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## TMA4275 LIFETIME ANALYSIS

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### RIGHT CENSORED DATA EXPONENTIAL AND WEIBULL MODELS

Row	C1	C2
1	0,35	1
2	0,50	0
3	0,75	0
4	1,00	1
5	1,30	1
6	1,80	1
7	3,00	0
8	3,15	0
9	4,85	0
10	5,50	1
11	5,50	0
12	6,25	0

Variable: C1  
Censoring Information  
Count  
Uncensored value 5  
Right censored value 7  
Censoring value: C2 = 0

Estimation Method: Maximum Likelihood Distribution: Exponential

#### Parameter Estimates

Parameter	Estimate	Standard Error	95,0% Normal CI	
			Lower	Upper
Shape	1,00000			
Scale	6,790	3,037	2,826	16,313

Log-Likelihood = -14,577

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## WEIBULL MODEL

Variable: C1

Censoring Information

Count

Uncensored value 5

Right censored value 7

Censoring value: C2 = 0

Estimation Method: Maximum Likelihood Distribution: Weibull

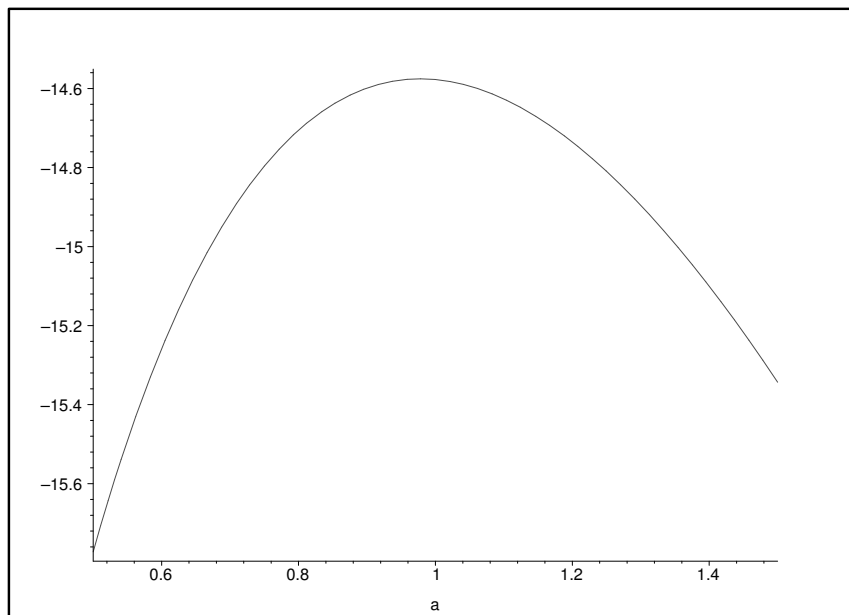
### Parameter Estimates

Parameter	Estimate	Standard Error	95,0% Normal CI	
			Lower	Upper
Shape	0,9780	0,3694	0,4665	2,0504
Scale	6,880	3,517	2,526	18,740

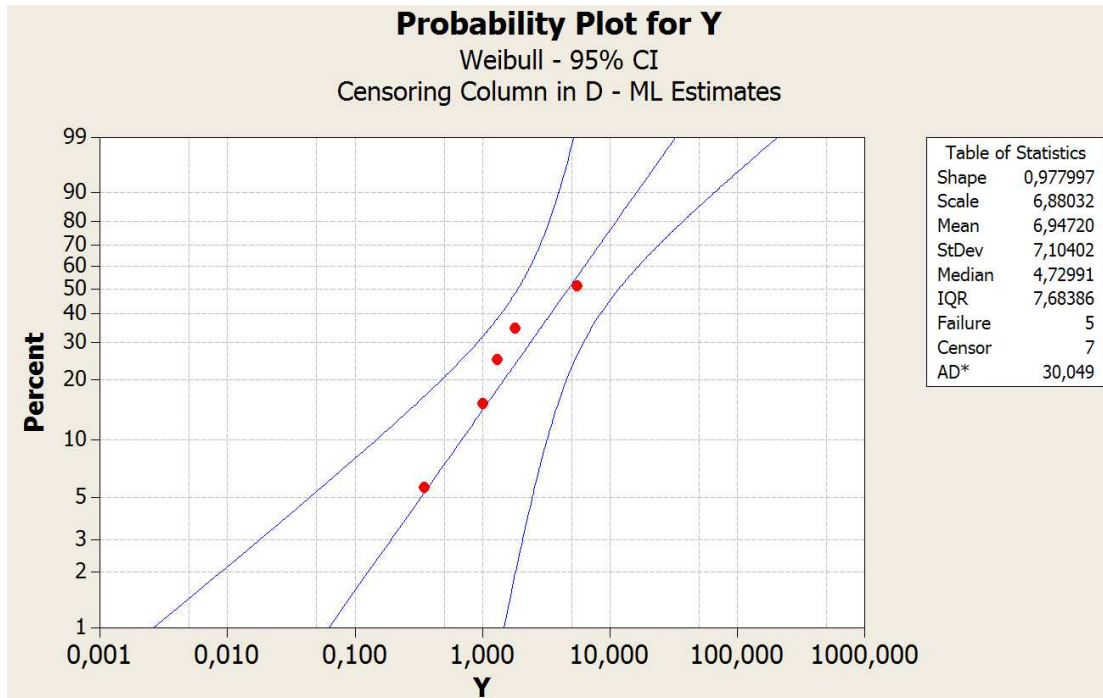
Log-Likelihood = -14,576

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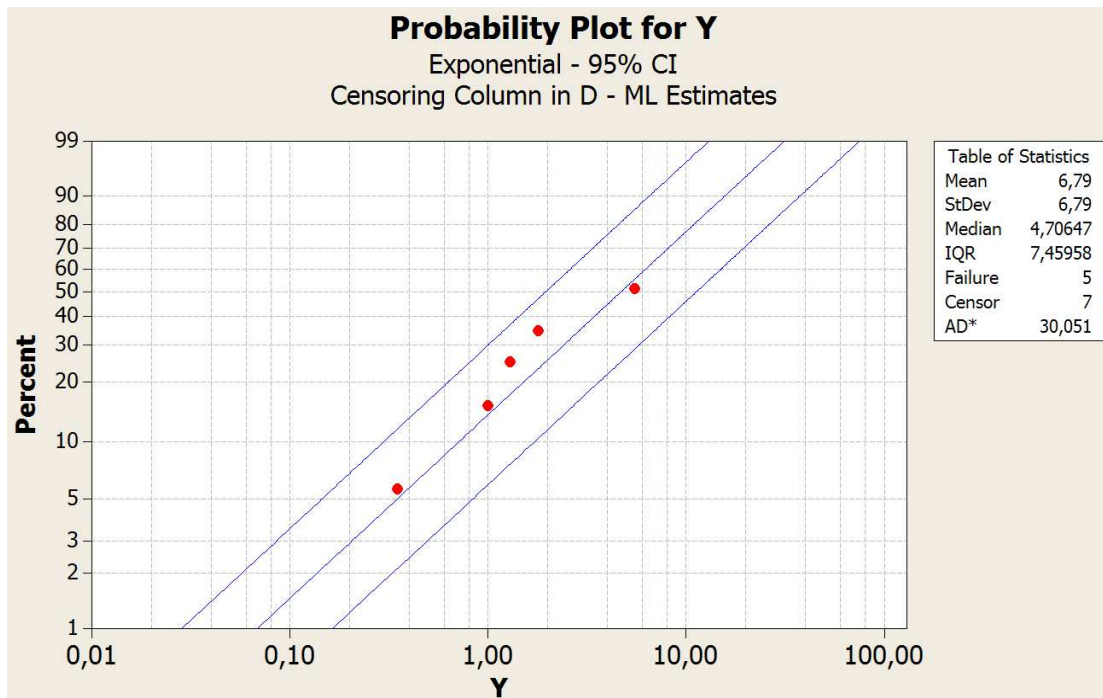
## PROFILE LOG LIKELIHOOD FOR SHAPE PARAMETER $\alpha$ IN WEIBULL DISTRIBUTION



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## Shock absorber data

Y = kilometers to failure, F = failure mode (0 is censoring)

Row	Y	F			
1	6700	1	19	14300	1
2	6950	0	20	17520	1
3	7820	0	21	17540	0
4	8790	0	22	17890	0
5	9120	2	23	18450	0
6	9660	0	24	18960	0
7	9820	0	25	18980	0
8	11310	0	26	19410	0
9	11690	0	27	20100	2
10	11850	0	28	20100	0
11	11880	0	29	20150	0
12	12140	0	30	20320	0
13	12200	1	31	20900	2
14	12870	0	32	22700	1
15	13150	2	33	23490	0
16	13330	0	34	23490	0
17	13470	0	35	26510	1
18	14040	0	36	27410	0
			37	27490	1
			38	27890	0
				28100	0

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## Shock Absorber Failure Data

First reported in O'Connor (1985).

- Failure times, in number of kilometers of use, of vehicle shock absorbers.
- Two failure modes, denoted by M1 and M2.
- One might be interested in the distribution of time to failure for mode M1, mode M2, or in the overall failure-time distribution of the part.

Here we do not differentiate between modes M1 and M2. We will estimate the distribution of time to failure by either mode M1 or M2.

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## Shock absorber data

Estimation Method: Maximum Likelihood Distribution: Lognormal

### Parameter Estimates

Parameter	Estimate	Standard Error	95,0% Normal CI	
			Lower	Upper
Location	10,1448	0,144175	9,86219	10,4273
Scale	0,530068	0,112683	0,349447	0,804047

Log-Likelihood = -124,609

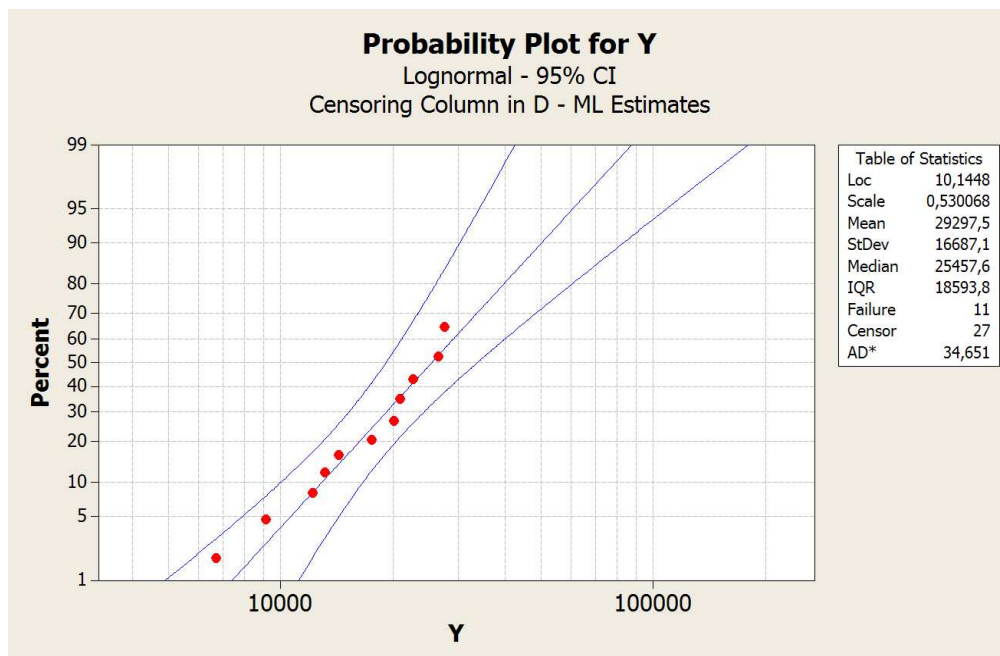
Goodness-of-Fit Anderson-Darling (adjusted) = 34,651

### Characteristics of Distribution

	Estimate	Standard Error	95,0% Normal CI	
			Lower	Upper
Mean(MTTF)	29297,5	5455,91	20338,3	42203,2
Standard Deviation	16687,1	6787,01	7519,35	37032,5
Median	25457,6	3670,36	19190,9	33770,7
First Quartile(Q1)	17805,2	2062,96	14188,1	22344,4
Third Quartile(Q3)	36399,0	7252,61	24631,2	53789,0
Interquartile Range(IQR)	18593,8	6115,60	9758,96	35426,9

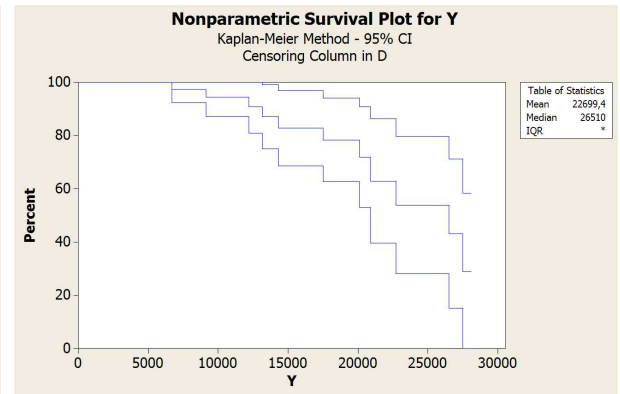
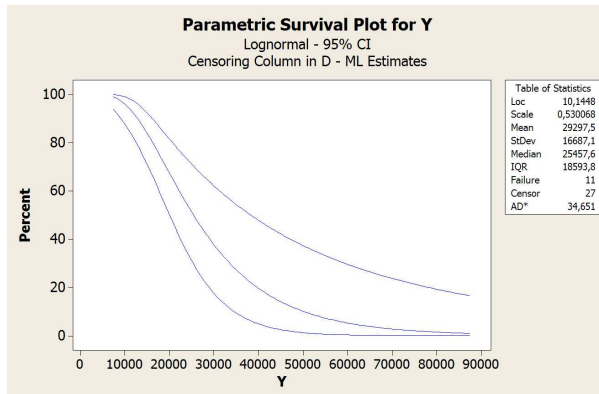
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## Shock absorber data



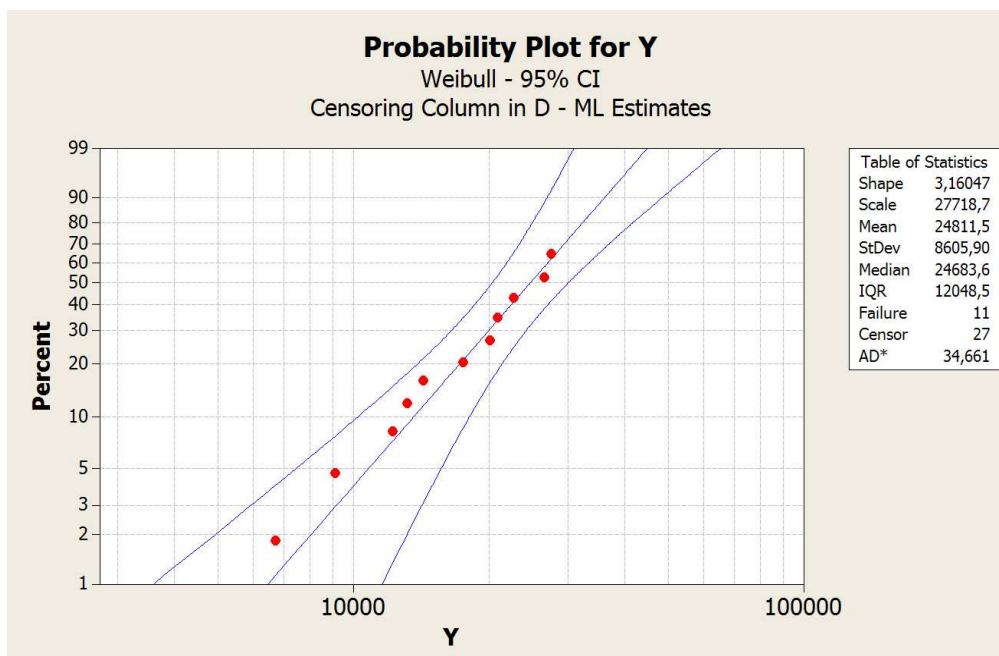
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## Shock absorber data



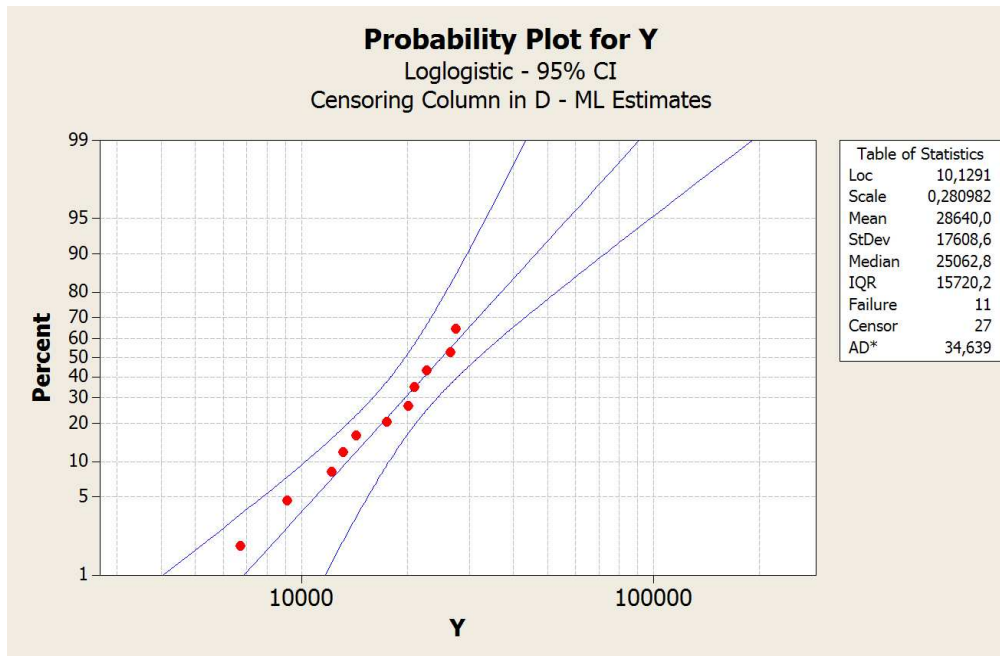
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## Shock absorber data



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## Shock absorber data



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**Shock absorber data:**  
Results for loglogistic (left), lognormal (middle), Weibull (right)

Loc	10,1291
Scale	0,280982
Mean	28640,0
StDev	17608,6
Median	25062,8
IQR	15720,2
Failure	11
Censor	27
AD*	34,639

Loc	10,1448
Scale	0,530068
Mean	29297,5
StDev	16687,1
Median	25457,6
IQR	18593,8
Failure	11
Censor	27
AD*	34,651

Shape	3,16047
Scale	27718,7
Mean	24811,5
StDev	8605,90
Median	24683,6
IQR	12048,5
Failure	11
Censor	27
AD*	34,661

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## Confidence Interval for the Mean Life of a New Insulating Material

- A life test for a new insulating material used 25 specimens which were tested simultaneously at a high voltage of 30 kV.
- The test was run until 15 of the specimens failed.
- The 15 failure times (hours) were recorded as:

1.08, 12.20, 17.80, 19.10, 26.00, 27.90, 28.20, 32.20, 35.90, 43.50, 44.00, 45.20, 45.70, 46.30, 47.80

Then  $TTT = 1.08 + \dots + 47.80 + 10 \times 47.80 = 950.88$  hours.

- The ML estimate of  $\theta$  and a 95% confidence interval are:

$$\begin{aligned}\hat{\theta} &= 950.88/15 = 63.392 \text{ hours} \\ \left[ \theta, \tilde{\theta} \right] &= \left[ \frac{2(950.88)}{\chi^2_{(.975;30)}}, \frac{2(950.88)}{\chi^2_{(.025;30)}} \right] = \left[ \frac{1901.76}{46.98}, \frac{1901.76}{16.79} \right] \\ &= [40.48, 113.26].\end{aligned}$$

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### Pike (1966) cancer data for rats

Row	Y	D
1	143	1
2	164	1
3	188	1
4	188	1
5	190	1
6	192	1
7	206	1
8	209	1
9	213	1
10	216	1
11	220	1
12	227	1
13	230	1
14	234	1
15	246	1
16	265	1
17	304	1
18	216	0
19	244	0

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# Pike (1966) cancer data for rats: 3-parameter Weibull

## Distribution Analysis: C1

Variable: C1

Censoring Information Count  
 Uncensored value 17  
 Right censored value 2

Censoring value: C2 = 0

Estimation Method: Maximum Likelihood

Distribution: 3-Parameter Weibull

### Parameter Estimates

Parameter	Estimate	Standard Error	95,0% Normal CI	
			Lower	Upper
Shape	2,71148	1,05876	1,26135	5,82878
Scale	108,383	32,5734	60,1367	195,335
Threshold	122,026	28,6924	65,7898	178,262

Log-Likelihood = -87,324

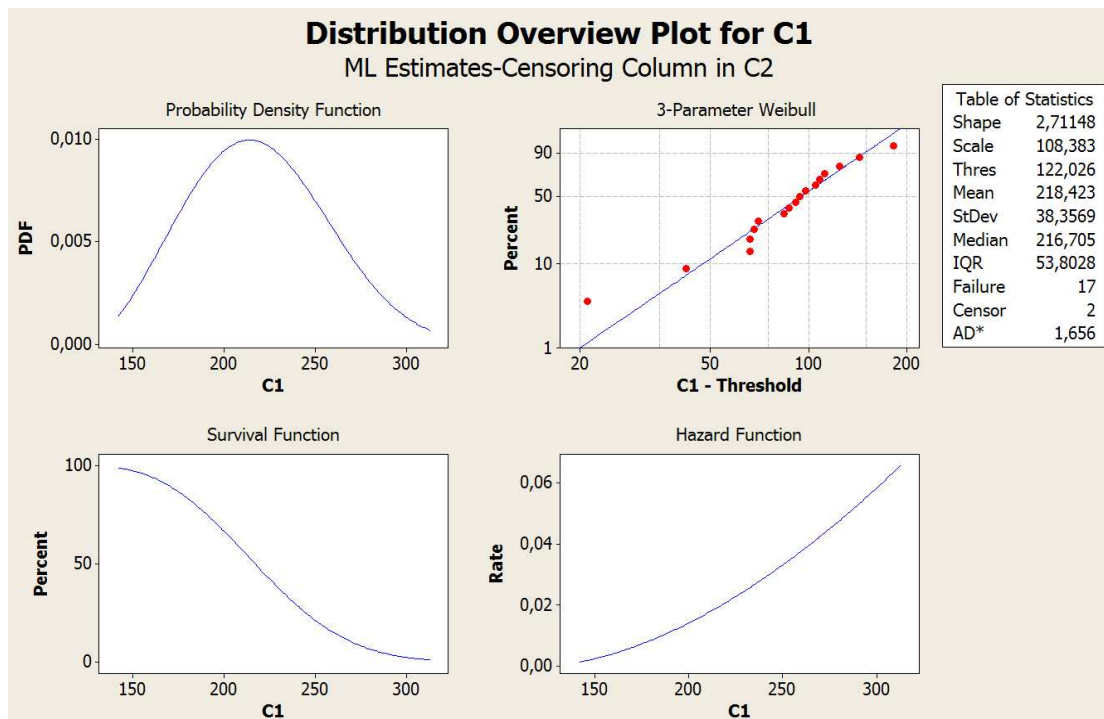
Goodness-of-Fit:

Anderson-Darling (adjusted) = 1,656

### Characteristics of Distribution

	Estimate	Standard Error	95,0% Normal CI	
			Lower	Upper
Mean (MTTF)	218,423	8,99156	201,492	236,777
Standard Deviation	38,3569	6,41597	27,6352	53,2383
Median	216,705	9,89384	198,156	236,991
First Quartile (Q1)	190,481	9,63934	172,495	210,342
Third Quartile (Q3)	244,284	11,0118	223,627	266,849
Interquartile Range (IQR)	53,8028	8,97770	38,7945	74,6172

# Pike (1966) cancer data for rats 3-parameter Weibull



**Pike 3-parameter Weibull: Profile log likelihood for  $\gamma$**

$\gamma$	$\hat{\theta}(\gamma)$	$\hat{\alpha}(\gamma)$	$\tilde{l}(\gamma)$
0	234.3	6.08	-88.233
60	173.2	4.49	-87.831
100	131.8	3.38	-87.467
110	121.2	3.08	-87.381
120	110.6	2.78	-87.327
122	108.4	2.71	-87.324
125	105.2	2.61	-87.330
130	99.7	2.44	-87.382
135	94.0	2.24	-87.542
140	88.0	1.99	-88.064
142	85.2	1.80	-88.773
143	81.1	1.00	-91.718