949	1	
6	85,6	112,968	0	Data from Nelson (1990):
7	85,8	152,680	1	
8	86,4	156,725	1	SUPER ALLOY DATA
9	86,7	138,114	0	
10	87,2	56,723	1	
11	87,3	121,075	1	
12	89,7	122,372	0	
13	91,3	112,002	1	
14	99,8	43,331	1	
15	100,1	12,076	1	
16	100,5	13,181	1	
17		18,067	1	
18	114,8	21,300	1	
19	116,4	15,616	1	
20	118,0	13,030	1	
21		8,489	1	
22	118,6	12,434	1	
23	-	9,750	1	
24	142,5	11,865	1	
25		6,705	1	
26	145,9	5,733	1	
				21

Plot of Y vs s

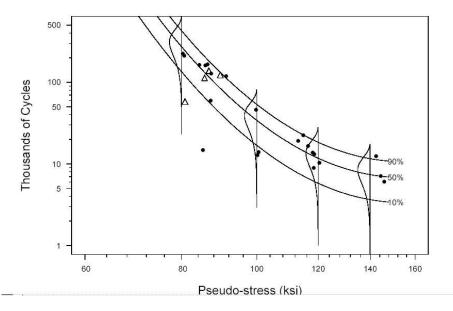




```
Regression with Life Data: Y versus x
Response Variable: Y
Censoring Information
                                     Count
Uncensored value
                                        22
                                          4
Right censored value
Censoring value: C = 0
Estimation Method: Maximum Likelihood
Distribution: Weibull
Regression Table
                       Standard
                                                       95,0% Normal CI
Predictor
                Coef
                                       Z
                          Error
                                             Ρ
                                                      Lower
                                                                  Upper
Intercept
              217,61
                          62,13
                                    3,50 0,000
                                                      95,83
                                                                 339,39
              -85,52
                          26,55
                                   -3,22 0,001
                                                    -137,55
                                                                 -33,49
x
               8,483
                          2,831
                                    3,00 0,003
                                                      2,934
                                                                 14,032
x*x
Shape
              2,6685
                         0,4777
                                                     1,8789
                                                                 3,7900
Log-Likelihood = -93,382
Anderson-Darling (adjusted) Goodness-of-Fit
Standardized Residuals = 0,9283; Cox-Snell Residuals = 0,9283
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Log-Quadratic Weibull Regression Model with Constant ($\beta = 1/\sigma$) Fit to the Fatigue Data $\log[\hat{t}_p(x)] = \hat{\mu}(x) + \Phi_{\text{SeV}}^{-1}(p)\hat{\sigma}, x = \log(\text{pseudo-stress})$



Regression with Life Data: Y versus x

Response Variable: Y

Table of Percentiles

Table Of	rercen	CITCO				
				Standard	95,0%	Normal CI
Percent	S	x	Percentile	Error	Lower	Upper
10	80	4,3820	133,3747	34,0579	80,8565	220,0048
10	100	4,6052	16,7928	3,4263	11,2577	25,0494
10	120	4,7875	5,7830	1,2364	3,8034	8,7929
10	140	4,9416	3,6458	0,8760	2,2766	<mark>5,8386</mark>
50	80	4,3820	270,1879	56,0580	179,9121	405,7621
50	100	4,6052	34,0186	4,3027	26,5494	43,5891
50	120	4,7875	11,7151	1,5950	8,9713	15,2980
50	140	4,9416	7,3856	1,2828	5,2547	10,3807
90	80	4,3820	423,6933	90,4646	278,8097	643,8659
90	100	4,6052	53,3461	6,8162	41,5281	68,5272
90	120	4,7875	18,3709	2,4567	14,1351	23,8760
90	140	4,9416	11,5817	1,9813	8,2824	16,1952

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ESTIMERT KOVARIANSMATRISE FOR $(\hat{eta}_0,\hat{eta}_1,\hat{eta}_2,\hat{\sigma})$

3860,37	-1649,17	175,82	-0,80
-1649,17	704,70	-75,15	0,33
175,82	-75,15	8,02	-0,03
-0,80	0,33	-0,03	0,23

