LECTURE WEEK 12 Supplement
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TMA4275 LIFETIME ANALYSIS

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Comparison of trend tests

main topic

Minitab provides five trend tests for data with multiple systems: MIL-hdbk-189 (TTT-based), MIL-hdbk-189 (Pooled), Laplace's (TTT-based), Laplace's (Pooled), and Anderson-Darling. The pooled Laplace and military handbook tests reduce to their respective TTT-based tests when there is only one system. These tests behave differently under the following two circumstances:

1 the data follow a non-monotonic trend

2 the data are from heterogeneous systems

Monotonic and non-monotonic trends

There is a trend in the pattern of times between failure if the times change in a systematic way. Trends can be:

- monotonic – times between failures are getting either consistently longer (decreasing trend) or consistently shorter (increasing trend)
- non-monotonic – times between failures alternate between increasing and decreasing trend (cyclic) or have a decreasing trend, no trend, and then increasing trend (bathtub)

The Anderson-Darling test will reject the null hypothesis in the presence of both monotonic and non-monotonic trends. The other tests will generally only detect monotonic trends. While the Anderson-Darling test is useful if you suspect the existence of a cyclic or other non-monotonic trend, the other tests are more powerful in the case of a monotonic trend.

Homogeneous and heterogeneous systems

The null hypothesis of no trend differs slightly for the different tests:

- The null hypothesis for the pooled tests (MIL-hdbk-189 and Laplace's) is that the data come from a homogeneous Poisson processes (HPP) with a possibly different MTBF for each system. Thus, rejecting the null hypothesis means that you can definitely conclude there is a trend in your data.
- The null hypothesis for the TTT-based tests (MIL-hdbk-189, Laplace's, and Anderson-Darling) is that the data come from a homogeneous Posson process (HPP) with the same MTBF for each system. Thus, rejecting the null hypothesis could mean that either there is a trend in your data or your data come from heterogeneous systems. Therefore, you should use TTT-based tests only when you are confident that your systems are homogeneous.

The table below summarizes the different null hypotheses associated with the trend tests.

	MIL-hdbk-189 (Pooled)	**MIL-hdbk-189 (TTT-based)**	**Laplace's (Pooled)**	**Laplace's (TTT-based)**	**Anderson-Darling**
Null Hypothesis	HPP (possibly different MTBFs)	HPP (equal MTBFs)	HPP (possibly different MTBFs)	HPP (equal MTBFs)	HPP (possibly different MTBFs)
Rejecting H_0 means...	monotonic trend	monotonic trend or systems are heterogeneous	monotonic trend	monotonic trend or systems are heterogeneous	monotonic trend or non-monotonic trend or systems are heterogeneous

See [12] for more information concerning these tests.

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TTT-based tests for trend in repairable systems data

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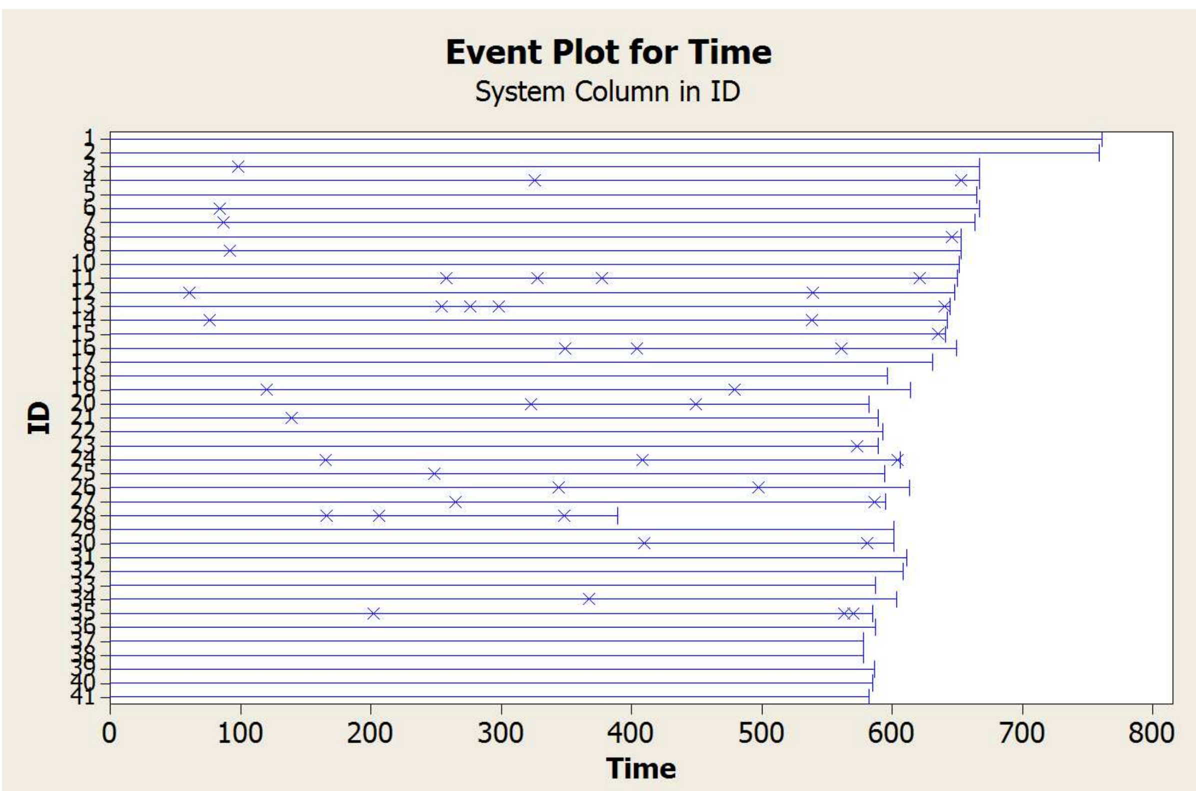
A major aspect of analysis of failure data for repairable systems is the testing for a possible trend in interfailure times. This paper reviews some important and popular graphical methods and tests for the nonhomogeneous Poisson process model. In particular, the total time on test (TTT) plot is considered, and trend tests based on the TTT-statistic are motivated and derived. In particular, a test based on the Anderson–Darling statistic is suggested. The tests are evaluated and compared in a simulation study, both with respect to the achievement of correct significance level and rejection power. The considered alternatives to 'no trend' are the log-linear, power law and a class of bathtub-shaped intensity functions. The simulation study involves single systems, as well as the case where several independent systems of the same kind are observed. © 1998 Elsevier Science Limited.

Valveseat Data



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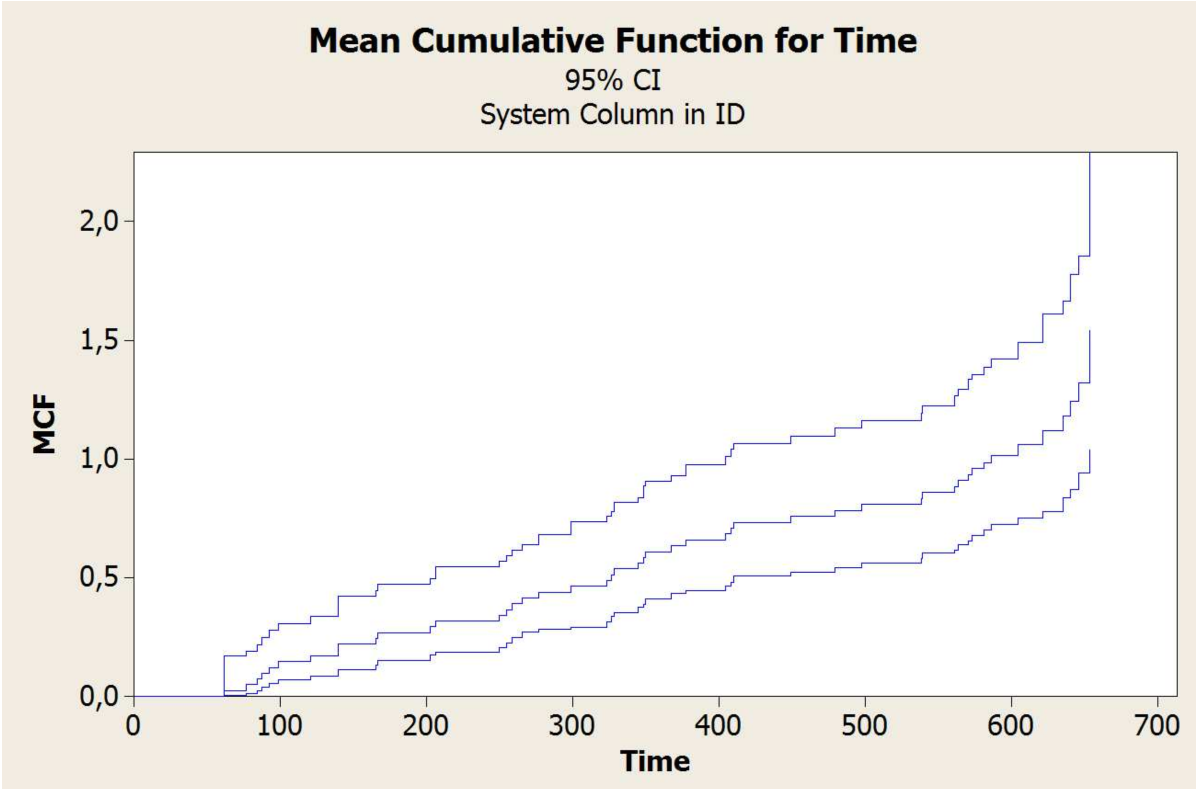


Valveseat Data

Trend Tests

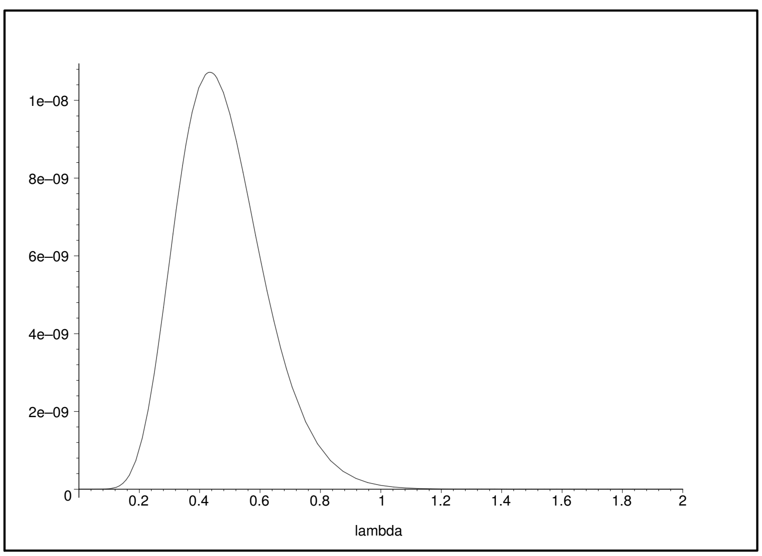
	MIL-Hdbk-189		Laplace's		
	TTT-based	Pooled	TTT-based	Pooled	Anderson-Darling
Test Statistic	80,28	66,15	0,46	2,38	0,80
P-Value	0,249	0,017	0,645	0,017	0,478
DF	96	96			

Life testing of $n = 13$ airplane components (Mann and Fertig, 1976), censored after failure number $r = 10$ (Type II-censoring), resulted in:

j	Time (Y_j)	Censor
1	0,22	1
2	0,50	1
3	0,88	1
4	1,00	1
5	1,32	1
6	1,33	1
7	1,54	1
8	1,76	1
9	2,50	1
10	3,00	1
11	3,00	0
12	3,00	0
13	3,00	0

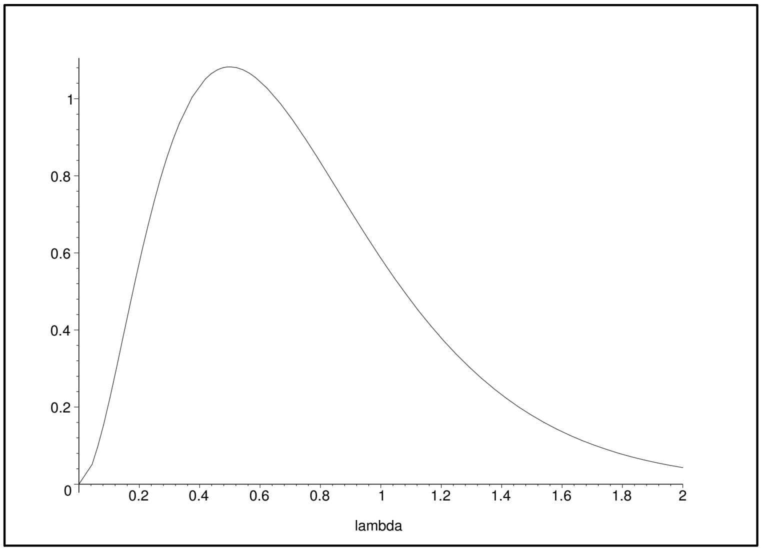
Let the model be that $T \sim$ eksp(λ), i.e. the likelihood-function is

$$L(\lambda|\text{data}) = \lambda^r e^{-\sum_{j=1}^{n} y_j} = \lambda^{10} e^{-23.05}$$



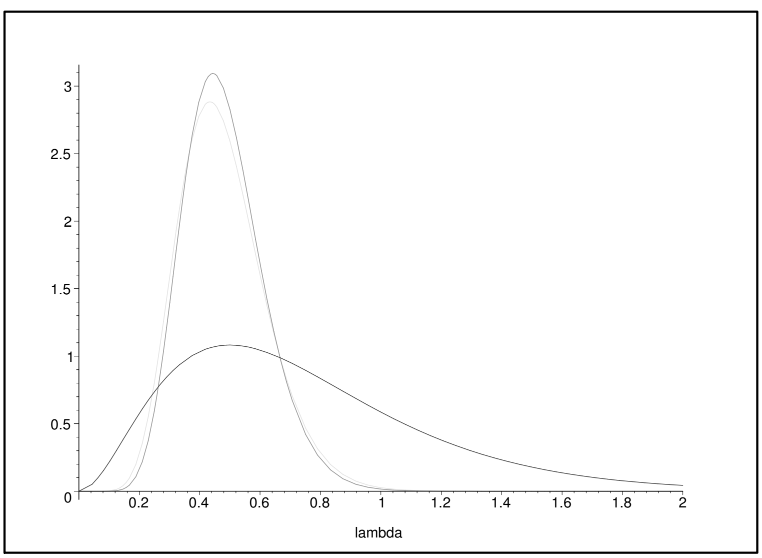
Likelihood-function for airplane component data

MLE: $\hat{\lambda} = 0.434$



Prior distribution for λ in airplane component data, $\Lambda \sim$ Gamma$(3, 4)$

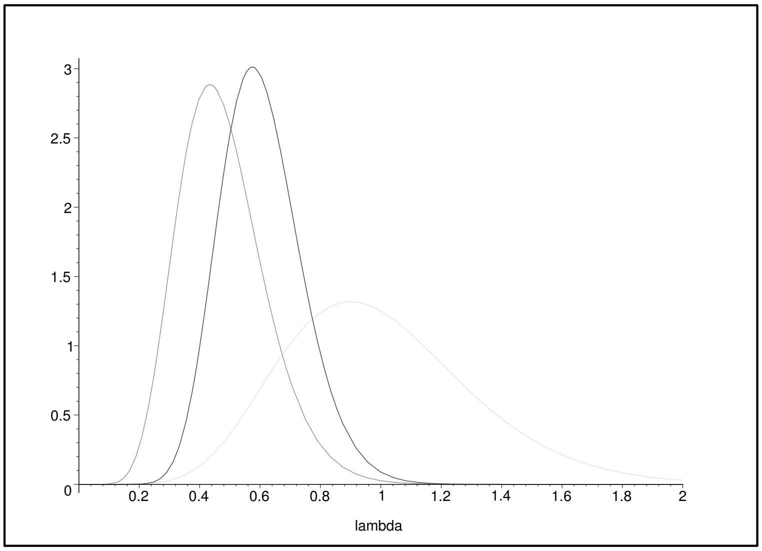
$E(\Lambda) = 0.75, \ SD(\Lambda) = 0.43320$



Prior distribution ("lowest"), Likelihood-function (normalized to density, "second highest") and Posterior distribution ("highest") for λ in airplane component data.

Posterior maximum is for $\lambda = 0.444$

Posterior expectation, i.e. Bayes-estimate is $\hat{\lambda}_B = 0.481$



Alternative prior distribution (Gamma(10,10), "lowest"), Likelihood function (normalized to density, "second highest") and Posterior distribution ("highest") for λ in airplane component data.

Posterior maximum is now for $\lambda = 0.575$.

Posterior expectation, i.e. Bayes-estimate is now $\hat{\lambda}_B = 0.6051$