LECTURE WEEK 11 Supplement
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# TMA4275 LIFETIME ANALYSIS

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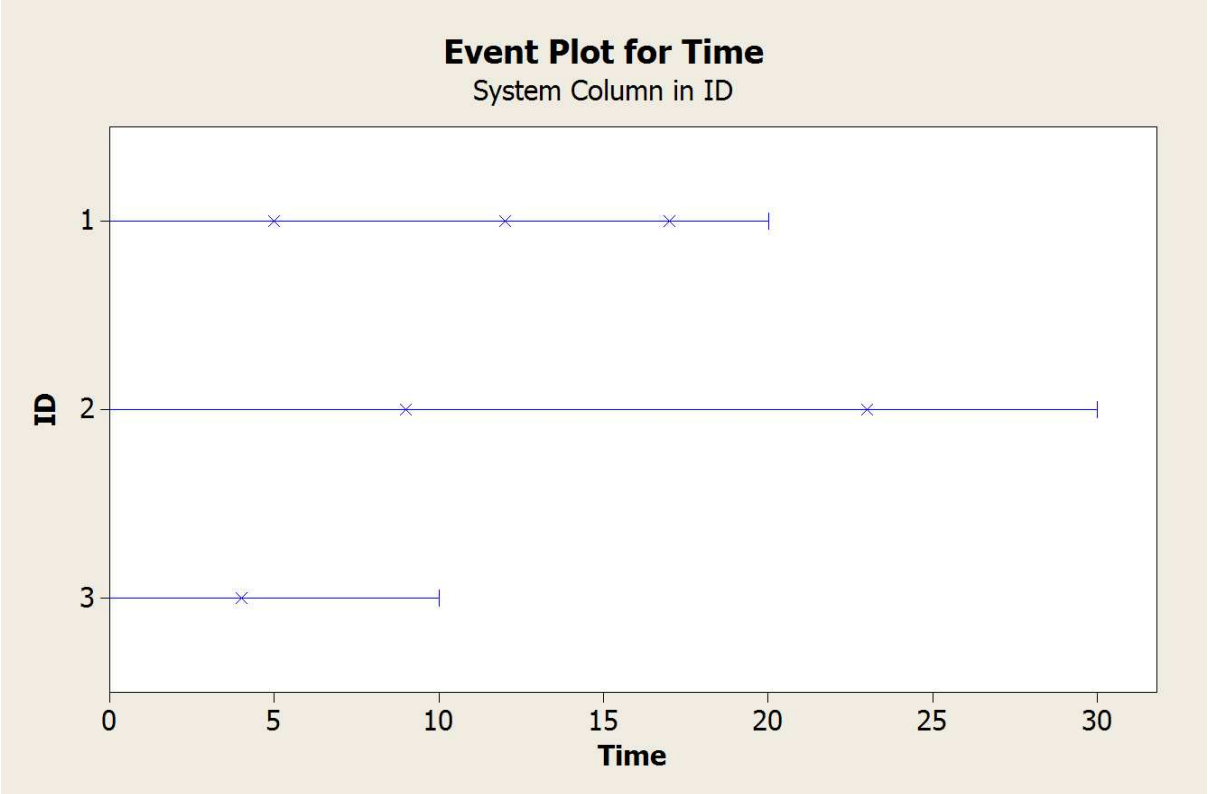
## Simple Example With 3 Systems

SimpleNHPP.MTW ***

| ↓ | C1 | C2 | C3 |
|---|---|---|---|
| | ID | Time | |
| 1 | 1 | 5 | |
| 2 | 1 | 12 | |
| 3 | 1 | 17 | |
| 4 | 1 | 20 | |
| 5 | 2 | 9 | |
| 6 | 2 | 23 | |
| 7 | 2 | 30 | |
| 8 | 3 | 4 | |
| 9 | 3 | 10 | |
| 10 | | | |

## Simple Example With 3 Systems






## Simple Example With 3 Systems

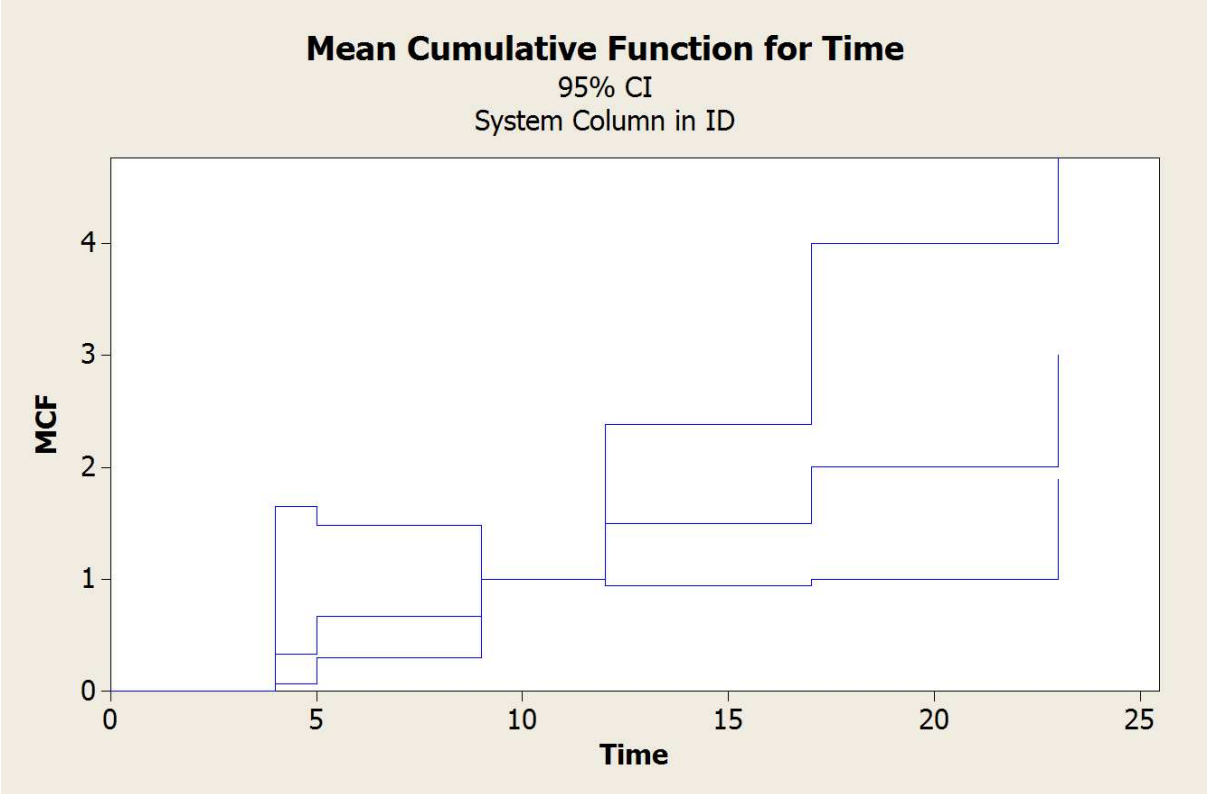
Results for: SimpleNHPP.MTW
Nonparametric Growth Curve: Time
System: ID

Nonparametric Estimates

Table of Mean Cumulative Function

| Time | Mean Cumulative Function | Standard Error | 95% Normal CI Lower | 95% Normal CI Upper | System |
|---|---|---|---|---|---|
| 4 | 0,33333 | 0,272166 | 0,06728 | 1,65151 | 3 |
| 5 | 0,66667 | 0,272166 | 0,29951 | 1,48392 | 1 |
| 9 | 1,00000 | 0,000000 | 1,00000 | 1,00000 | 2 |
| 12 | 1,50000 | 0,353553 | 0,94506 | 2,38079 | 1 |
| 17 | 2,00000 | 0,707107 | 1,00020 | 3,99922 | 1 |
| 23 | 3,00000 | 0,707107 | 1,89013 | 4,76158 | 2 |

## Simple Example With 3 Systems






## Nelson-Aalen estimator for Cumulative ROCOF $W(t)$

1. Order all failure times as $t_1 < t_2 < \ldots t_n$.
2. Let $d_j(t_i) = \#$ events in system $j$ at $t_i$.
3. Let $d(t_i) = \sum_{j=1}^{m} d_j(t_i) = \#$ events in all systems at $t_i$.
4. Let $Y_j(t) = \begin{cases} 1 & \text{if system } j \text{ is under observation at time } t \\ 0 & \text{otherwise} \end{cases}$
5. Let $Y(t) = \sum_{j=1}^{m} Y_j(t) = \#$ systems under observation at time $t$ .

Then

Under general assumptions: $\widehat{W}(t) = \sum_{t_i \le t} \frac{d(t_i)}{Y(t_i)}$.

Assuming NHPP: Var $\widehat{W}(t) = \sum_{t_i \le t} \frac{d(t_i)}{\{Y(t_i)\}^2}$

Under general assumptions (MINITAB): Var $\widehat{W}(t) = \sum_{j=1}^{m} \left\{ \sum_{t_i \le t} \frac{Y_j(t_i)}{Y(t_i)} \left[ d_j(t_i) - \frac{d(t_i)}{Y(t_i)} \right] \right\}^2$

**Illustration of last formula for Simple NHPP Example (Compare with MINITAB Output):**

$$\begin{aligned}
\text{Var}\,\widehat{\widehat{W}}(4) &= \left\{\frac{1}{3}\left[0-\frac{1}{3}\right]\right\}^2+\left\{\frac{1}{3}\left[0-\frac{1}{3}\right]\right\}^2+\left\{\frac{1}{3}\left[1-\frac{1}{3}\right]\right\}^2 \\
&= \frac{6}{81}=0.2722^2
\end{aligned}$$

$$\begin{aligned}
\text{Var}\,\widehat{\widehat{W}}(5) &= \left\{\frac{1}{3}\left[0-\frac{1}{3}\right]+\frac{1}{3}\left[1-\frac{1}{3}\right]\right\}^2 \\
&+ \left\{\frac{1}{3}\left[0-\frac{1}{3}\right]+\frac{1}{3}\left[0-\frac{1}{3}\right]\right\}^2 \\
&+ \left\{\frac{1}{3}\left[1-\frac{1}{3}\right]+\frac{1}{3}\left[0-\frac{1}{3}\right]\right\}^2 \\
&= \frac{6}{81}=0.2722^2
\end{aligned}$$



$$\begin{aligned}
\text{Var}\,\widehat{\widehat{W}}(9) &= \left\{\frac{1}{3}\left[0-\frac{1}{3}\right]+\frac{1}{3}\left[1-\frac{1}{3}\right]+\frac{1}{3}\left[0-\frac{1}{3}\right]\right\}^2 \\
&+ \left\{\frac{1}{3}\left[0-\frac{1}{3}\right]+\frac{1}{3}\left[0-\frac{1}{3}\right]+\frac{1}{3}\left[1-\frac{1}{3}\right]\right\}^2 \\
&+ \left\{\frac{1}{3}\left[1-\frac{1}{3}\right]+\frac{1}{3}\left[0-\frac{1}{3}\right]+\frac{1}{3}\left[0-\frac{1}{3}\right]\right\}^2 \\
&= 0
\end{aligned}$$

$$\begin{aligned}
\text{Var}\,\widehat{\widehat{W}}(12) &= \left\{\frac{1}{3}\left[0-\frac{1}{3}\right]+\frac{1}{3}\left[1-\frac{1}{3}\right]+\frac{1}{3}\left[0-\frac{1}{3}\right]+\frac{1}{2}\left[1-\frac{1}{2}\right]\right\}^2 \\
&+ \left\{\frac{1}{3}\left[0-\frac{1}{3}\right]+\frac{1}{3}\left[0-\frac{1}{3}\right]+\frac{1}{3}\left[1-\frac{1}{3}\right]+\frac{1}{2}\left[0-\frac{1}{2}\right]\right\}^2 \\
&+ \left\{\frac{1}{3}\left[1-\frac{1}{3}\right]+\frac{1}{3}\left[0-\frac{1}{3}\right]+\frac{1}{3}\left[0-\frac{1}{3}\right]\right\}^2 \\
&= \frac{1}{8}=0.3536^2
\end{aligned}$$

## Simple Example With 3 Systems

**Power Law NHPP Model:** $W(t;\alpha,\theta) = (t/\theta)^{\alpha}$

**Results for: SimpleNHPP.MTW**

**Parametric Growth Curve: Time**

System: ID

Model: Power-Law Process
Estimation Method: Maximum Likelihood

Parameter Estimates

| Parameter | Estimate | Standard Error | 95% Normal CI Lower | Upper |
|---|---|---|---|---|
| Shape | 1,19423 | 0,445 | 0,323015 | 2,06545 |
| Scale | 11,3803 | 4,840 | 1,89335 | 20,8672 |

Test for Equal Shape Parameters
Bartlett's Modified Likelihood Ratio Chi-Square

| | |
|---|---|
| Test Statistic | 0,06 |
| P-Value | 0,972 |
| DF | 2 |

Trend Tests

| | MIL-Hdbk-189 TTT-based | MIL-Hdbk-189 Pooled | Laplace's TTT-based | Laplace's Pooled | Anderson-Darling |
|---|---|---|---|---|---|
| Test Statistic | 9,03 | 8,89 | 0,28 | 0,31 | 0,28 |
| P-Value | 0,599 | 0,576 | 0,781 | 0,756 | 0,954 |
| DF | 12 | 12 | | | |



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