LECTURE WEEK 7 Supplement
Spring 2005
February 22 and 25

TMA4275 LIFETIME ANALYSIS

Bo Lindqvist

Department of Mathematical Sciences
NTNU

bo@math.ntnu.no
http://www.math.ntnu.no/~bo/

Inferences for 3-Parameter Weibull or Lognormal Distributions Assuming that Threshold γ is Known

- If γ can be assumed to be known, we can subtract γ from all times and fit the two-parameter Weibull distribution to estimate μ and σ.

- Need to adjust inferences accordingly (e.g., add γ back into estimates of quantiles or subtract γ from times before computing probabilities).

- Similar methods can be used for other distributions for positive random variables.

- ML works if used correctly.

Fitting the Three-Parameter Weibull Distribution Likelihood with Unknown Threshold γ

Two possible problems with ML that need to be avoided

- If the smallest observation is an exact failure, there may be paths in the parameter space leading to infinite likelihood when the **density approximation likelihood** is used.

  Using the **correct** likelihood will allow one to avoid this problem.

- For some data sets the ML estimate of σ will approach 0 (on the boundary of the parameter space).

  This problem can be avoided by extending to the parameter space to allow values of $\sigma \leq 0$. For the 3-parameter Weibull (lognormal), use the SEV (NORM) GETS distribution.

The Three-Parameter Weibull Distribution Likelihood for Right Censored Data

- The likelihood has the form

$$
\begin{aligned}
L(\mu,\sigma,\gamma) &= \prod_{i=1}^{n} L_i(\mu,\sigma,\gamma;\text{data}_i) \\
&= \prod_{i=1}^{n} \{f(t_i;\mu,\sigma,\gamma)\}^{\delta_i}\{1-F(t_i;\mu,\sigma,\gamma)\}^{1-\delta_i} \\
&= \prod_{i=1}^{n} \left\{\frac{1}{\sigma(t_i-\gamma)}\phi_{\text{sev}}\left[\frac{\log(t_i-\gamma)-\mu}{\sigma}\right]\right\}^{\delta_i} \\
&\quad \times\left\{1-\Phi_{\text{sev}}\left[\frac{\log(t_i-\gamma)-\mu}{\sigma}\right]\right\}^{1-\delta_i}
\end{aligned}
$$

- Problem: when $\gamma \to t_{(1)}$ and $\sigma \to 0$, $L(\mu,\sigma,\gamma) \to \infty$.

- **Solution:** Do not use the density approximation; use the correct likelihood (based on small intervals).

Truncated Data

Important to distinguish between truncated data and censored data.

Examples of left truncation:

- Ultrasonic inspection of material. Signal amplitude only trusted when above limit τ_L.
- Life data with pretest screening. Electronic component is burn-in tested for 1000 hours. Only the ones that passed this test are observed later. The number of components failing at burn-in is unknown.
- Brake pad wear.
  - W, proportion of wear at the end of life, and distance driven, V, in thousands of km, was recorded on automobiles that came in for service.
  - Time of failure for each pad set to Y = V/W.
  - Units having already had a pad replacement are omitted from the data.
  - Each observed unit is left-truncated at its observation time.

Examples of right truncation:

- Casting for automobile engine mounts. Pore size distribution below 10 microns are recorded.
- Warranty data of household appliances. Only the ones used can be reported.

Loglikelihood for truncated exponential data