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TMA4275 LIFETIME ANALYSIS

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TTT-plot

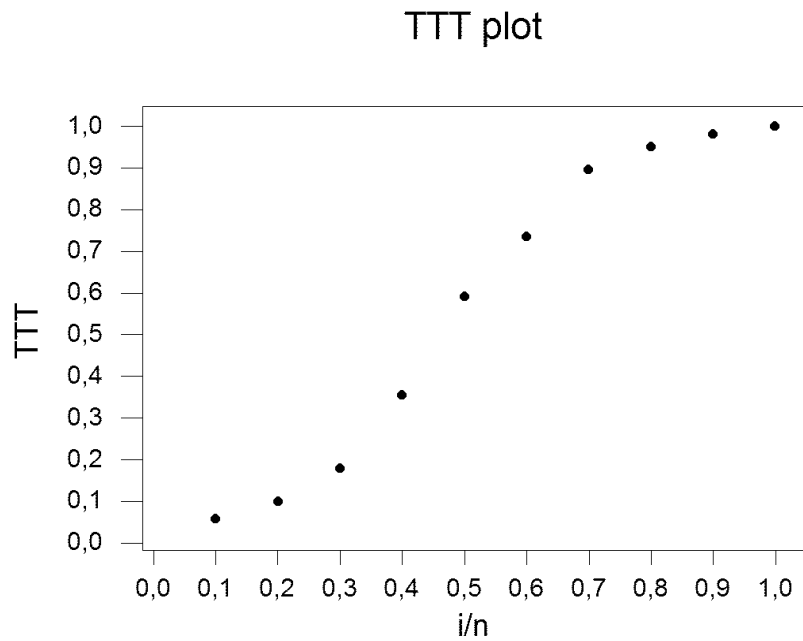
See Table 11.5/9.1.

Complete (i.e. uncensored) data.

Row	Time	TTT interval	TTT cum	i/n	TTT
1	6,3	$10 \cdot 6,3 = 63,0$	63,0	0,1	0,05943
2	11,0	$9 \cdot 4,7 = 42,3$	105,3	0,2	0,09934
3	21,5	$8 \cdot 10,5 = 84,0$	189,3	0,3	0,17858
4	48,4	$7 \cdot 27,9 = 188,3$	377,6	0,4	0,35623
5	90,1	$6 \cdot 41,7 = 250,2$	627,8	0,5	0,59226
6	120,2	$5 \cdot 30,1 = 150,5$	778,3	0,6	0,73425
7	163,0	$4 \cdot 42,8 = 171,2$	949,5	0,7	0,89575
8	182,5	$3 \cdot 19,5 = 58,5$	1008,0	0,8	0,95094
9	198,0	$2 \cdot 15,5 = 31,0$	1039,0	0,9	0,98019
10	219,0	$1 \cdot 21,0 = 21,0$	1060,0	1,0	1,00000

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TTT-plot (Data from Table 11.5/9.1)



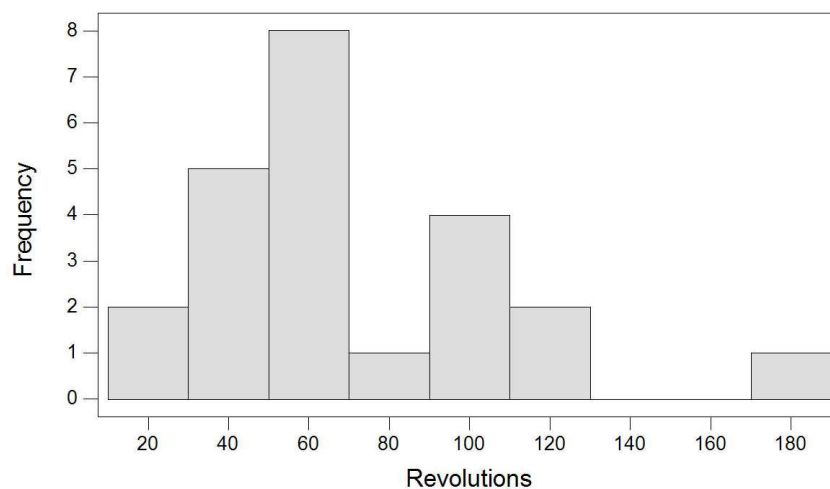
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BALL BEARINGS FAILURE DATA

Data: Millions of revolutions to fatigue failure for 23 units

17,88	28,92	33,00	41,52	42,12	45,60	48,40	51,84
51,96	54,12	55,56	67,80	68,64	68,64	68,88	84,12
93,12	98,64	105,12	105,84	127,92	128,04	173,40	

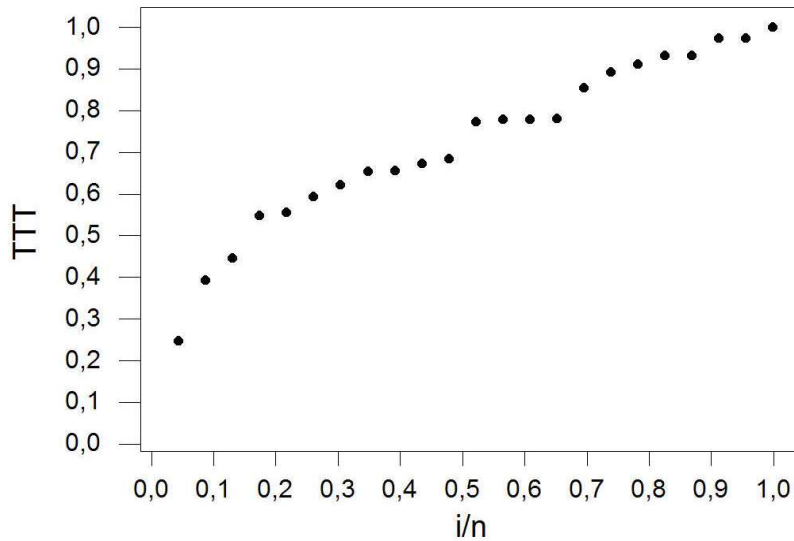
Histogram of Revolutions



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BALL BEARINGS FAILURE DATA

TTT plot



BALL BEARINGS FAILURE DATA

Row	Time	Censor	No at risk	Total time	Cum total time	Plot total time	i/n	TTT
1	17,88	1	23	411,24	411,24	411,24	0,04348	0,24757
2	28,92	1	22	242,88	654,12	654,12	0,08696	0,39379
3	33,00	1	21	85,68	739,80	739,80	0,13043	0,44537
4	41,52	1	20	170,40	910,20	910,20	0,17391	0,54796
5	42,12	1	19	11,40	921,60	921,60	0,21739	0,55482
6	45,60	1	18	62,64	984,24	984,24	0,26087	0,59253
7	48,40	1	17	47,60	1031,84	1031,84	0,30435	0,62119
8	51,84	1	16	55,04	1086,88	1086,88	0,34783	0,65432
9	51,96	1	15	1,80	1088,68	1088,68	0,39130	0,65540
10	54,12	1	14	30,24	1118,92	1118,92	0,43478	0,67361
11	55,56	1	13	18,72	1137,64	1137,64	0,47826	0,68488
12	67,80	1	12	146,88	1284,52	1284,52	0,52174	0,77330
13	68,64	1	11	9,24	1293,76	1293,76	0,56522	0,77887
14	68,64	1	10	0,00	1293,76	1293,76	0,60870	0,77887
15	68,88	1	9	2,16	1295,92	1295,92	0,65217	0,78017
16	84,12	1	8	121,92	1417,84	1417,84	0,69565	0,85357
17	93,12	1	7	63,00	1480,84	1480,84	0,73913	0,89149
18	98,64	1	6	33,12	1513,96	1513,96	0,78261	0,91143
19	105,12	1	5	32,40	1546,36	1546,36	0,82609	0,93094
20	105,84	1	4	2,88	1549,24	1549,24	0,86957	0,93267
21	127,92	1	3	66,24	1615,48	1615,48	0,91304	0,97255
22	128,04	1	2	0,24	1615,72	1615,72	0,95652	0,97269
23	173,40	1	1	45,36	1661,08	1661,08	1,00000	1,00000

TTT AND BARLOW-PROSCHAN FOR CENSORED DATA
Example 11.15/9.8 in HR

Row	Time	Censor	No at risk	Total time	Cum total time	
1	0,35	1	12	4,20	4,20	4,2
2	0,50	0	11	1,65	5,85	10,6
3	0,75	0	10	2,50	8,35	13,0
4	1,00	1	9	2,25	10,60	16,5
5	1,30	1	8	2,40	13,00	33,2
6	1,80	1	7	3,50	16,50	
7	3,00	0	6	7,20	23,70	
8	3,15	0	5	0,75	24,45	
9	4,85	0	4	6,80	31,25	
10	5,50	1	3	1,95	33,20	
11	5,50	0	2	0,00	33,20	
12	6,25	0	1	0,75	33,95	

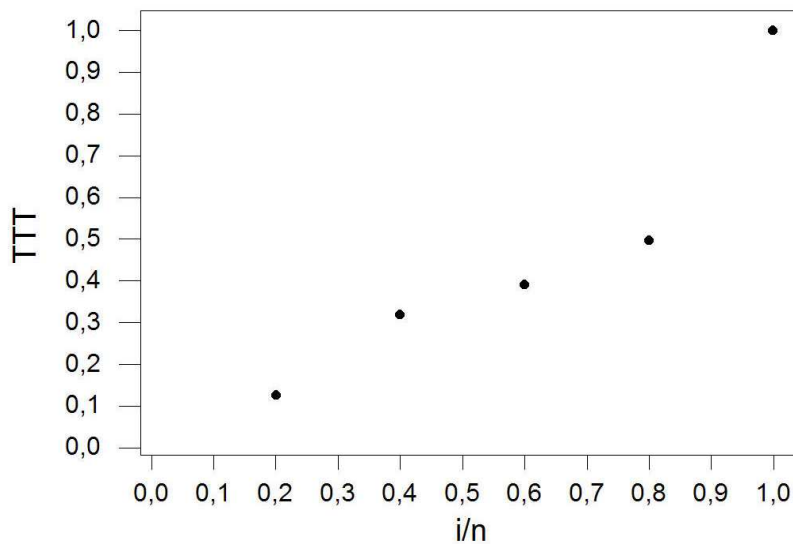
Row	i/n	TTT
1	0,2	0,12651
2	0,4	0,31928
3	0,6	0,39157
4	0,8	0,49699
5	1,0	1,00000

BARLOW-PROSCHAN'S TEST:
 $W = (4.2+10.6+13.0+16.5)/33.2 = 1.33$
alt. $W = 0.13 + 0.32 + 0.39 + 0.50 = 1.34$
(k-1 = 4)

Example 11.15/9.8

NOTE! This is not the same plot as is suggested for censored data in the book (Chap. 11.3.8/p. 410)

TTT-plot censored data



Row	Time	Censor	No at risk	Total time	Cum total time	Plot total time
1	31,7	1	16	507,2	507,2	507,2
2	39,2	1	15	112,5	619,7	619,7
3	57,5	1	14	256,2	875,9	875,9
4	65,0	0	13	97,5	973,4	983,0
5	65,8	1	12	9,6	983,0	1029,2
6	70,0	1	11	46,2	1029,2	1274,3
7	75,0	0	10	50,0	1079,2	1286,1
8	75,2	0	9	1,8	1081,0	
9	87,5	0	8	98,4	1179,4	
10	88,3	0	7	5,6	1185,0	
11	94,2	0	6	35,4	1220,4	
12	101,7	0	5	37,5	1257,9	
13	105,8	1	4	16,4	1274,3	
14	109,2	0	3	10,2	1284,5	
15	110,0	1	2	1,6	1286,1	
16	130,0	0	1	20,0	1306,1	

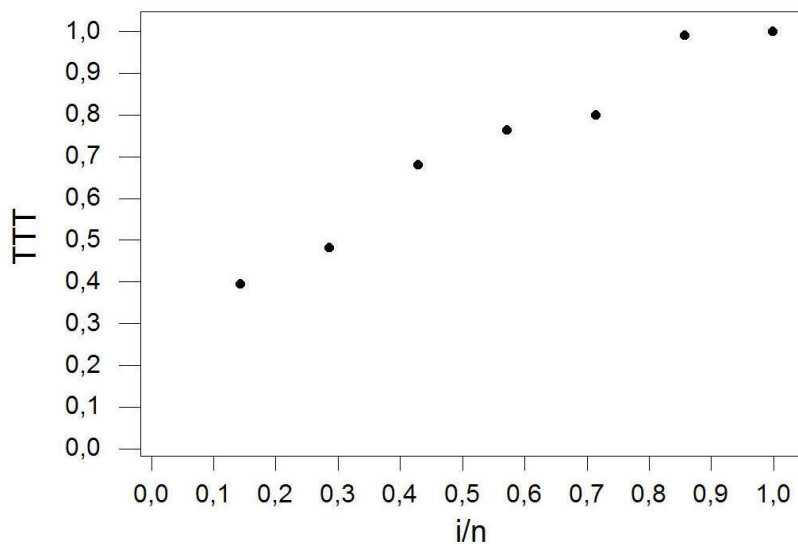
Row	i/n	TTT
1	0,14286	0,39437
2	0,28571	0,48184
3	0,42857	0,68105
4	0,57143	0,76433
5	0,71429	0,80025
6	0,85714	0,99082
7	1,00000	1,00000

Data from Table 11.1/9.3
 BARLOW-PROSCHAN'S TEST:

$$W = 0.39+0.48+0.68+0.76+0.80+0.99 = 4.10 \quad (k-1 = 6)$$

Table 11.1/9.3

TTT-plot censored data



Leukemia Data

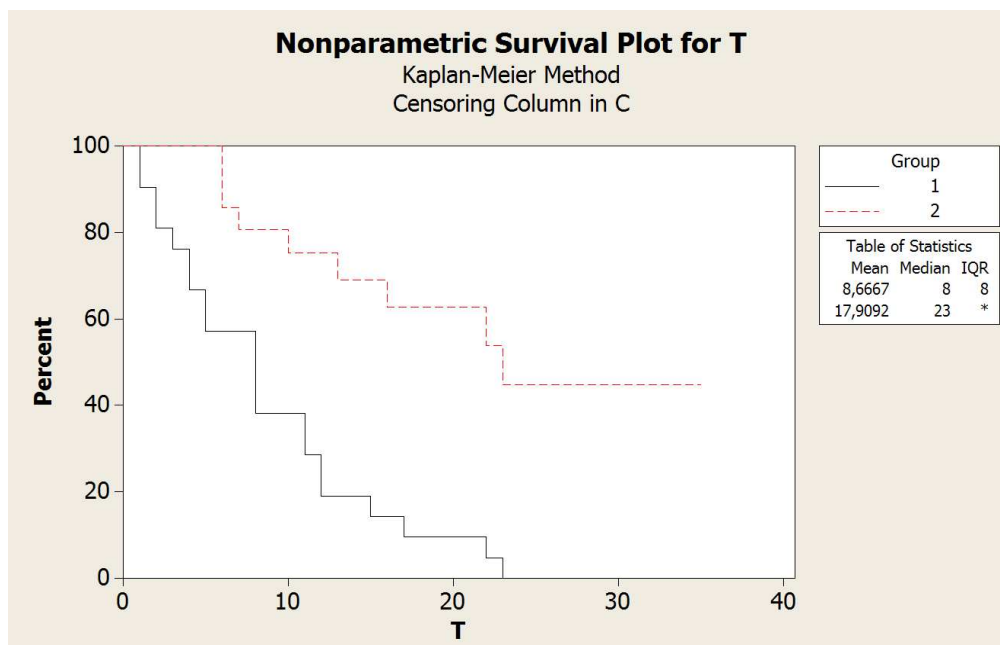
Remission times for 42 children with acute leukemia (free of symptoms).

PL=Placebo, MP=6MP treatment, Y=time, C=censoring (0 means censored)

Row	YPL	CPL	YMP	CMP
1	1	1	6	1
2	1	1	6	1
3	2	1	6	1
4	2	1	6	0
5	3	1	7	1
6	4	1	9	0
7	4	1	10	1
8	5	1	10	0
9	5	1	11	0
10	8	1	13	1
11	8	1	16	1
12	8	1	17	0
13	8	1	19	0
14	11	1	20	0
15	11	1	22	1
16	12	1	23	1
17	12	1	25	0
18	15	1	32	0
19	17	1	32	0
20	22	1	34	0
21	23	1	35	0

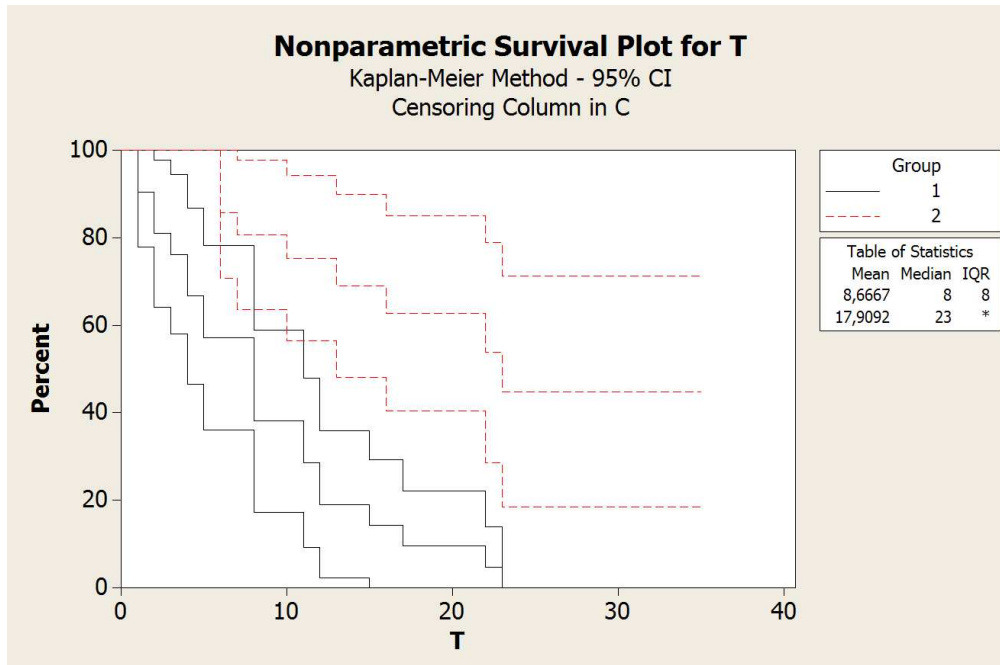
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Example: Leukemia data (1=Placebo, 2=6MP)



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Example: Leukemia data (1=Placebo, 2=6MP, with confidence limits)



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Logrank Test for Leukemia Data

C = Control group (Placebo)

B = Treatment group (6MP)

Time	RiskC	RiskB	Risk	FailC	FailB	Fail	EC	EB
1	21	21	42	2	0	2	$(2/42) * 21 = 1$	$(2/42) * 21 = 1$
2	19	21	40	2	0	2	$(2/40) * 19 = 0.95$	$(2/40) * 21 = 1.05$
3	17	21	38	1	0	1	$(1/38) * 17 = 0.447$	$(1/38) * 21 = 0.553$
4	16	21	37	2	0	2	$(2/37) * 16 = 0.865$	$(2/37) * 21 = 1.135$
13	4	12	16	0	1	1	$(1/16) * 4 = 0.25$	$(1/16) * 12 = 0.75$
23	1	6	7	1	1	2	$(2/7) * 1 = 0.286$	$(2/7) * 6 = 1.714$
Total				21	9		10.749	19.251

Test statistic:

$$\frac{(O_C - E_C)^2}{E_C} + \frac{(O_B - E_B)^2}{E_B}$$

$$= \frac{(21 - 10.749)^2}{10.749} + \frac{(9 - 19.251)^2}{19.251} = 5.46 + 9.77 = 15.33$$

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Breast cancer vs birth length

