LECTURE WEEK 6 Supplement
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TMA4275 LIFETIME ANALYSIS

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Shock absorber data

Y = kilometers to failure, F = failure mode (0 is censoring)

Row	Y	F	Row	Y	F
1	6700	1	19	14300	1
2	6950	0	20	17520	1
3	7820	0	21	17540	0
4	8790	0	22	17890	0
5	9120	2	23	18450	0
6	9660	0	24	18960	0
7	9820	0	25	18980	0
8	11310	0	26	19410	0
9	11690	0	27	20100	2
10	11850	0	28	20100	0
11	11880	0	29	20150	0
12	12140	0	30	20320	0
13	12200	1	31	20900	2
14	12870	0	32	22700	1
15	13150	2	33	23490	0
16	13330	0	34	26510	1
17	13470	0	35	27410	0
18	14040	0	36	27490	1
			37	27890	0
			38	28100	0

Shock Absorber Failure Data

First reported in O'Connor (1985).

- Failure times, in number of kilometers of use, of vehicle shock absorbers.
- Two failure modes, denoted by M1 and M2.
- One might be interested in the distribution of time to failure for mode M1, mode M2, or in the overall failure-time distribution of the part.

  Here we do not differentiate between modes M1 and M2. We will estimate the distribution of time to failure by either mode M1 or M2.

Shock absorber data

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Estimation Method: Maximum Likelihood
Distribution:   Lognormal

Parameter Estimates

                     Standard    95,0% Normal CI
Parameter  Estimate     Error     Lower     Upper
Location    10,1448  0,144175   9,86219   10,4273
Scale      0,530068
0,112683  0,349447  0,804047

Log-Likelihood = -124,609

Goodness-of-Fit Anderson-Darling (adjusted) = 34,651

Characteristics of Distribution

                                 Standard   95,0% Normal CI
                       Estimate     Error    Lower    Upper
Mean(MTTF)              29297,5   5455,91  20338,3  42203,2
Standard Deviation      16687,1   6787,01  7519,35  37032,5
Median                  25457,6   3670,36  19190,9  33770,7
First Quartile(Q1)      17805,2   2062,96  14188,1  22344,4
Third Quartile(Q3)      36399,0   7252,61  24631,2  53789,0
Interquartile Range(IQR)  18593,8   6115,60  9758,96  35426,9
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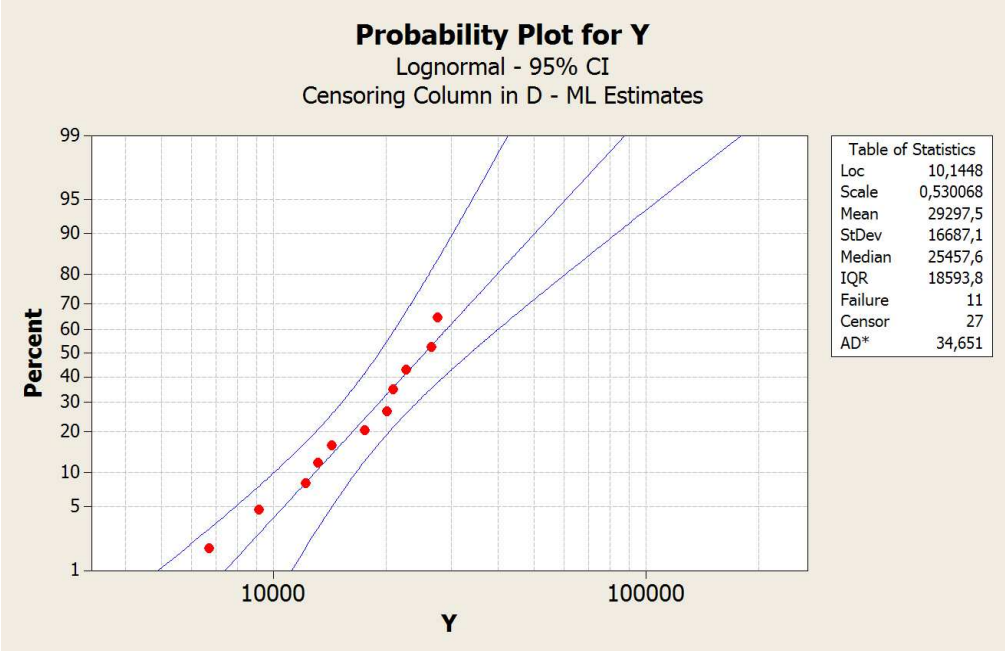
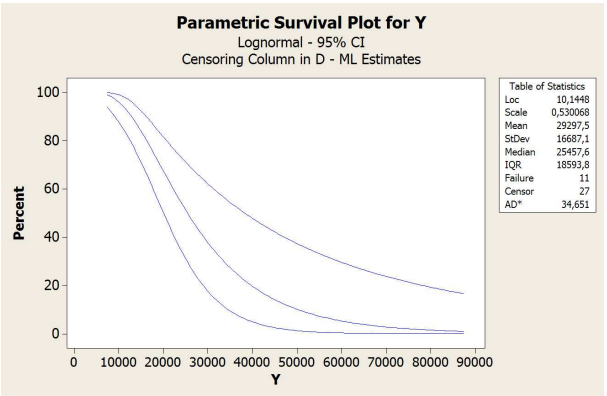
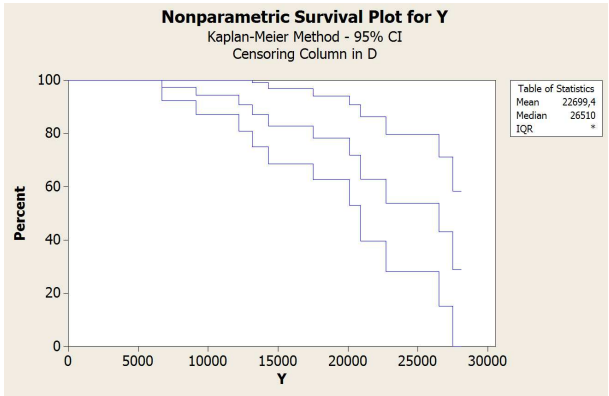
Shock absorber data

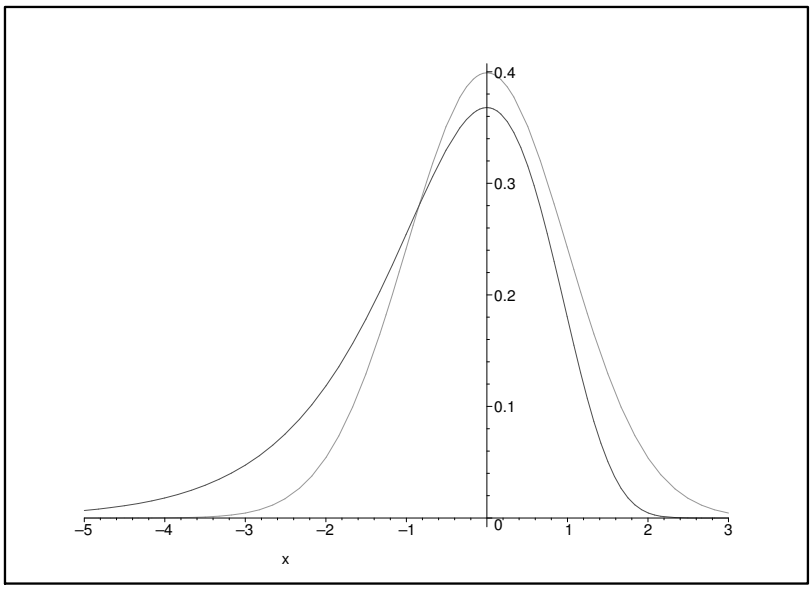


Shock absorber data

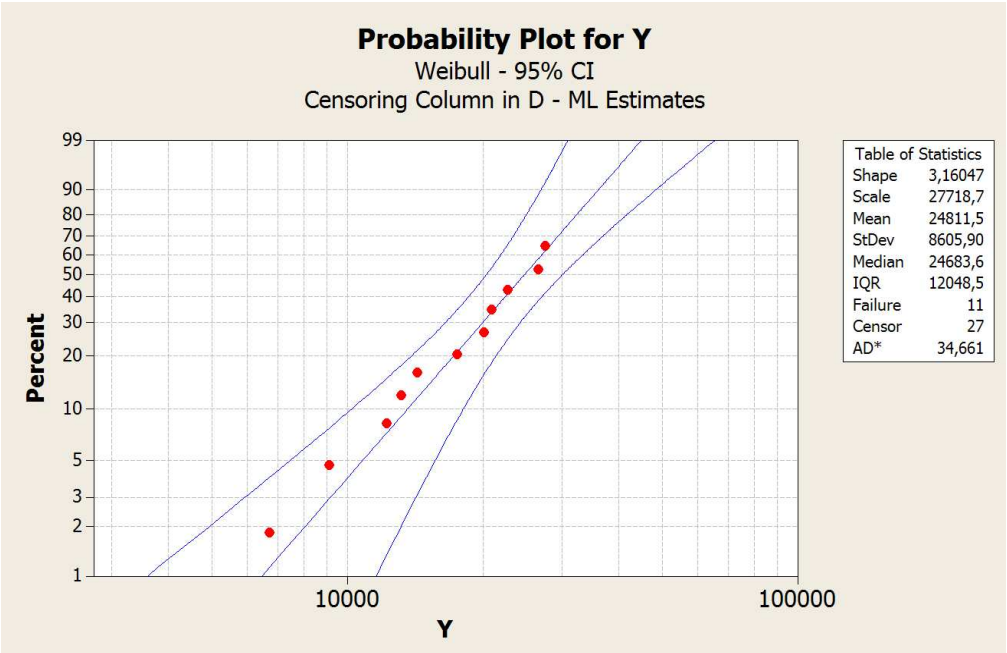




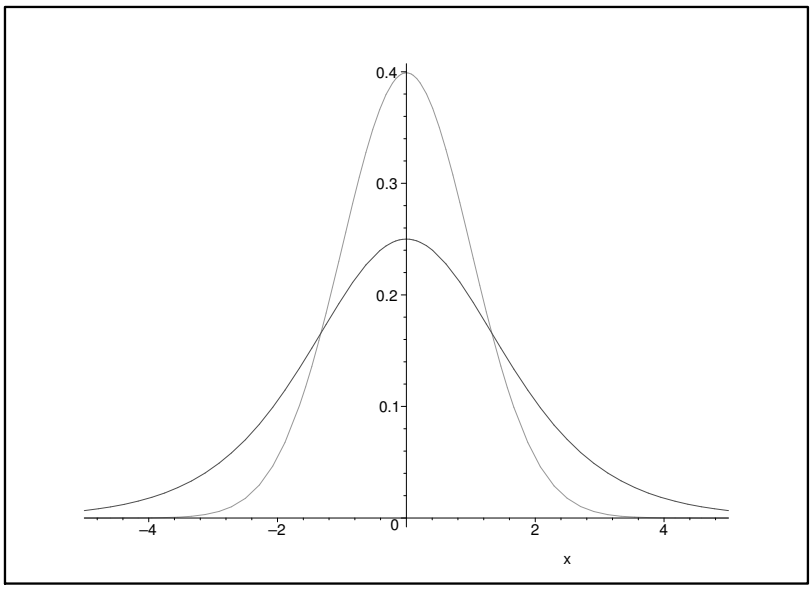
GUMBEL-DISTRIBUTION AND NORMAL DISTRIBUTION WITH $\mu = 0, \sigma = 1$



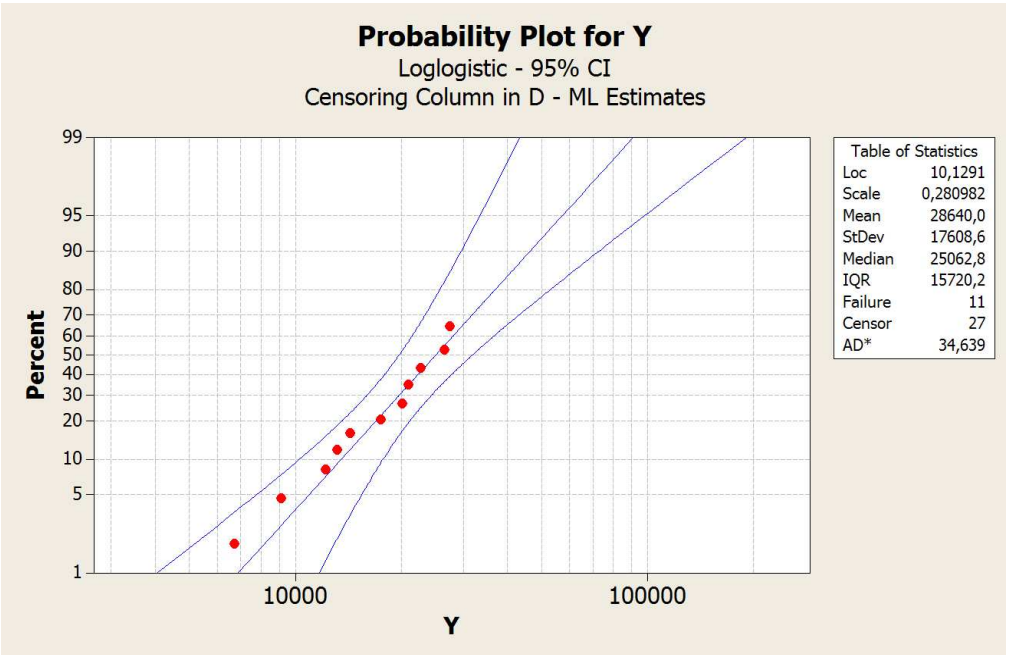
Shock absorber data



LOGISTIC DISTRIBUTION AND NORMAL DISTRIBUTION WITH $\mu = 0, \sigma = 1$



Shock absorber data



Shock absorber data:
Results for loglogistic (left), lognormal (middle), Weibull (right)

Table of Statistics	
Loc	10,1291
Scale	0,280982
Mean	28640,0
StDev	17608,6
Median	25062,8
IQR	15720,2
Failure	11
Censor	27
AD*	34,639

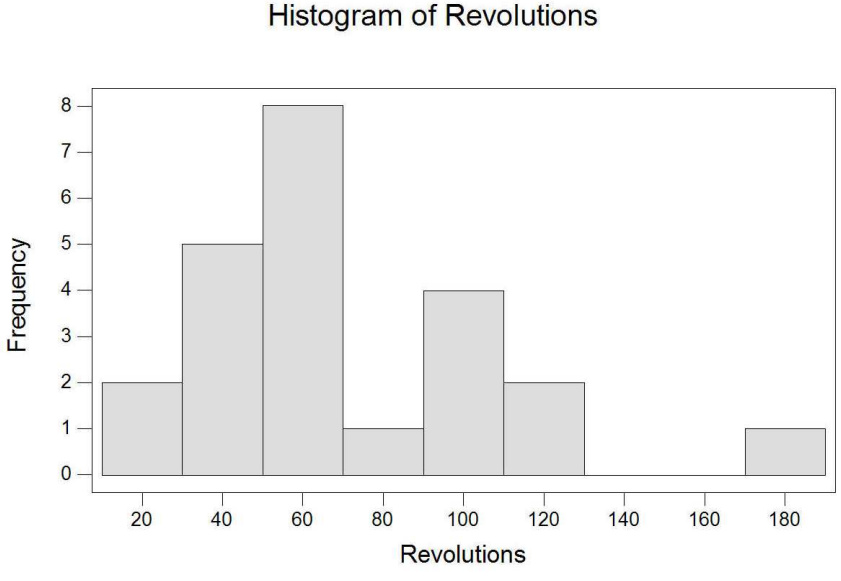
Table of Statistics	
Loc	10,1448
Scale	0,530068
Mean	29297,5
StDev	16687,1
Median	25457,6
IQR	18593,8
Failure	11
Censor	27
AD*	34,651

Table of Statistics	
Shape	3,16047
Scale	27718,7
Mean	24811,5
StDev	8605,90
Median	24683,6
IQR	12048,5
Failure	11
Censor	27
AD*	34,661

BALL BEARINGS FAILURE DATA

Data: Millions of revolutions to fatigue failure for 23 units (No censoring)

17,88	28,92	33,00	41,52	42,12	45,60	48,40	51,84
51,96	54,12	55,56	67,80	68,64	68,64	68,88	84,12
93,12	98,64	105,12	105,84	127,92	128,04	173,40	



Example: Ball Bearing Data

Data from fatigue endurance tests for deep–groove ball bearings from four major bearing companies (Lawless 1982).

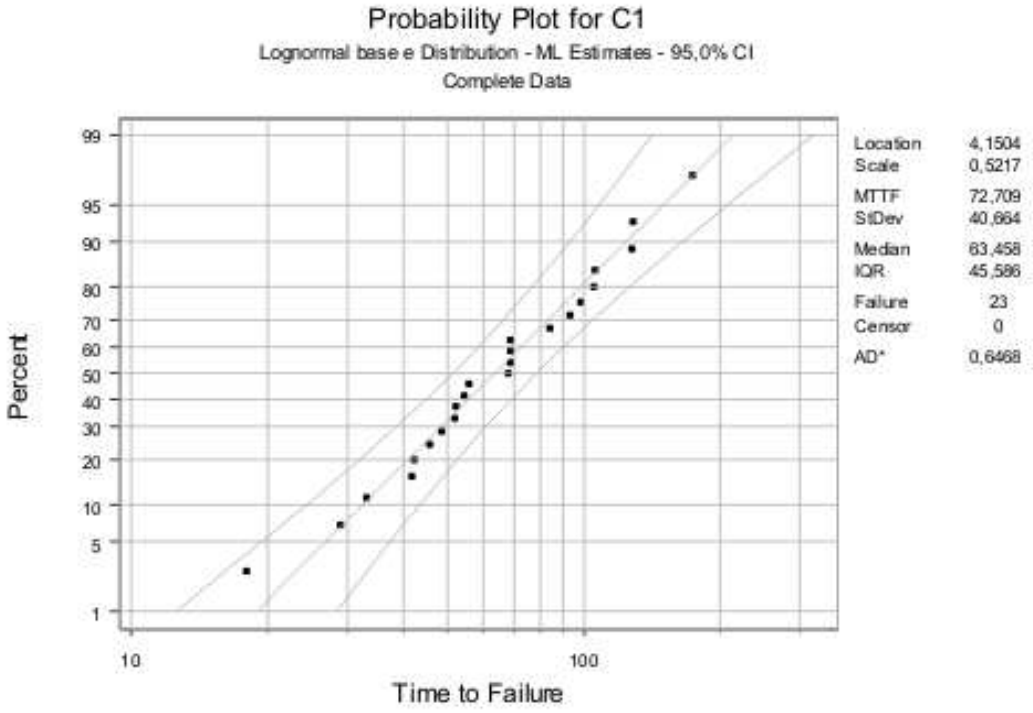
The data: Millions of revolutions to failure for each of $n = 23$ bearings before fatigue failure.

Main objectives of the study: Determine **best** values of the parameters in equation relating fatigue to load.

Motivation:

- Fatigue affects service life of ball bearings.
- Disagreement in the industry on the appropriate parameter values to use.

Ball Bearings Data: Lognormal distribution



Distribution Analysis: C1

Variable: C1

Censoring Information	Count
Uncensored value	23

Estimation Method: Maximum Likelihood
Distribution: Lognormal base e

Parameter Estimates

Parameter	Estimate	Standard Error	95,0% Normal CI Lower	Upper
Location	4,1504	0,1088	3,9372	4,3636
Scale	0,52169	0,07692	0,39076	0,69649

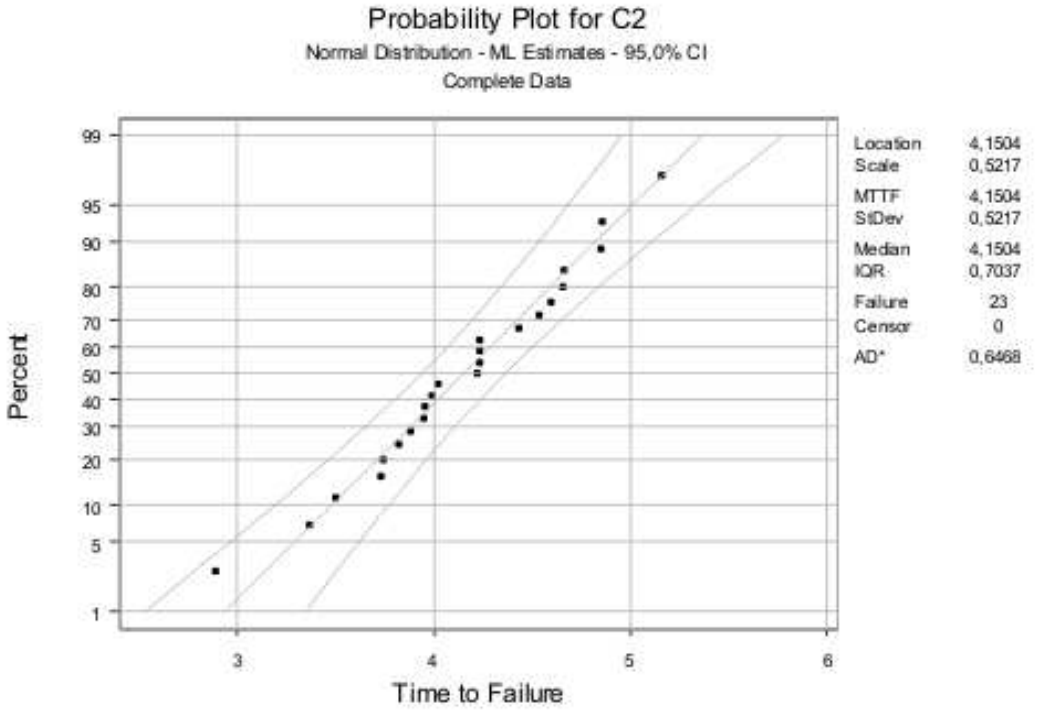
Log-Likelihood = -113,129

Goodness-of-Fit
Anderson-Darling (adjusted) = 0,6468

Characteristics of Distribution

	Estimate	Standard Error	95,0% Normal CI Lower	Upper
Mean(MTTF)	72,7087	8,4302	57,9288	91,2596
Standard Deviation	40,6644	9,5646	25,6452	64,4798
Median	63,4583	6,9029	51,2738	78,5383
First Quartile(Q1)	44,6347	5,3793	35,2442	56,5271
Third Quartile(Q3)	90,2203	10,8731	71,2393	114,2587
Interquartile Range(IQR)	45,5857	8,5755	31,5284	65,9105

Taking Log of Ball Bearings Data and fit to Normal distribution



Variable: C2

Censoring Information	Count
Uncensored value	23

Estimation Method: Maximum Likelihood
Distribution: Normal

Parameter Estimates

Parameter	Estimate	Standard Error	95,0% Normal CI Lower	Upper
Location	4,1504	0,1088	3,9372	4,3636
Scale	0,52169	0,07692	0,39076	0,69649

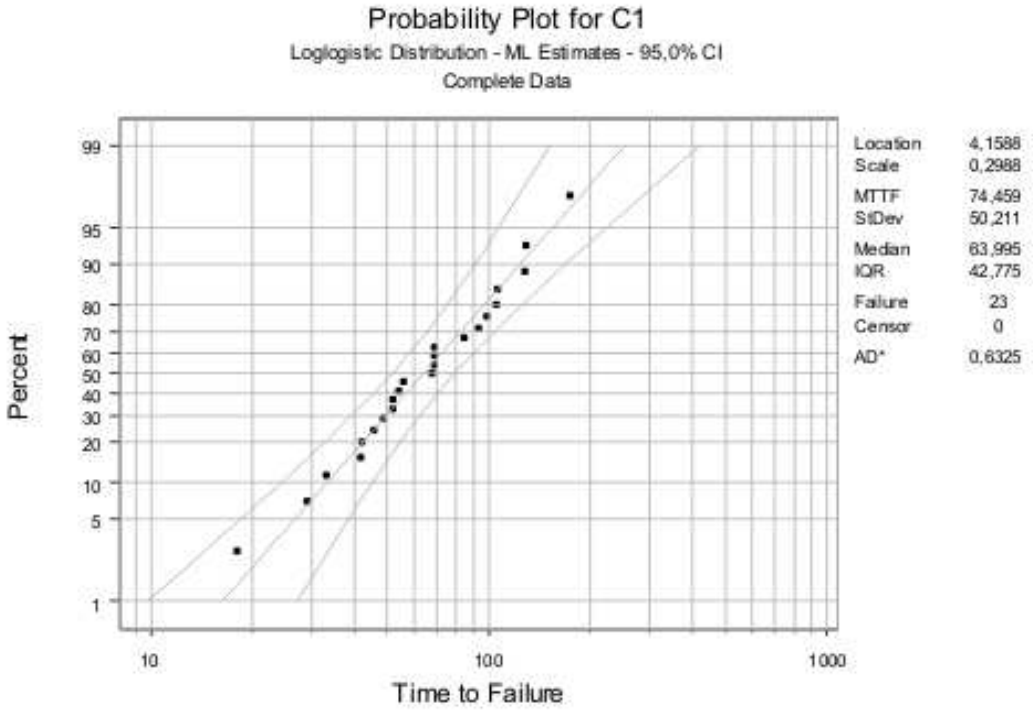
Log-Likelihood = -17,670

Goodness-of-Fit
Anderson-Darling (adjusted) = 0,6468

Characteristics of Distribution

	Estimate	Standard Error	95,0% Normal CI Lower	Upper
Mean(MTTF)	4,1504	0,1088	3,9372	4,3636
Standard Deviation	0,5217	0,07692	0,3908	0,6965
Median	4,1504	0,1088	3,9372	4,3636
First Quartile(Q1)	3,7985	0,1205	3,5623	4,0347
Third Quartile(Q3)	4,5023	0,1205	4,2660	4,7385
Interquartile Range(IQR)	0,7037	0,1038	0,5271	0,9395

Ball Bearings Data: Loglogistic distribution



Variable: C1

Censoring Information	Count
Uncensored value	23

Estimation Method: Maximum Likelihood
Distribution: Loglogistic

Parameter Estimates

Parameter	Estimate	Standard Error	95,0% Normal CI Lower	Upper
Location	4,1588	0,1090	3,9451	4,3725
Scale	0,29881	0,05153	0,21312	0,41897

Log-Likelihood = -113,373

Goodness-of-Fit
Anderson-Darling (adjusted) = 0,6325

Characteristics of Distribution

	Estimate	Standard Error	95,0% Normal CI Lower	Upper
Mean(MTTF)	74,4589	9,0701	58,6447	94,5377
Standard Deviation	50,2113	17,0541	25,8044	97,7034
Median	63,9947	6,9784	51,6801	79,2438
First Quartile(Q1)	46,0866	5,6542	36,2363	58,6144
Third Quartile(Q3)	88,8616	10,9338	69,8199	113,0963
Interquartile Range(IQR)	42,7750	8,9647	28,3658	64,5036

Confidence Interval for the Mean Life of a New Insulating Material

- A life test for a new insulating material used 25 specimens which were tested simultaneously at a high voltage of 30 kV.
- The test was run until 15 of the specimens failed.
- The 15 failure times (hours) were recorded as:

  1.08, 12.20, 17.80, 19.10, 26.00, 27.90, 28.20, 32.20, 35.90, 43.50, 44.00, 45.20, 45.70, 46.30, 47.80

  Then $TTT = 1.08 + \cdots + 47.80 + 10 \times 47.80 = 950.88$ hours.
- The ML estimate of θ and a 95% confidence interval are:

$$\begin{aligned} \hat{\theta} &= 950.88/15 = 63.392 \text{ hours} \\ \left[\underline{\theta}, \tilde{\theta}\right] &= \left[\frac{2(950.88)}{\chi^2_{(.975;30)}}, \frac{2(950.88)}{\chi^2_{(.025;30)}}\right] = \left[\frac{1901.76}{46.98}, \frac{1901.76}{16.79}\right] \\ &= [40.48, \quad 113.26]. \end{aligned}$$