

LECTURE WEEK 5

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TMA4275 LIFETIME ANALYSIS

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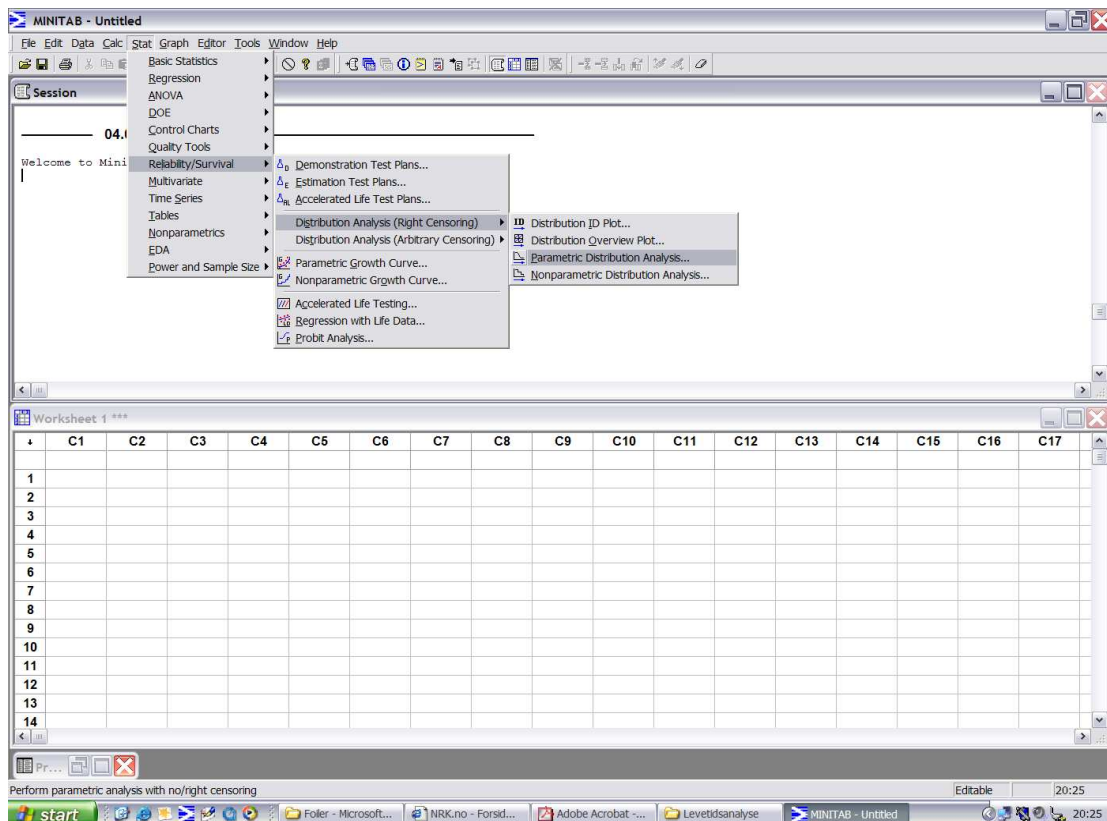
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PARAMETRIC LIFETIME ANALYSIS IN MINITAB



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DATA OPTIONS

RIGHT CENSORING:

Y_i	δ_i
Observed time	Cens. status 1: Lifetime 0: Censoring

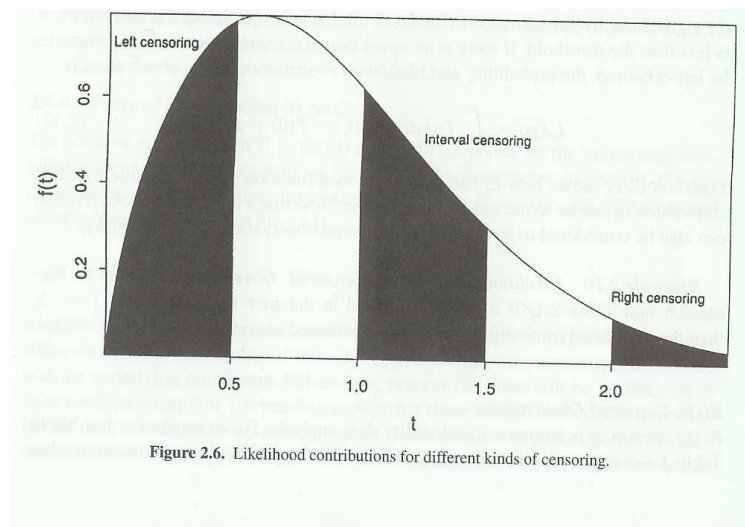
ARBITRARY CENSORING:

Start variable A_i	End variable B_i	
1.7	1.7	Exact lifetime 1.7
2.0	*	Right censoring at time 2.0, i.e. lifetime is > 2.0
*	0.5	Left censoring at time 0.5, i.e. lifetime is < 0.5
1.0	1.5	Interval censoring: Lifetime between 1.0 and 1.5

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LIKELIHOOD CONTRIBUTION

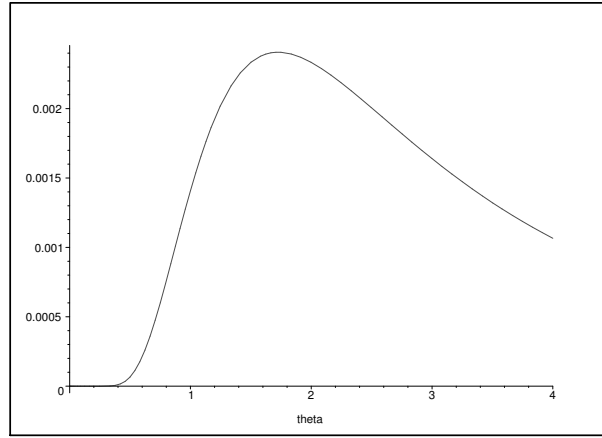
Obs. type	Start variable A_i	End variable B_i	Likelihood contribution
Exact lifetime	1.7	1.7	$f(1.7; \theta)$
Right censoring	2.0	*	$1 - F(2.0; \theta)$
Left censoring	*	0.5	$F(0.5; \theta)$
Interval censoring	1.0	1.5	$F(1.5; \theta) - F(1.0; \theta)$



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LIKELIHOOD FOR MODEL $f(t; \theta) = (1/\theta)e^{-t/\theta}$

$$L(\theta) = \left(\frac{1}{\theta}e^{-1.7/\theta}\right) \cdot (e^{-2.0/\theta}) \cdot (1 - e^{-0.5/\theta}) \cdot (e^{-1.0/\theta} - e^{-1.5/\theta})$$



Maximum likelihood estimate: $\hat{\theta} = 1.725$

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ARBITRARY CENSORED DATA: MINITAB OUTPUT

Distribution Analysis, Start = A and End = B

Variable Start: A End: B

Censoring Information	Count
Uncensored value	1
Right censored value	1
Interval censored value	1
Left censored value	1

Estimation Method: Maximum Likelihood

Distribution: Exponential

Parameter Estimates

Parameter	Estimate	Standard Error	95,0% Normal CI	
			Lower	Upper
Mean	1,72529	0,998421	0,554978	5,36353

Log-Likelihood = -6,029

Goodness-of-Fit

Anderson-Darling (adjusted) = 4,933

Characteristics of Distribution

	Estimate	Standard Error	95,0% Normal CI	
			Lower	Upper
Mean (MTTF)	1,72529	0,998421	0,554978	5,36353
Standard Deviation	1,72529	0,998421	0,554978	5,36353
Median	1,19588	0,692053	0,384682	3,71771
First Quartile (Q1)	0,496336	0,287228	0,159657	1,54299
Third Quartile (Q3)	2,39177	1,38411	0,769363	7,43543
Interquartile Range (IQR)	1,89543	1,09688	0,609706	5,89244

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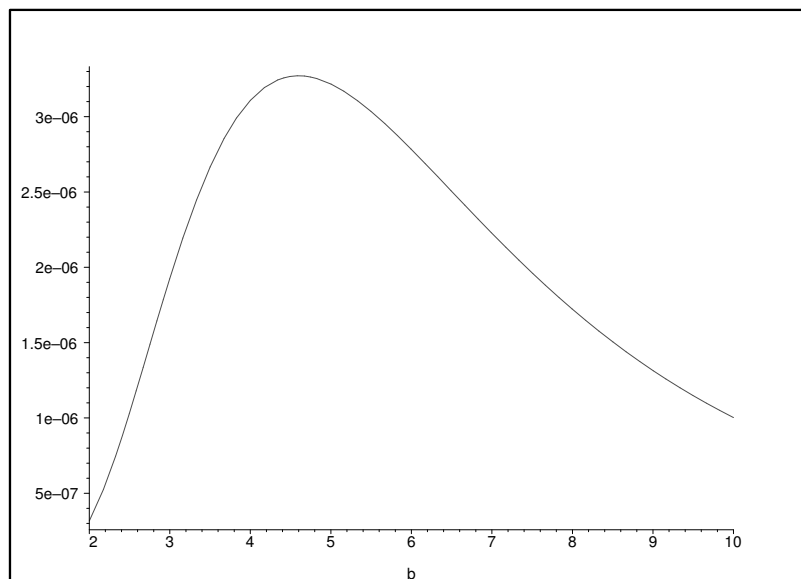
EXAMPLE: RIGHT CENSORED DATA

Worksheet 1 ***

↓	C1	C2
	Y	D
1	0,6	0
2	0,8	1
3	2,1	1
4	3,2	1
5	3,3	0
6	4,4	1
7	8,6	1

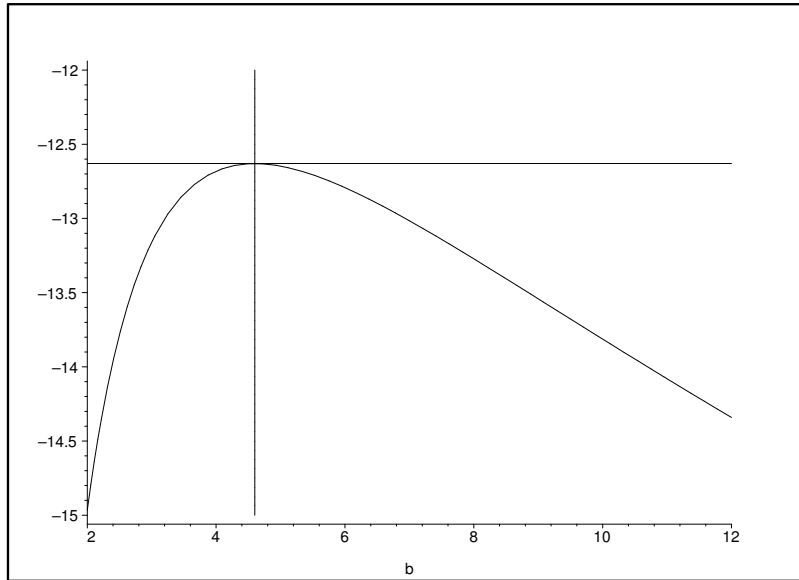
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LIKELIHOOD FUNCTION FOR MODEL $f(t; \theta) = (1/\theta)e^{-t/\theta}$



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LOG-LIKELIHOOD FUNCTION



Maximum likelihood estimate: $\hat{\theta} = 4.6$

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Distribution Analysis: Y

Variable: Y

Censoring Information	Count
Uncensored value	5
Right censored value	2

Censoring value: D = 0

Estimation Method: Maximum Likelihood

Distribution: Exponential

Parameter Estimates

Parameter	Estimate	Standard Error	95,0% Normal CI	
			Lower	Upper
Mean	4,6	2,05718	1,91465	11,0516

Log-Likelihood = -12,630

Goodness-of-Fit

Anderson-Darling (adjusted) = 3,767

Characteristics of Distribution

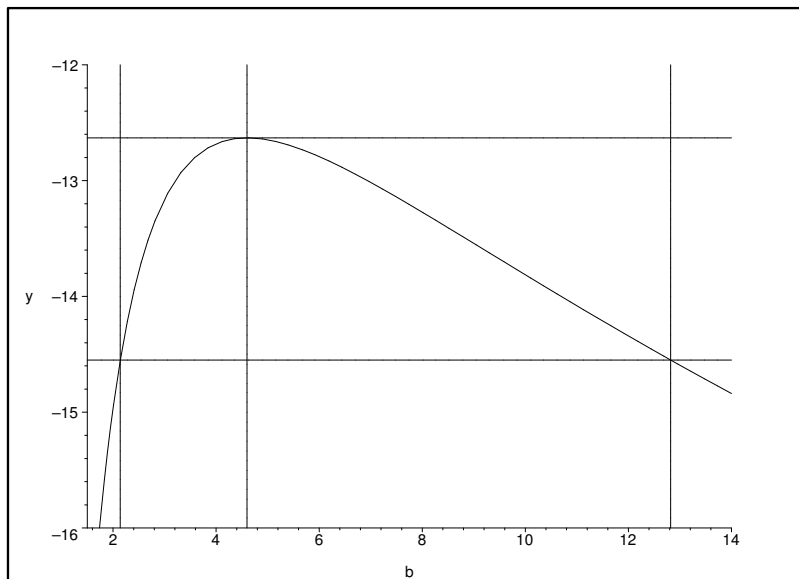
	Estimate	Standard Error	95,0% Normal CI	
			Lower	Upper
Mean (MTTF)	4,6	2,05718	1,91465	11,0516
Standard Deviation	4,6	2,05718	1,91465	11,0516
Median	3,18848	1,42593	1,32713	7,66041
First Quartile (Q1)	1,32334	0,591815	0,550810	3,17936
Third Quartile (Q3)	6,37695	2,85186	2,65427	15,3208
Interquartile Range (IQR)	5,05362	2,26005	2,10346	12,1415

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Table of Percentiles

Percent	Percentile	Standard Error	95,0% Normal CI	
			Lower	Upper
1	0,0462315	0,0206754	0,0192429	0,111073
2	0,0929325	0,0415607	0,0386811	0,223273
3	0,140112	0,0626601	0,0583187	0,336624
4	0,187781	0,0839783	0,0781597	0,451150
5	0,235949	0,105520	0,0982086	0,566875
6	0,284627	0,127289	0,118470	0,683825
7	0,333825	0,149291	0,138947	0,802025
8	0,383555	0,171531	0,159646	0,921504
9	0,433829	0,194014	0,180572	1,04229
10	0,484658	0,216746	0,201728	1,16441
20	1,02646	0,459047	0,427241	2,46610
30	1,64070	0,733745	0,682907	3,94184
40	2,34980	1,05086	0,978051	5,64546
50	3,18848	1,42593	1,32713	7,66041
60	4,21494	1,88498	1,75437	10,1265
70	5,53827	2,47679	2,30518	13,3059
80	7,40341	3,31091	3,08151	17,7869
90	10,5919	4,73684	4,40864	25,4473
91	11,0765	4,95358	4,61037	26,6117
92	11,6184	5,19588	4,83588	27,9134
93	12,2326	5,47058	5,09155	29,3892
94	12,9417	5,78770	5,38669	31,0928
95	13,7804	6,16277	5,73577	33,1078
96	14,8068	6,62182	6,16301	35,5739
97	16,1302	7,21363	6,71382	38,7532
98	17,9953	8,04775	7,49015	43,2343
99	21,1838	9,47368	8,81728	50,8947

LOG-LIKELIHOOD FUNCTION



Probability Plot for Y
Exponential - 95% CI
Censoring Column in D - ML Estimates

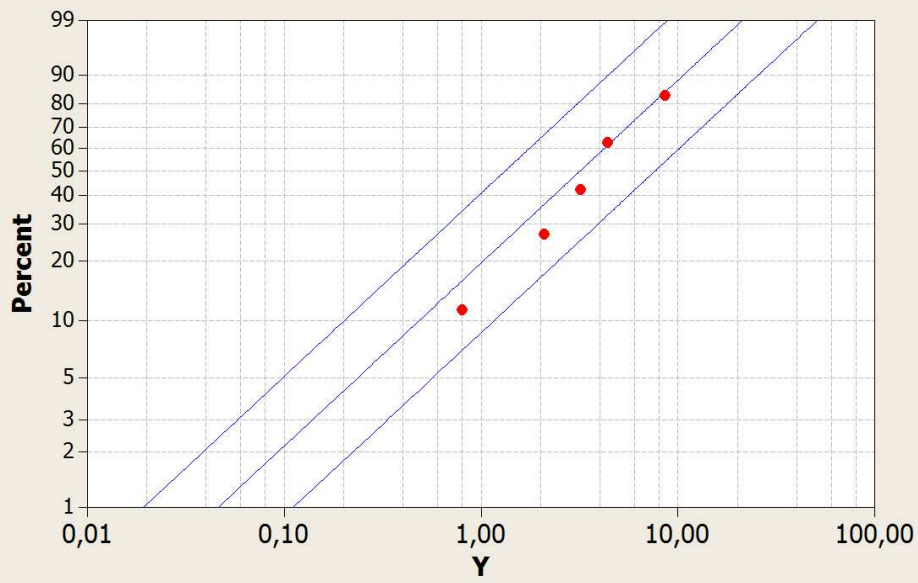


Table of Statistics	
Mean	4,6
StDev	4,6
Median	3,18848
IQR	5,05362
Failure	5
Censor	2
AD*	3,767