

FORELESNING 9

Våren 2004

18. mars

# **TMA4275 LEVETIDSANALYSE**

Bo Lindqvist

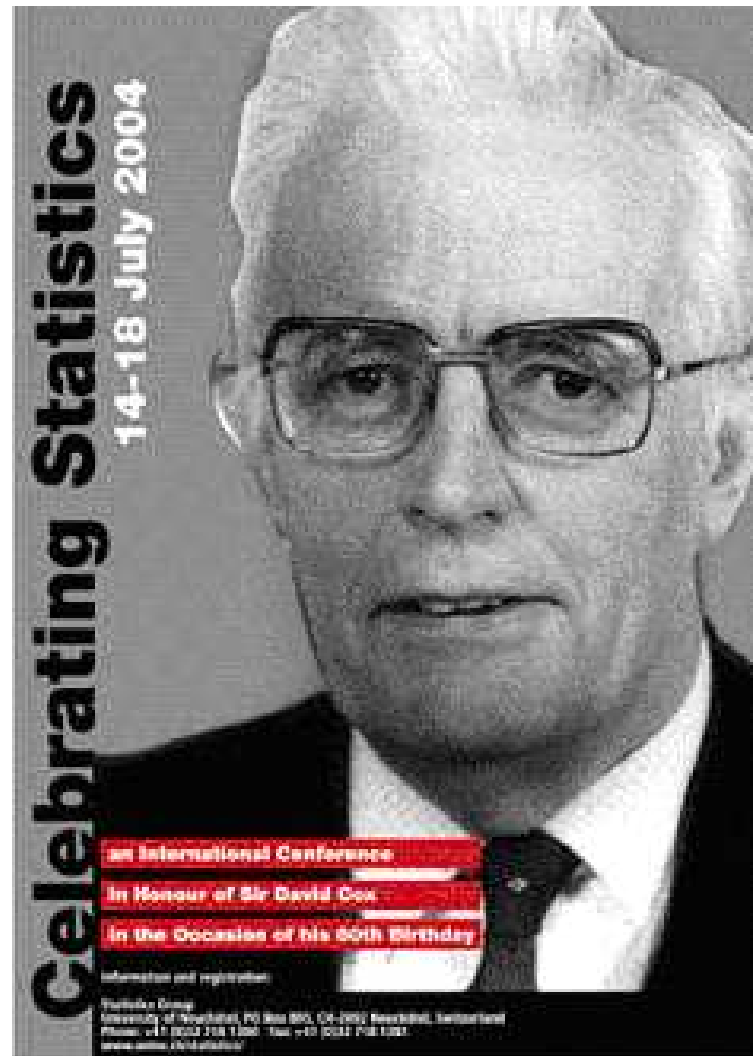
*Institutt for matematiske fag*

*NTNU*

bo@math.ntnu.no

<http://www.math.ntnu.no/~bo/>

## Sir David Cox (1924–)



## EXAMPLE COX-REGRESSION

$j$	$Y_j$	$x_j$	$\delta_j$
1	5	12	0
2	10	10	1
3	40	3	0
4	80	5	0
5	120	3	1
6	400	4	1
7	600	1	0

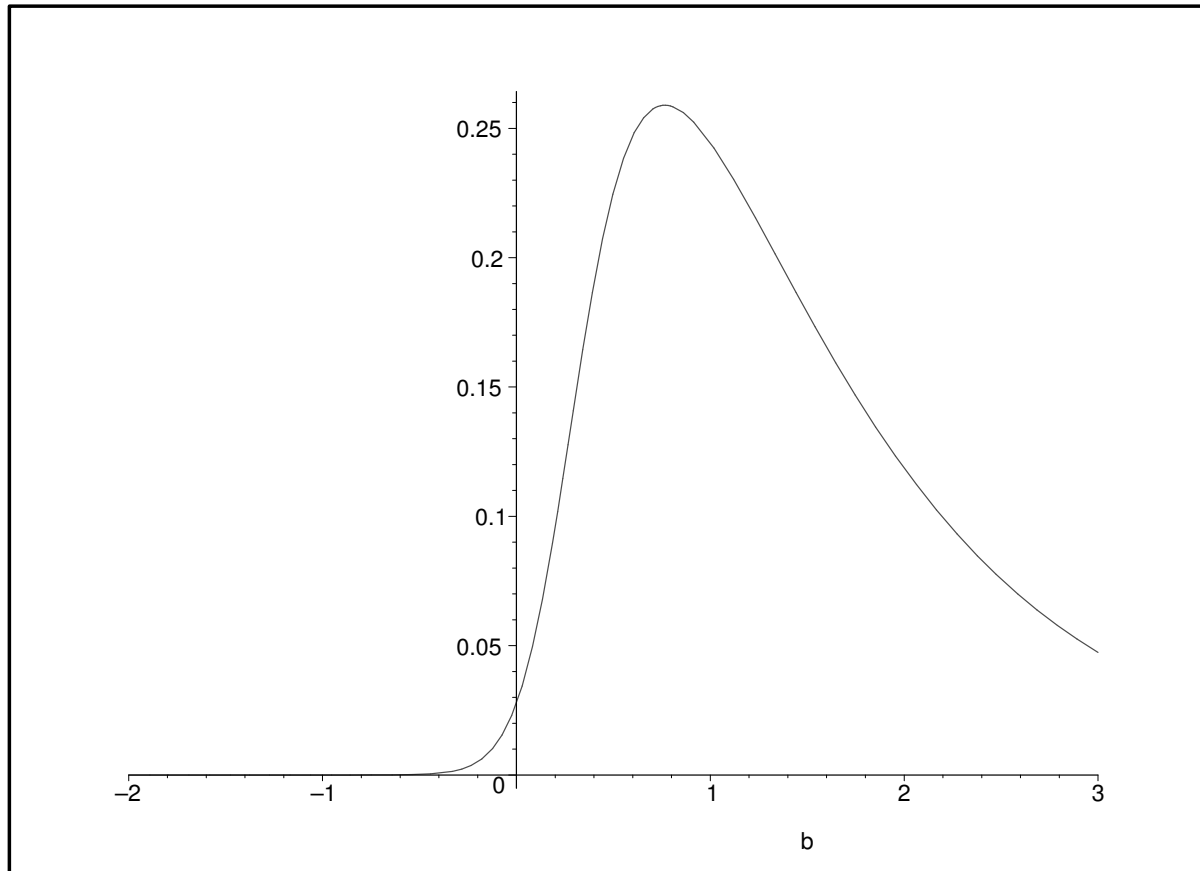
Model:

- $\lambda(t|x) = \lambda_0(t) \exp\{\beta x\}$

Partial likelihood:

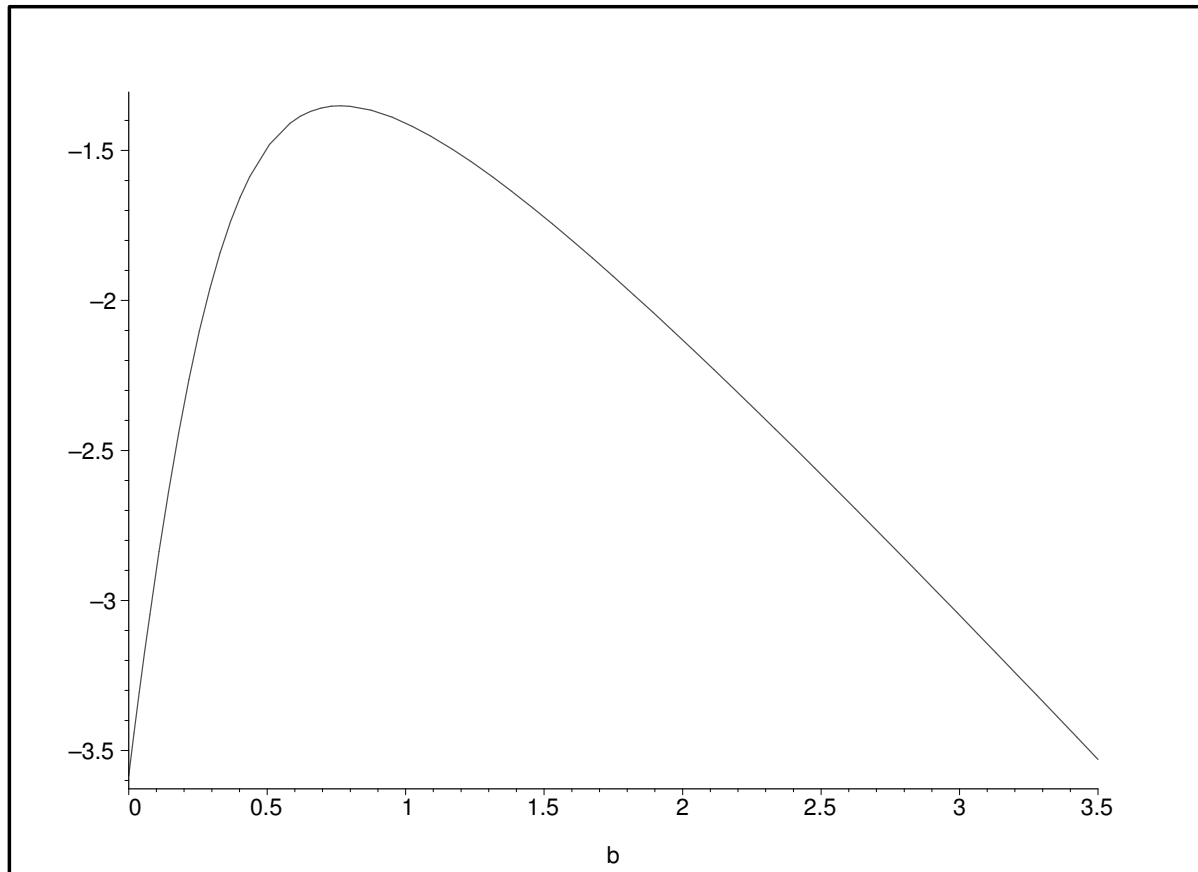
$$L(\beta) = \frac{e^{10\beta}}{e^{10\beta} + e^{3\beta} + e^{5\beta} + e^{3\beta} + e^{4\beta} + e^{\beta}} \cdot \frac{e^{3\beta}}{e^{3\beta} + e^{4\beta} + e^{\beta}} \cdot \frac{e^{4\beta}}{e^{4\beta} + e^{\beta}}$$

Cox' partial likelihood  $L(\beta)$  in the example:



Maximum likelihood estimate:  $\hat{\beta} = 0.765$ .

Cox' partial log-likelihood  $l(\beta)$  in the example:



Maximum likelihood estimate:  $\hat{\beta} = 0.765$ .

95% likelihood confidence interval: (0.1, 3.2).

# Weibull regression (Cox-example)

## Regression with Life Data: Y versus x

Response Variable: Y

Censoring Information Count  
 Uncensored value 3  
 Right censored value 4

Censoring value: d = 0

Estimation Method: Maximum Likelihood

Distribution: Weibull

Relationship with accelerating variable(s): Linear

### Regression Table

Predictor	Coef	Standard Error	Z	P	95,0% Normal CI	
					Lower	Upper
Intercept	7,58636	0,548229	13,84	0,000	6,51185	8,66087
x	-0,468235	0,0842830	-5,56	0,000	-0,633427	-0,303044
Shape	2,05563	0,872169			0,894943	4,72167

	C1	C2	C3	C4	C5	C6	C7	C8
	Y	x	d					
1	5	12	0					
2	10	10	1					
3	40	3	0					
4	80	5	0					
5	120	3	1					
6	400	4	1					
7	600	1	0					

Data fra Ansell & Phillips (s. 63)

Table 3.2. Lifetimes (in cycles) of sodium sulphur batteries

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Batch 1	164	164	218	230	263	467	538	639	669
	917	1148	1678+	1678+	1678+	1678+			
Batch 2	76	82	210	315	385	412	491	504	522
	646+	678	775	884	1131	1446	1824	1827	2248
	2385	3077							

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Note: Lifetimes with + are right censored observations, not failures.

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C1=Obs. tider, C2=sensurering, C3="batch" nr.

Ansell32.MTW ***												
↓	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12
1	164	1	1									
2	164	1	1									
3	218	1	1									
4	230	1	1									
5	263	1	1									
6	467	1	1									
7	538	1	1									
8	639	1	1									
9	669	1	1									
10	917	1	1									
11	1148	1	1									
12	1678	0	1									
13	1678	0	1									
14	1678	0	1									
15	1678	0	1									
16	76	1	2									
17	82	1	2									
18	210	1	2									
19	315	1	2									
20	385	1	2									
21	412	1	2									
22	491	1	2									
23	504	1	2									
24	522	1	2									
25	646	0	2									



## MINITAB-analyse av dataene fra Ansell & Phillips:

**Nonparametric Distribution Analysis-Right Censoring**

C1  
C2  
C3

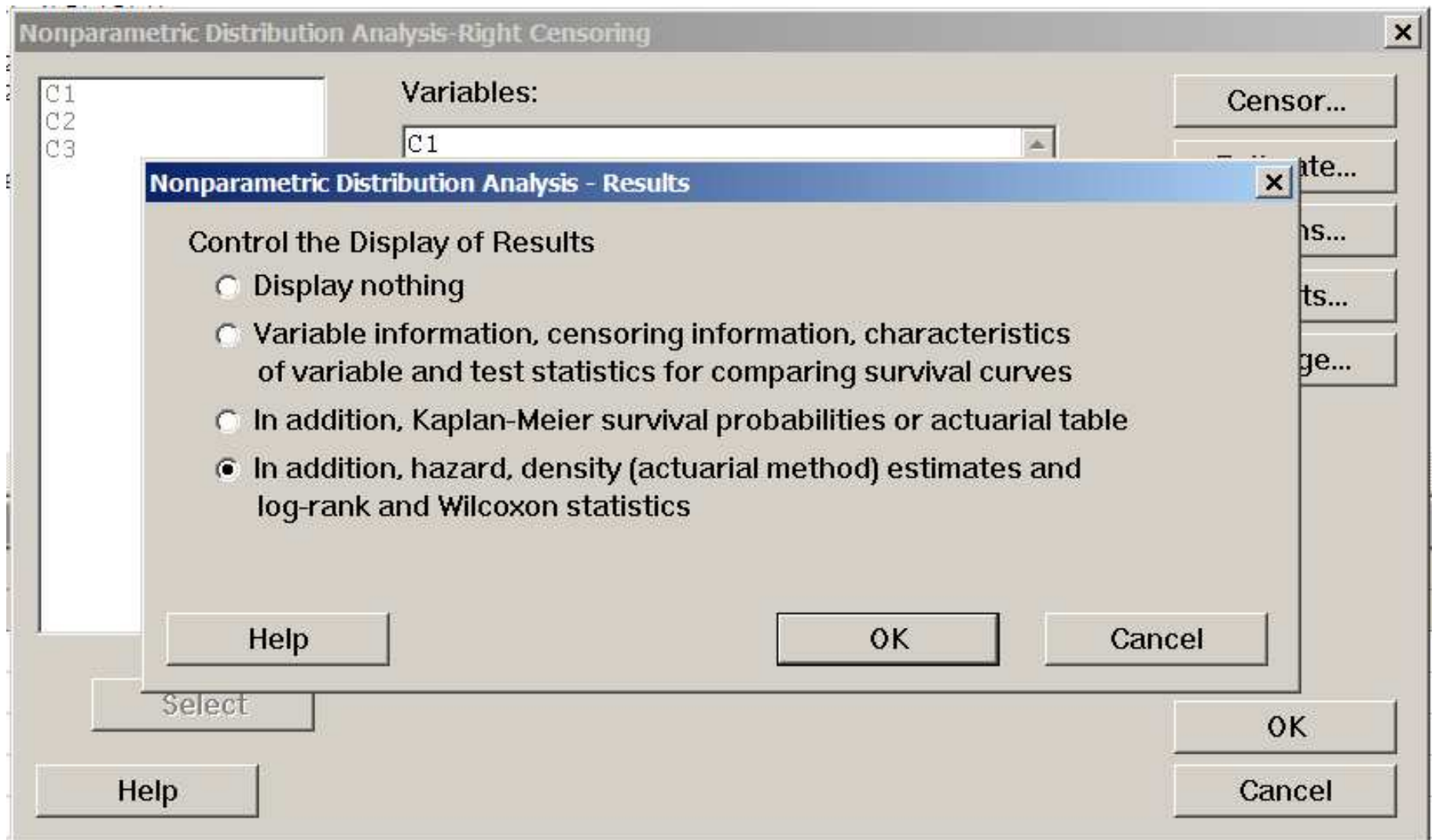
Variables:  
C1

Frequency columns (optional):

By variable: c3  
Enter number of levels: 2

Select  
Help

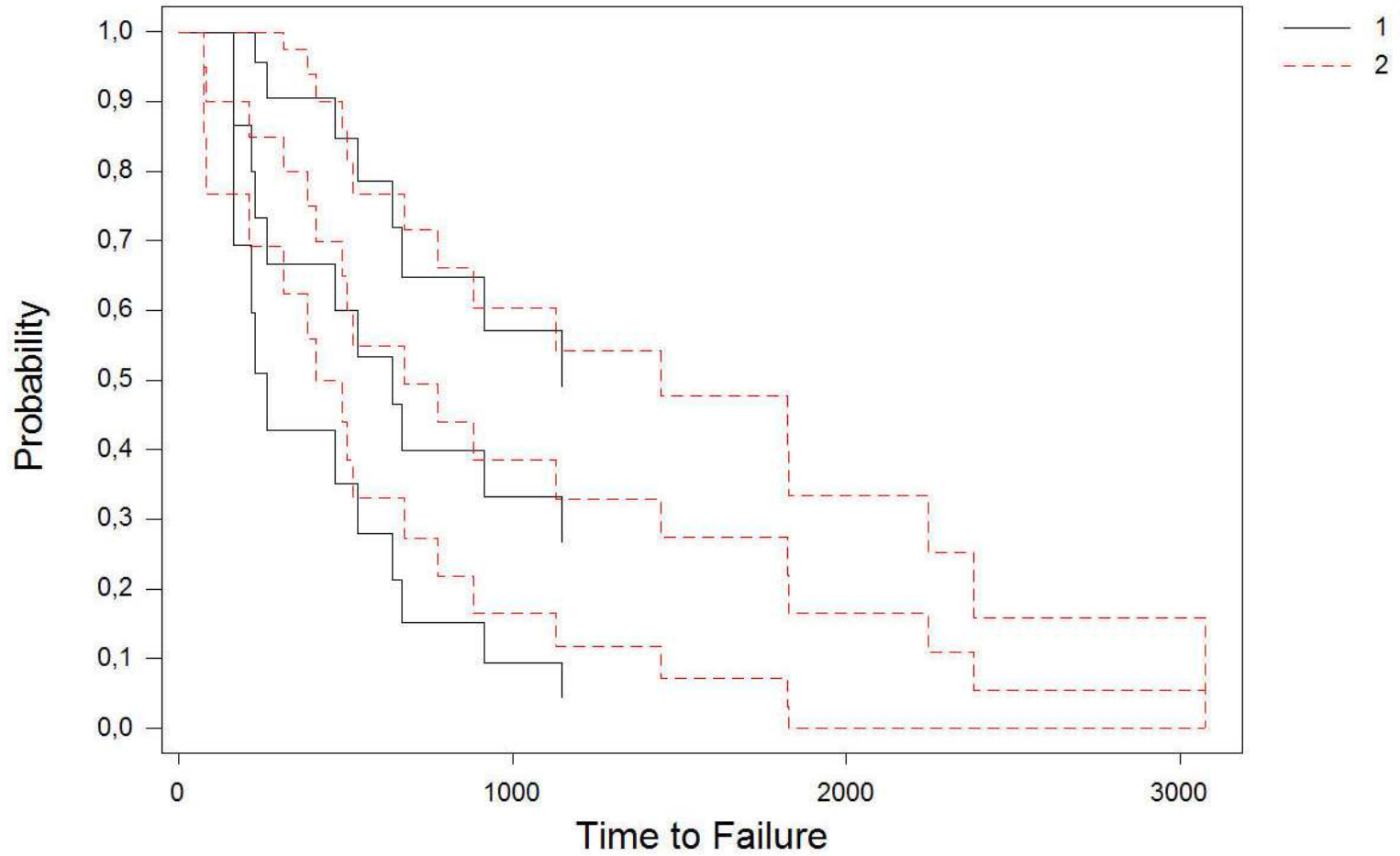
Censor...  
Estimate...  
Graphs...  
Results...  
Storage...  
OK  
Cancel



# Nonparametric Survival Plot for C1

Kaplan-Meier Method - 95,0% CI

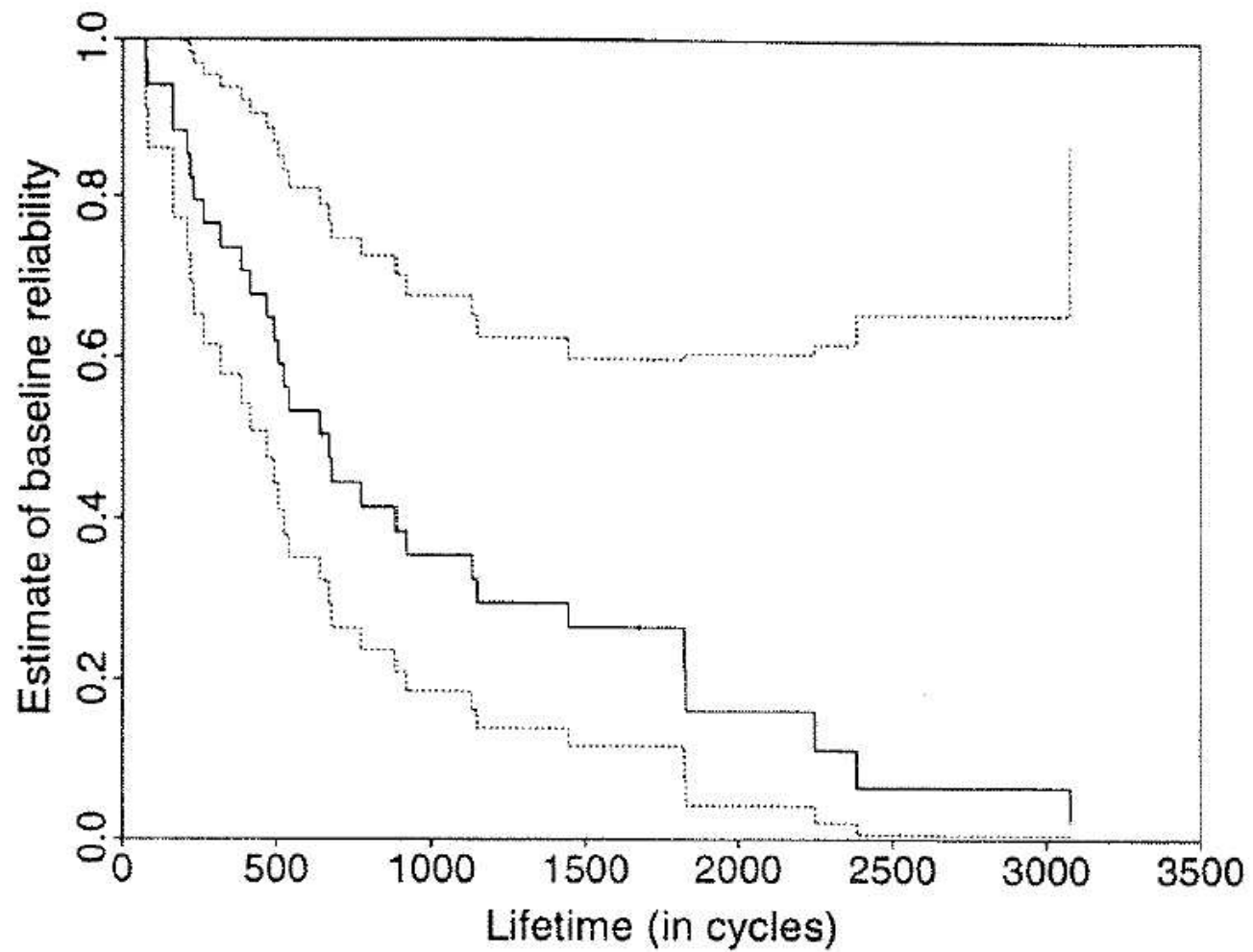
Censoring Column in C2



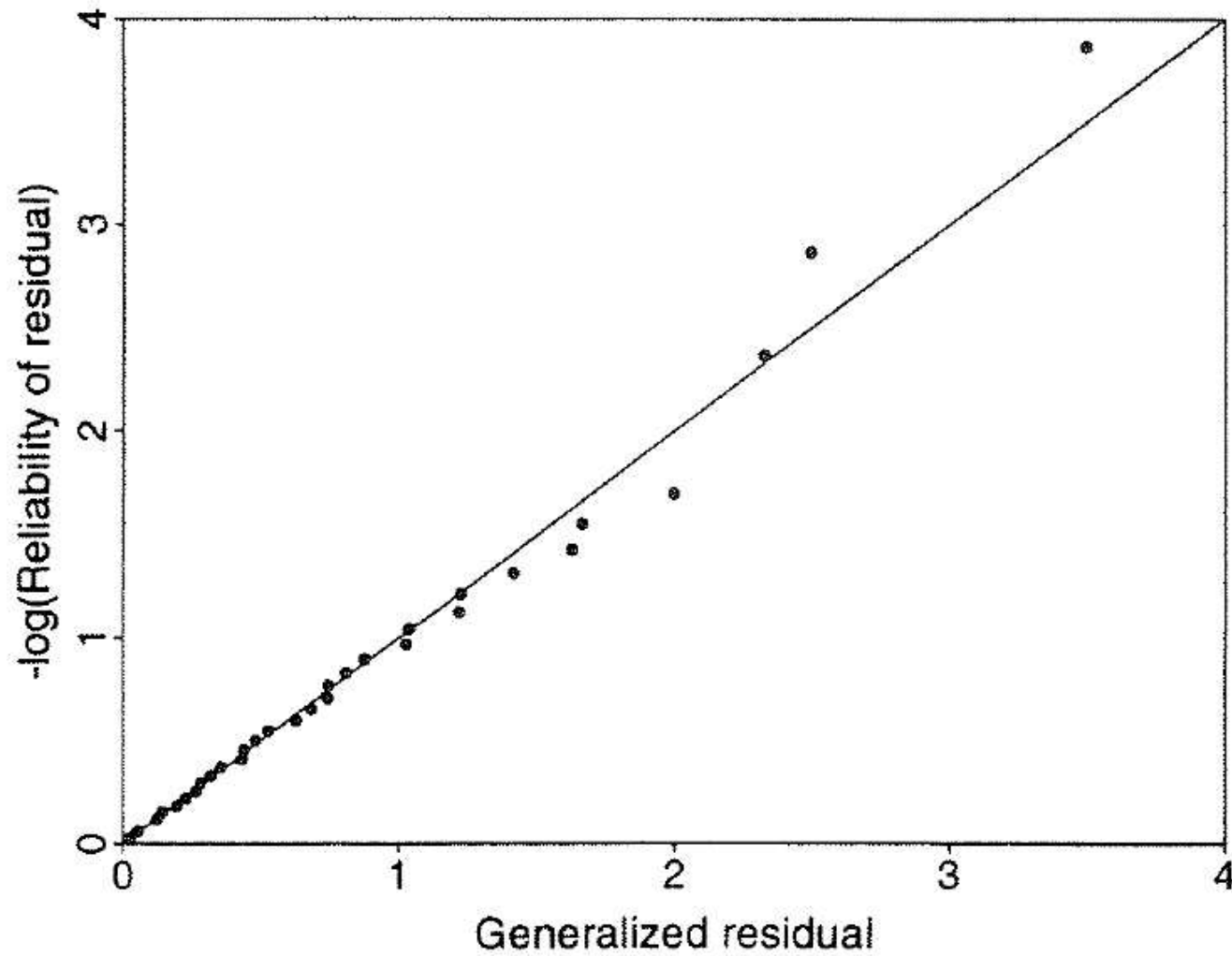
**MINITAB-resultat (blant mye annen output):**

Test Statistics Method			
Chi-Square	DF	P-Value	
Log-Rank	0,04855	1	0,8256

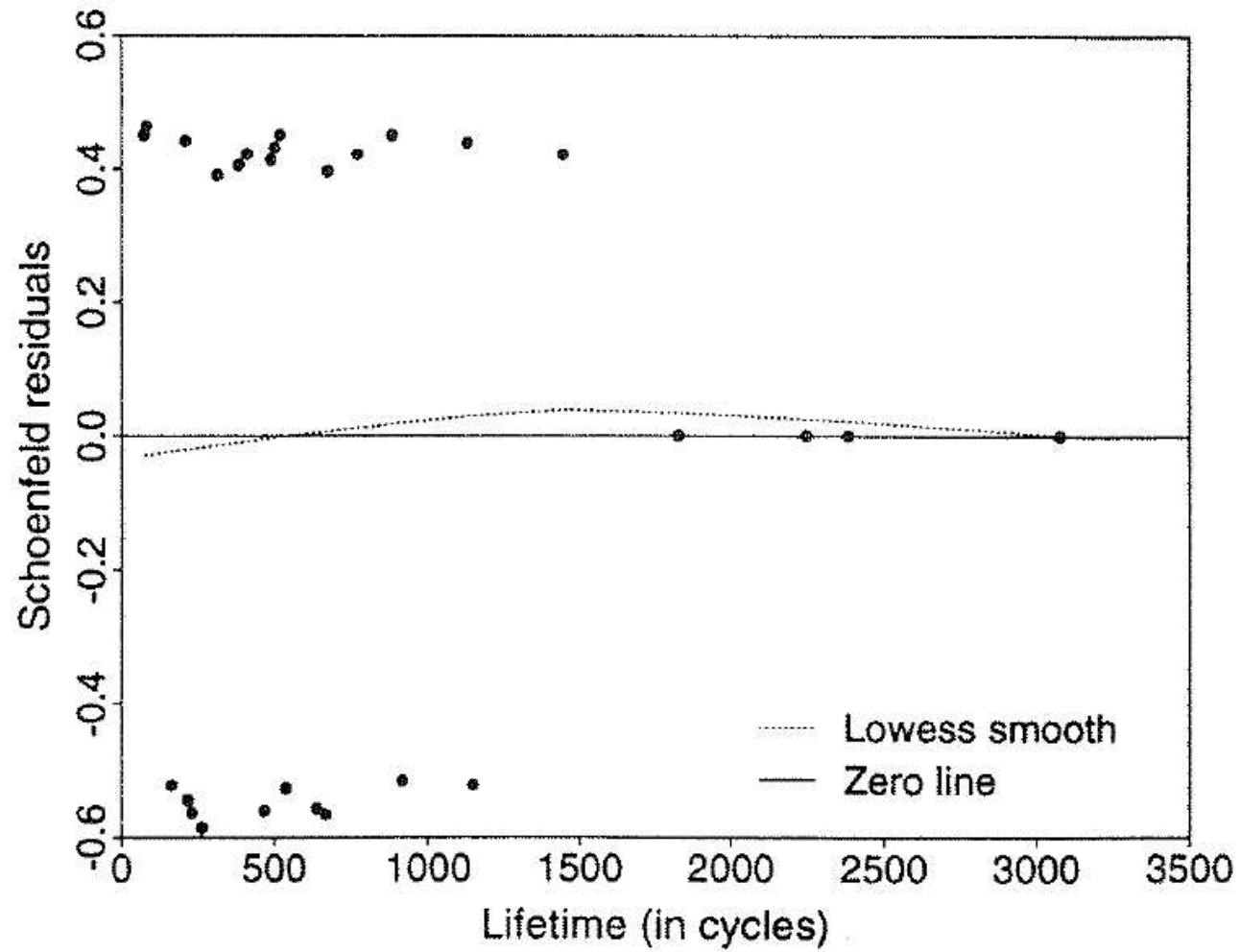
**Dette stemmer overens med resultatet s. 72 (Example 3.5.2) i Ansell & Phillips.**



**Fig. 3.3.** Plot of the baseline reliability function for the proportional hazards model for the sodium sulphur battery data with 95% confidence limits



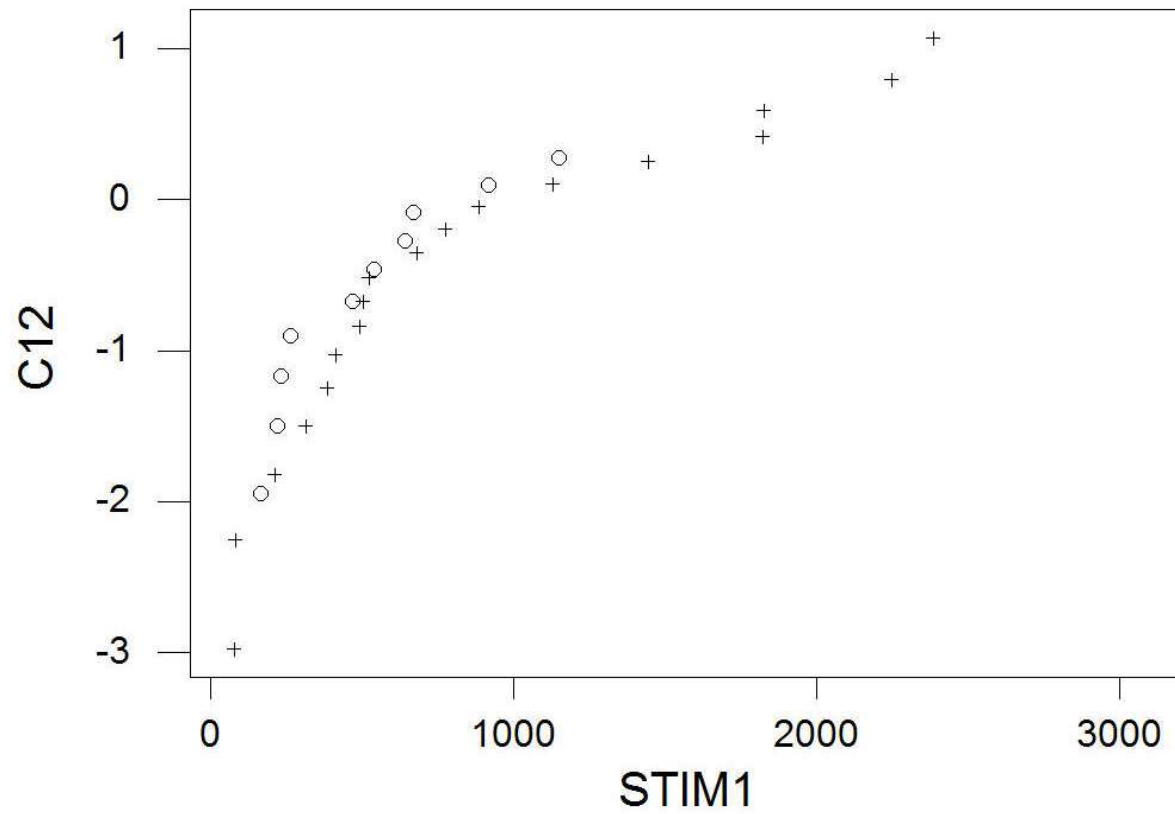
**Fig. 3.5.** Plot of the generalized residuals of the proportional hazards model for the sodium sulphur battery data



**Fig. 3.7.** Plot of the Schoenfeld residuals for batch of the proportional hazards model for the sodium sulphur battery data

# LOG MINUS LOG PLOTT MOT "t" FOR A&P DATA (s.63)

Parallele plott tyder på Proportional Hazard





# LOG MINUS LOG PLOTT MOT "log t" FOR A&P DATA (s.63)

Rette linjer tyder på Weibull-fordeling

