

FORELESNING 8

Våren 2004

11. mars

TMA4275 LEVETIDSANALYSE

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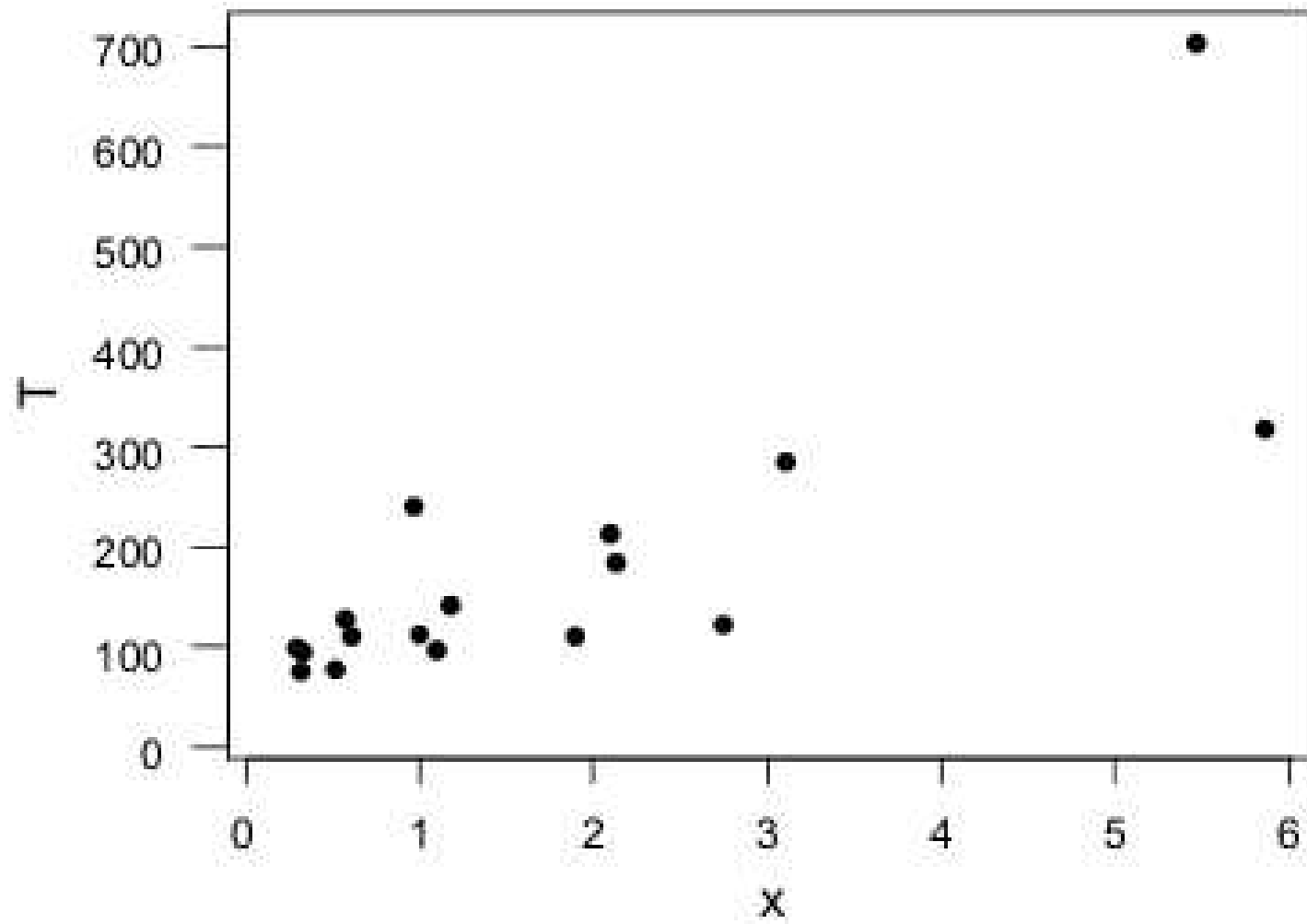
<http://www.math.ntnu.no/~bo/>

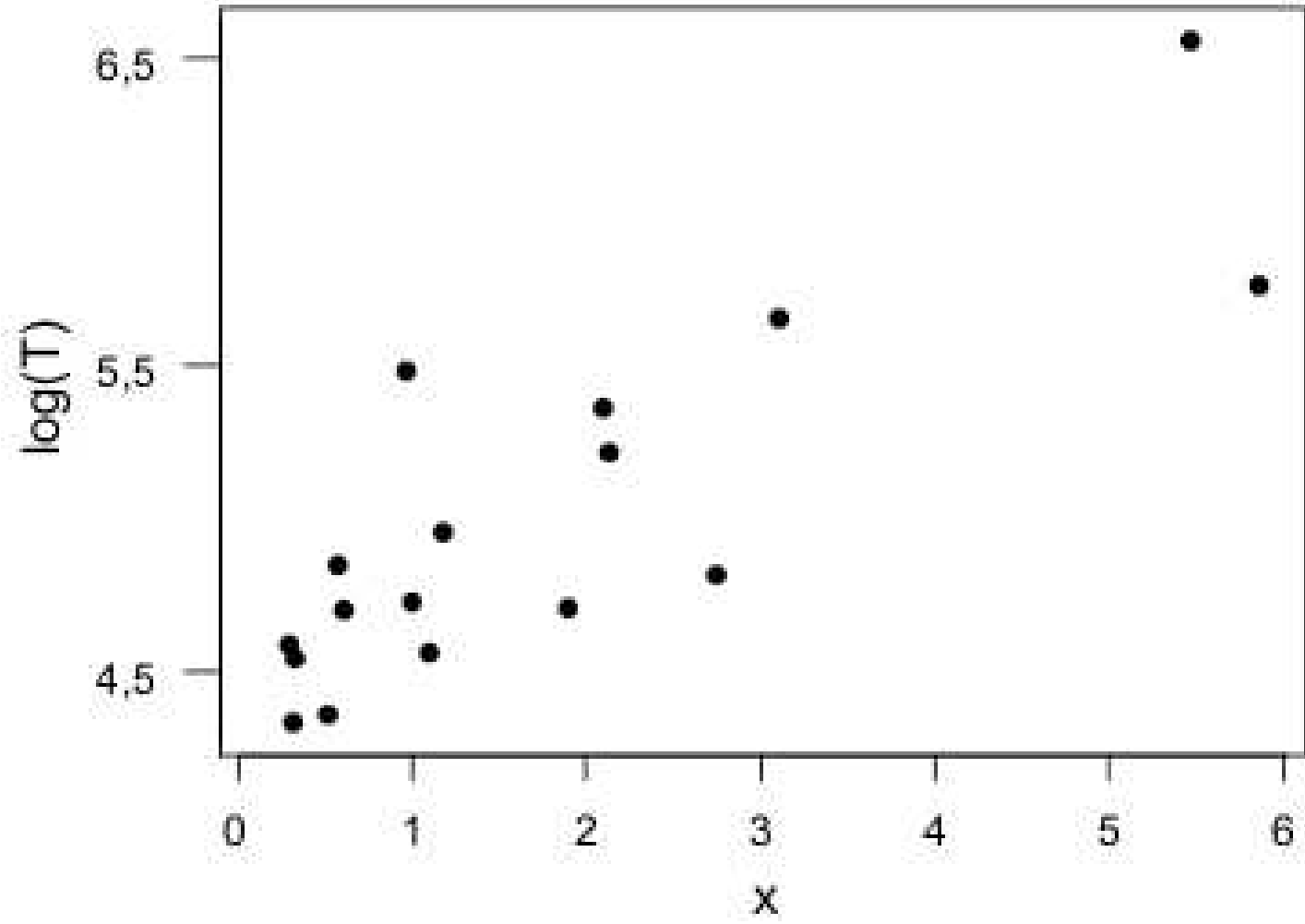
COMPUTER PROGRAM EXECUTION TIME vs SYSTEM LOAD

Data: 17 observasjoner av (T,x)

- Tid for å gjennomføre en dataintensiv oppgave
- Info fra Unix "uptime"-kommando
- Prediksjon ønskes for framtidig fler-steps oppgave

Seconds (T)	Load (x)	Seconds (T)	Load (x)
123	2,74	110	,60
704	5,47	213	2,10
184	2,13	284	3,10
113	1,00	317	5,86
94	,32	142	1,18
76	,31	127	,57
78	,51	96	1,10
98	,29	111	1,89
240	,96		





MINITAB - Untitled

File Edit Manip Calc Stat Graph Editor Window Help

Session

01.03

Welcome to Minitab
Saving file as: C:\Program Files\Minitab\Minitab 16\Sample\Levetidsanalyse\Minitabplot\C11.MTW

Results for: C11.MTW

Plot C1 * C2

Plot T * x

MTB > let c3=log(c1)
MTB > Plot c3*c2;
SUBC> Symbol;
SUBC> ScFrame;
SUBC> ScAnnotation.

Plot log(T) * x

MTB >

Stat

- Basic Statistics
- Regression
- ANOVA
- DOE
- Control Charts
- Quality Tools
- Reliability/Survival
 - Distribution ID Plot-Right Cens...
 - Distribution Overview Plot-Right Cens...
 - Parametric Dist Analysis-Right Cens...
 - Nonparametric Dist Analysis-Right Cens...
 - Distribution ID Plot-Arbitrary Cens...
 - Distribution Overview Plot-Arbitrary Cens...
 - Parametric Dist Analysis-Arbitrary Cens...
 - Nonparametric Dist Analysis-Arbitrary Cens...
 - Accelerated Life Testing...
 - Regression with Life Data...
 - Probit Analysis...
- Multivariate
- Time Series
- Tables
- Nonparametrics
- EDA
- Power and Sample Size

C11.MTW ***

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15	C16	C17
	T	x	log(T)														
1	123	2,74	4,81218														
2	704	5,47	6,55678														
3	184	2,13	5,21494														
4	113	1,00	4,72739														
5	94	0,32	4,54329														
6	76	0,31	4,33073														
7	78	0,51	4,35671														
8	98	0,29	4,58497														
9	240	0,96	5,48064														
10	110	0,60	4,70048														
11	213	2,10	5,36129														
12	284	3,10	5,64897														

Projec...

Perform a regression analysis on life data with more than two predictors

20:35

Start Internet Exp... Foller WinEdt 5.3 - [C:\... Yap 0.99a - [Forel... Corel PHOTO-PAIN... MINITAB - Untit... 20:35

MINITAB - Untitled

File Edit Manip Calc Stat Graph Editor Window Help

Session

01.03.2003 20:16:11

Welcome to Minitab, press F1 for help.
 Saving file as: C:\Documents and Settings\Bo Lindqvist\My Documents\Jobb\Fag\Levetidsanalyse\Minitabplot\C11.MTW

Results for: C11.MTW

Plot C1 * C2

Plot T * x

```

MTB > let c3=log(c1)
MTB > Plot c3*c2;
SUBC> Symbol;
SUBC> ScFrame;
SUBC> ScAnnotation.
  
```

Plot log(T) * x

```

MTB >
  
```

Regression with Life Data

C1	T	
C2	x	
C3	log(T)	

Responses are uncens/right censored data
 Responses are uncens/arbitrarily censored data

Variables/ Start variables: c1
 End variables:

Freq. columns: (optional)

Model: c2

Factors (optional):

Assumed distribution: Lognormal base e

Buttons: Censor..., Estimate..., Graphs..., Results..., Options..., Storage..., OK, Cancel, Select, Help

↓	C1	C2	C3
	T	x	log(T)
1	123	2,74	4,81218
2	704	5,47	6,55678
3	184	2,13	5,21494
4	113	1,00	4,72739
5	94	0,32	4,54329
6	76	0,31	4,33073
7	78	0,51	4,35671
8	98	0,29	4,58497
9	240	0,96	5,48064
10	110	0,60	4,70048
11	213	2,10	5,36129
12	284	3,10	5,64897

Projec... [] [x]

Welcome to Minitab, press F1 for help.

20:39

Start | 3 Internet Expl... | Foller | WinEdt 5.3 - [C:\... | Yap 0.99a - [Forel... | Corel PHOTO-PAIN... | MINITAB - Untit... | 20:39

Regression with Life Data: T versus x

|
Response Variable: T

Censoring Information Count
Uncensored value 17

Estimation Method: Maximum Likelihood
Distribution: Lognormal base e

Regression Table

Predictor	Coef	Standard Error	Z	P	95,0% Normal CI	
					Lower	Upper
Intercept	4,4936	0,1112	40,39	0,000	4,2756	4,7116
x	0,29075	0,04595	6,33	0,000	0,20069	0,38080
Scale	0,31247	0,05359			0,22327	0,43730

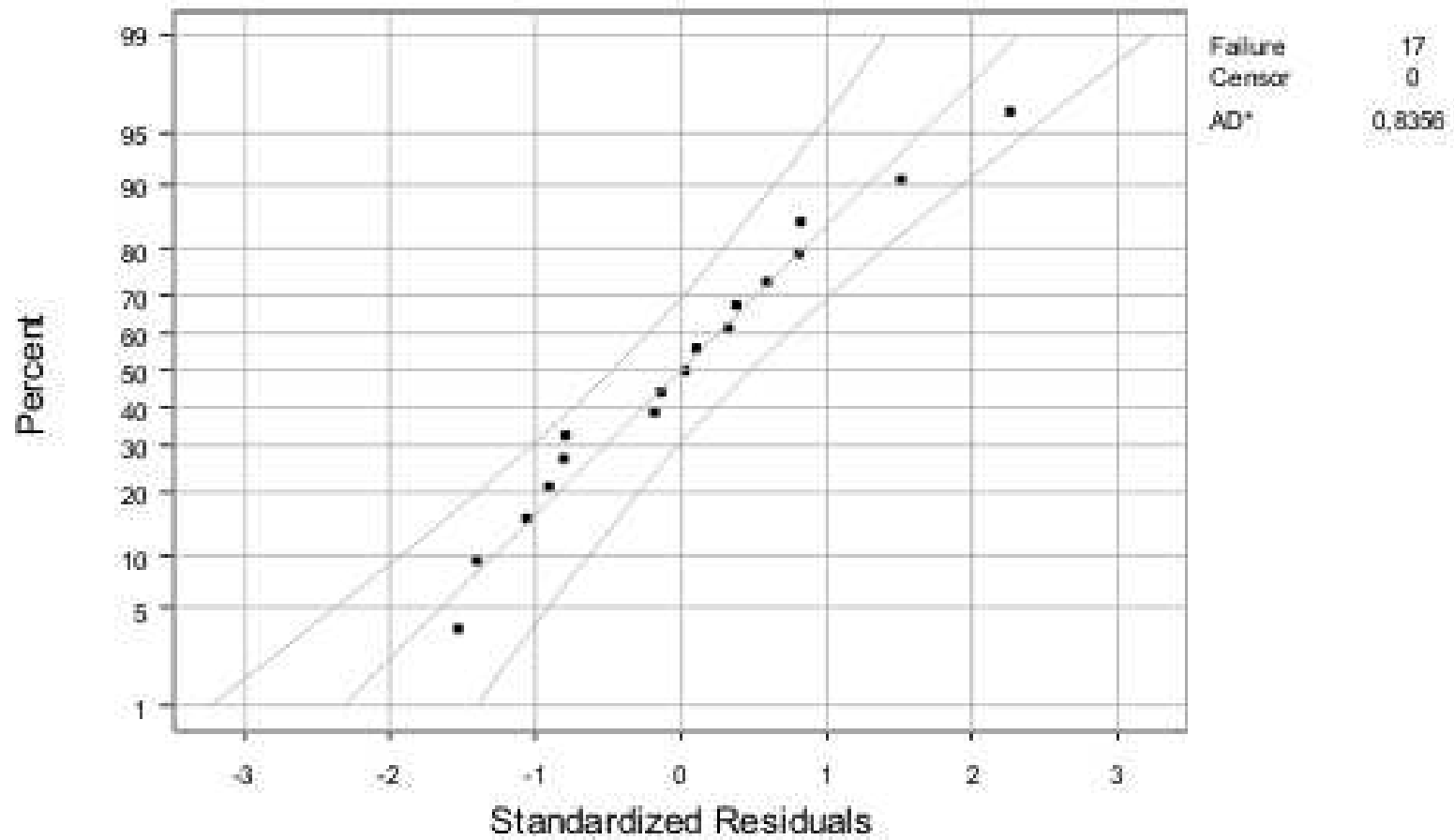
Log-Likelihood = -89,498

Anderson-Darling (adjusted) Goodness-of-Fit

Standardized Residuals = 0,8356; Cox-Snell Residuals = 0,8170

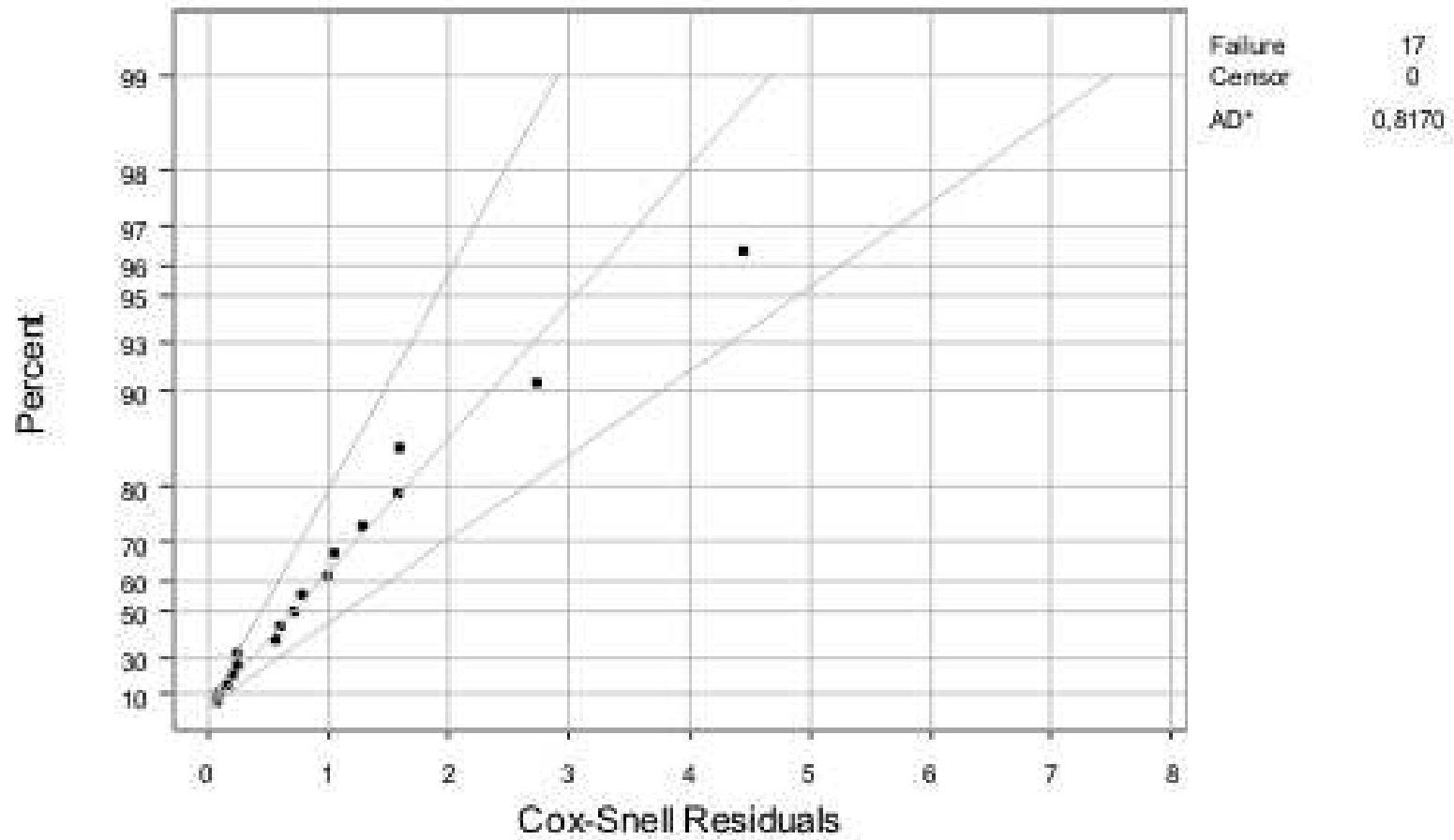
Probability Plot for SResids of C1

Normal Distribution - ML Estimates - 95.0% CI
Complete Data



Probability Plot for CSResids of C1

Exponential Distribution - ML Estimates - 95.0% CI
Complete Data



Regression with Life Data: C1 versus C2

Response Variable: C1

Censoring Information	Count
Uncensored value	17

Estimation Method: Maximum Likelihood
Distribution: Weibull

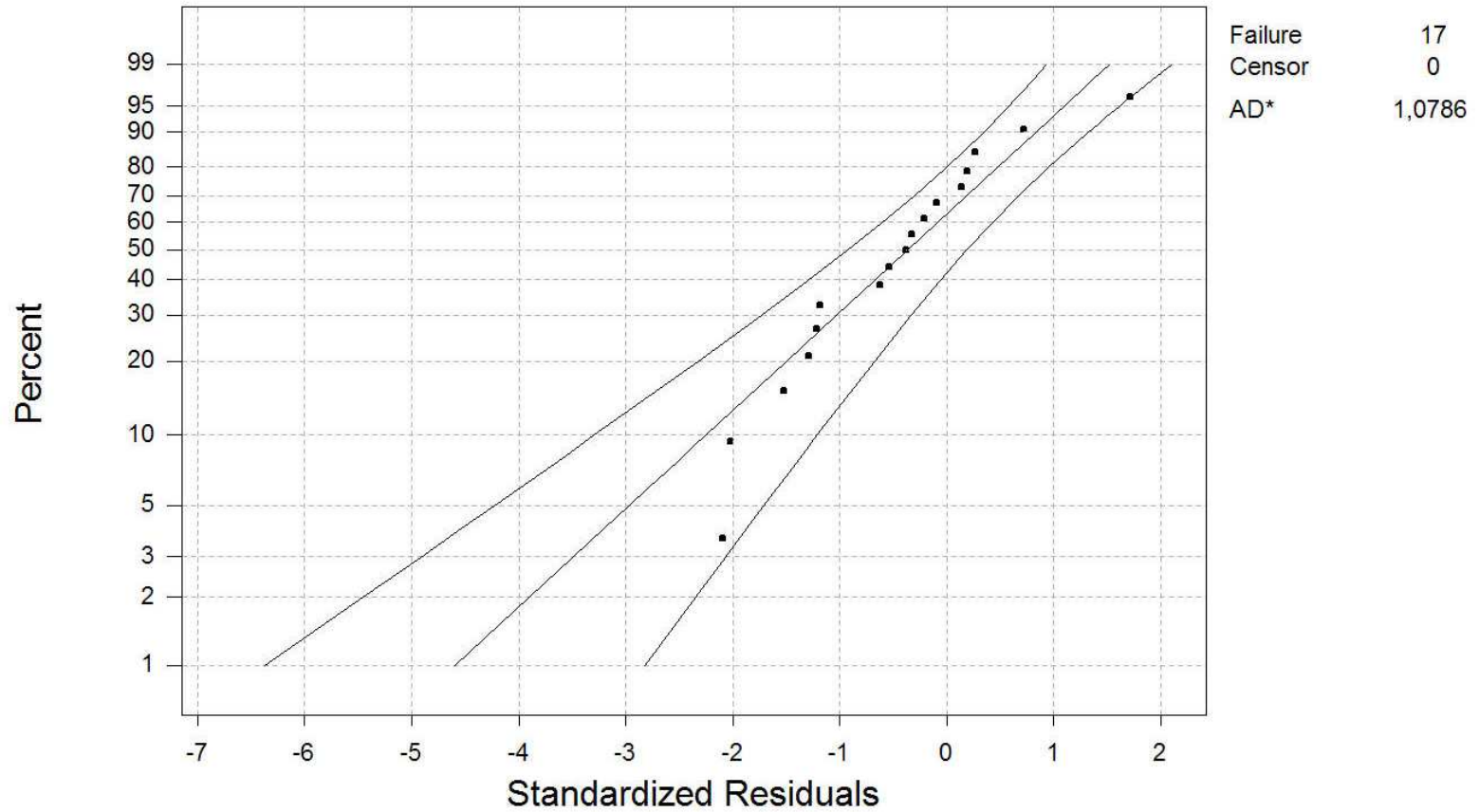
Regression Table

Predictor	Coef	Standard Error	Z	P	95,0% Normal CI	
					Lower	Upper
Intercept	<u>4,6182</u>	0,1219	37,88	0,000	4,3792	4,8572
C2	<u>0,31118</u>	0,04939	6,30	0,000	0,21437	0,40799
Shape	<u>3,0604</u>	0,5245			2,1873	4,2820

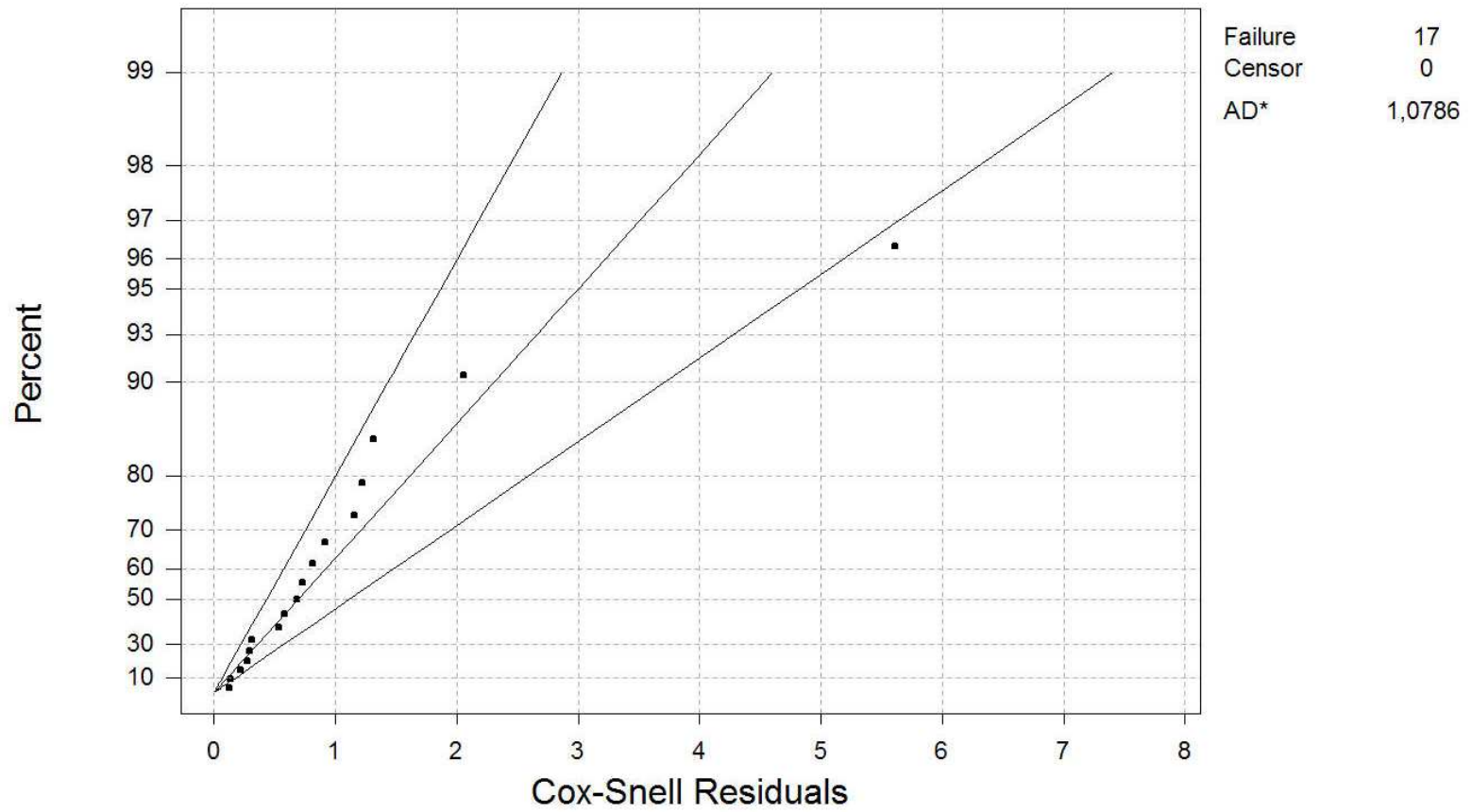
Log-Likelihood = -91,504

Anderson-Darling (adjusted) Goodness-of-Fit

Probability Plot for SResids of C1
Extreme value Distribution - ML Estimates - 95,0% CI
Complete Data



Probability Plot for CSResids of C1
Exponential Distribution - ML Estimates - 95,0% CI
Complete Data



Ordinary residuals

$$y_i - x'_i \hat{\beta}$$

where

y_i is the i th response value

x'_i is the vector of predictor values associated with the i th response value

$\hat{\beta}$ represents the estimated regression coefficients

Standardized residuals

$$\frac{y_i - x'_i \hat{\beta}}{\hat{\sigma}}$$

where $\hat{\sigma}$ is the estimated scale parameter.

Cox-Snell residuals

$$-\ln(\hat{R}(y_i))$$

where

$\hat{R}(y_i)$ is the estimated survival (reliability) probability for the response value y_i

$\ln(x)$ is the natural log of x

Lognormal Distribution Simple Regression Model with Constant Shape Parameter $\beta = 1/\sigma$

- The lognormal simple regression model is

$$\Pr(T \leq t) = F(t; \mu, \sigma) = F(t; \beta_0, \beta_1, \sigma) = \Phi_{\text{nor}} \left[\frac{\log(t) - \mu}{\sigma} \right]$$

where $\mu = \mu(x) = \beta_0 + \beta_1 x$ and σ does not depend on x .

- The failure-time log quantile function

$$\log[t_p(x)] = \mu(x) + \Phi_{\text{nor}}^{-1}(p)\sigma$$

is linear in x .

Notice that

$$\frac{t_p(x)}{t_p(0)} = \exp(\beta_1 x)$$

implies that this regression model is a scale accelerated failure time (SAFT) model with $\mathcal{AF}(x) = \exp(-\beta_1 x)$.

Likelihood for Lognormal Distribution Simple Regression Model with Right Censored Data

The likelihood for n independent observations has the form

$$\begin{aligned} L(\beta_0, \beta_1, \sigma) &= \prod_{i=1}^n L_i(\beta_0, \beta_1, \sigma; \text{data}_i) \\ &= \prod_{i=1}^n \left\{ \frac{1}{\sigma t_i} \phi_{\text{nor}} \left[\frac{\log(t_i) - \mu_i}{\sigma} \right] \right\}^{\delta_i} \left\{ 1 - \Phi_{\text{nor}} \left[\frac{\log(t_i) - \mu_i}{\sigma} \right] \right\}^{1-\delta_i} \end{aligned}$$

where $\text{data}_i = (x_i, t_i, \delta_i)$, $\mu_i = \beta_0 + \beta_1 x_i$,

$$\delta_i = \begin{cases} 1 & \text{exact observation} \\ 0 & \text{right censored observation} \end{cases}$$

$\phi_{\text{nor}}(z)$ is the standardized normal pdf and $\Phi_{\text{nor}}(z)$ is the corresponding normal cdf.

The parameters are $\theta = (\beta_0, \beta_1, \sigma)$.

Estimated Parameter Variance-Covariance Matrix

Local (observed information) estimate

$$\begin{aligned}\widehat{\Sigma}_{\hat{\theta}} &= \begin{bmatrix} \widehat{\text{Var}}(\hat{\beta}_0) & \widehat{\text{Cov}}(\hat{\beta}_0, \hat{\beta}_1) & \widehat{\text{Cov}}(\hat{\beta}_0, \hat{\sigma}) \\ \widehat{\text{Cov}}(\hat{\beta}_1, \hat{\beta}_0) & \widehat{\text{Var}}(\hat{\beta}_1) & \widehat{\text{Cov}}(\hat{\beta}_1, \hat{\sigma}) \\ \widehat{\text{Cov}}(\hat{\sigma}, \hat{\beta}_0) & \widehat{\text{Cov}}(\hat{\sigma}, \hat{\beta}_1) & \widehat{\text{Var}}(\hat{\sigma}) \end{bmatrix} \\ &= \begin{bmatrix} -\frac{\partial^2 \mathcal{L}(\beta_0, \beta_1, \sigma)}{\partial \beta_0^2} & -\frac{\partial^2 \mathcal{L}(\beta_0, \beta_1, \sigma)}{\partial \beta_0 \partial \beta_1} & -\frac{\partial^2 \mathcal{L}(\beta_0, \beta_1, \sigma)}{\partial \beta_0 \partial \sigma} \\ -\frac{\partial^2 \mathcal{L}(\beta_0, \beta_1, \sigma)}{\partial \beta_1 \partial \beta_0} & -\frac{\partial^2 \mathcal{L}(\beta_0, \beta_1, \sigma)}{\partial \beta_1^2} & -\frac{\partial^2 \mathcal{L}(\beta_0, \beta_1, \sigma)}{\partial \beta_1 \partial \sigma} \\ -\frac{\partial^2 \mathcal{L}(\beta_0, \beta_1, \sigma)}{\partial \sigma \partial \beta_0} & -\frac{\partial^2 \mathcal{L}(\beta_0, \beta_1, \sigma)}{\partial \sigma \partial \beta_1} & -\frac{\partial^2 \mathcal{L}(\beta_0, \beta_1, \sigma)}{\partial \sigma^2} \end{bmatrix}^{-1}\end{aligned}$$

Partial derivatives are evaluated at $\hat{\beta}_0, \hat{\beta}_1, \hat{\sigma}$.

Standard Errors and Confidence Intervals for Parameters

- Lognormal ML estimates for the computer time experiment were $\hat{\theta} = (\hat{\beta}_0, \hat{\beta}_1, \hat{\sigma}) = (4.49, .290, .312)$ and an estimate of the variance-covariance matrix for $\hat{\theta}$ is

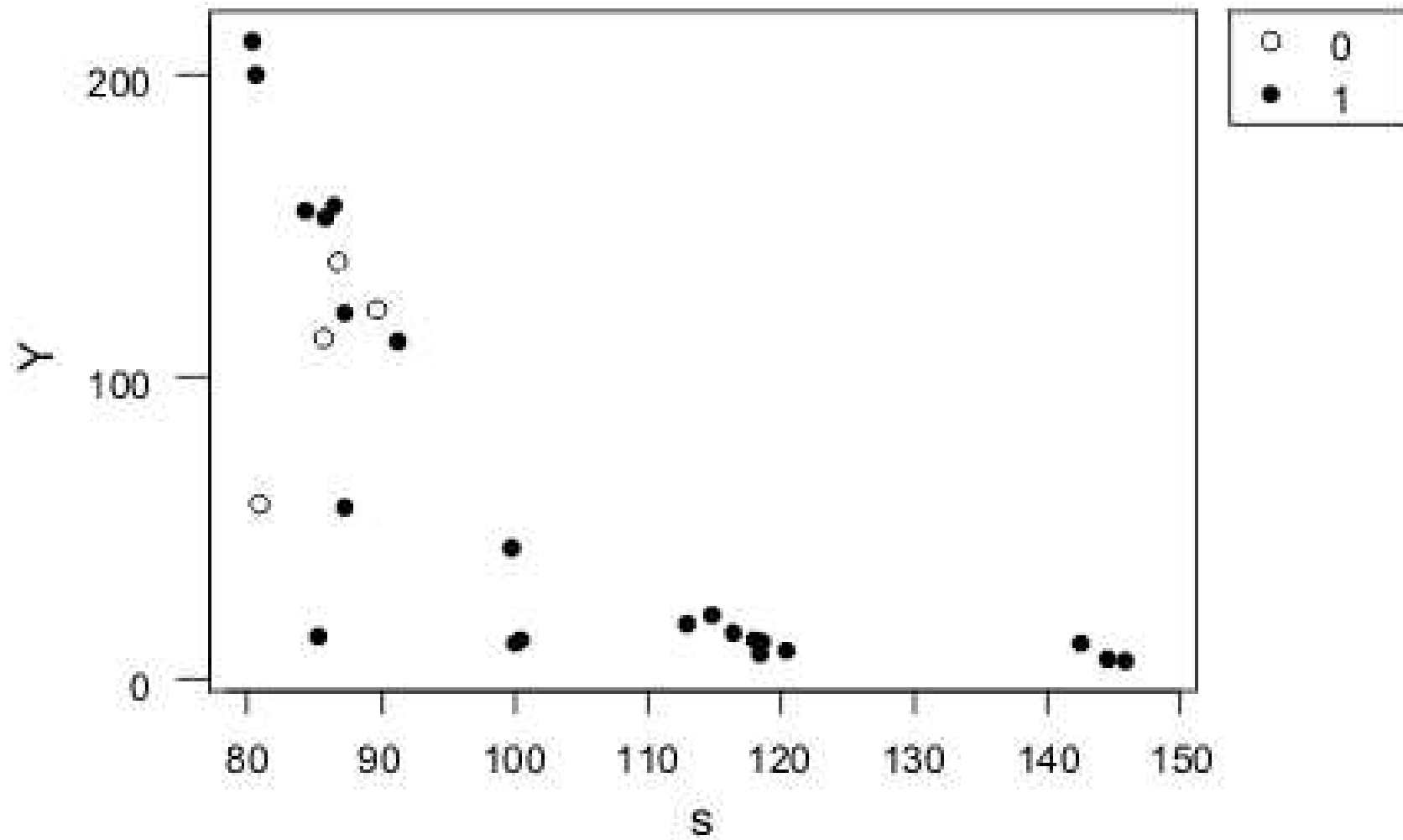
$$\hat{\Sigma}_{\hat{\theta}} = \begin{bmatrix} .012 & -.0037 & 0 \\ -.0037 & .0021 & 0 \\ 0 & 0 & .0029 \end{bmatrix}.$$

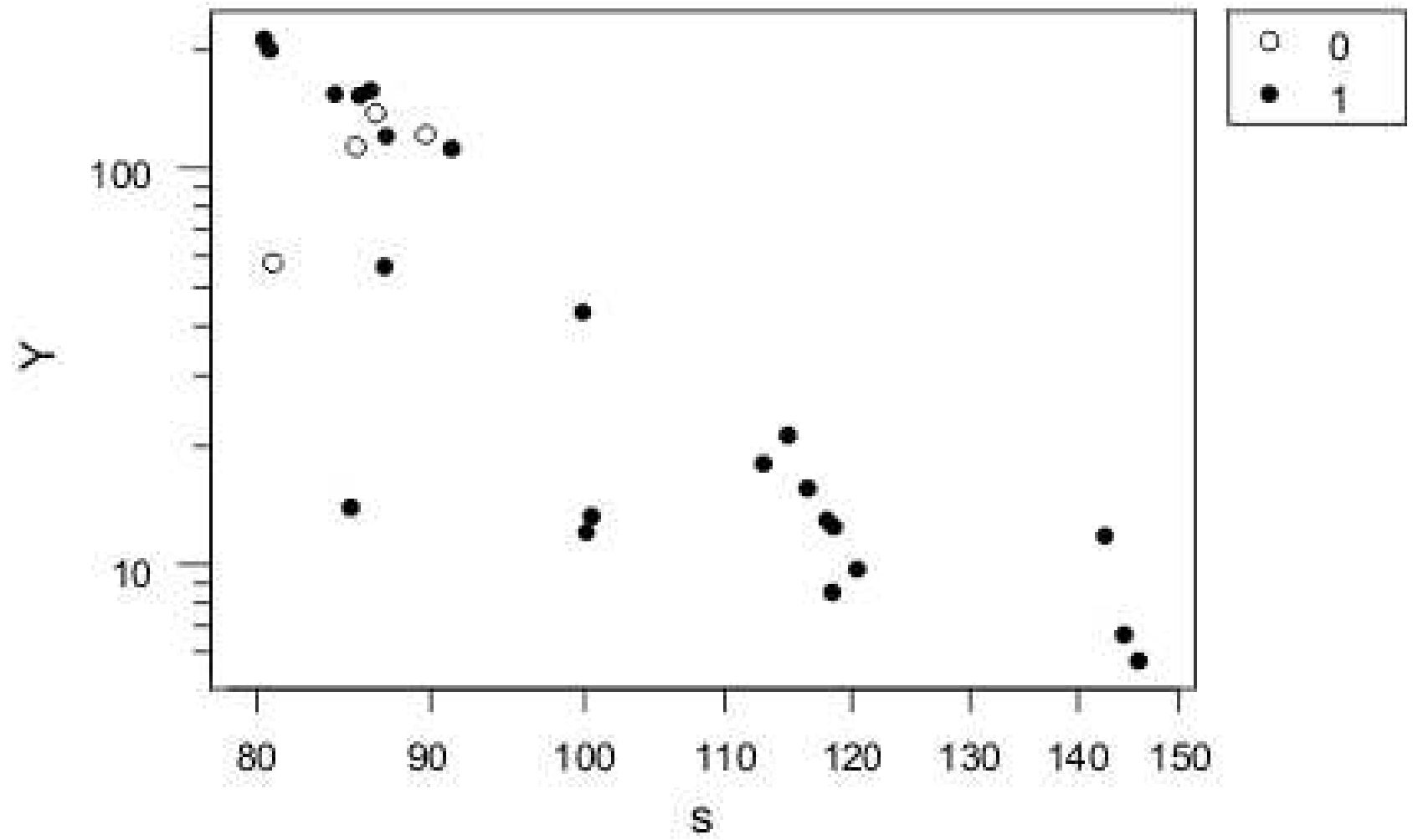
- Normal-approximation confidence interval for the computer execution time regression slope is

$$[\underline{\beta}_1, \tilde{\beta}_1] = \hat{\beta}_1 \pm z_{(.975)} \widehat{se}_{\hat{\beta}_1} = .290 \pm 1.96(.046) = [.20, .38]$$

where $\widehat{se}_{\hat{\beta}_1} = \sqrt{.0021} = .046$.

Row	Pseudo- stress	k-Cycles	Status (1=failed, 0=censored)	
i	s	Y	C	DATA DESCRIPTION:
1	80,3	211,629	1	Low-Cycle Fatigue Life of Nickel-Base
2	80,6	200,027	1	Superalloy Specimens
3	80,8	57,923	0	(in units of thousands of cycles
4	84,3	155,000	1	to failure).
5	85,2	13,949	1	
6	85,6	112,968	0	Data from Nelson (1990).
7	85,8	152,680	1	
8	86,4	156,725	1	
9	86,7	138,114	0	
10	87,2	56,723	1	
11	87,3	121,075	1	
12	89,7	122,372	0	
13	91,3	112,002	1	
14	99,8	43,331	1	
15	100,1	12,076	1	
16	100,5	13,181	1	
17	113,0	18,067	1	
18	114,8	21,300	1	
19	116,4	15,616	1	
20	118,0	13,030	1	
21	118,4	8,489	1	
22	118,6	12,434	1	
23	120,4	9,750	1	
24	142,5	11,865	1	
25	144,5	6,705	1	
26	145,9	5,733	1	





Regression with Life Data: Y versus x

Response Variable: Y

Censoring Information	Count
Uncensored value	22
Right censored value	4

Censoring value: C = 0

Estimation Method: Maximum Likelihood
Distribution: Weibull

Regression Table

Predictor	Coef	Standard Error	Z	P	95,0% Normal CI	
					Lower	Upper
Intercept	31,432	2,008	15,65	0,000	27,496	35,368
x	-5,9600	0,4329	-13,77	0,000	-6,8085	-5,1116
Shape	2,2105	0,3894			1,5651	3,1221

Log-Likelihood = -97,155

Anderson-Darling (adjusted) Goodness-of-Fit

Standardized Residuals = 1,0768; Cox-Snell Residuals = 1,0768

Regression with Life Data: Y versus x

Response Variable: Y

Censoring Information	Count
Uncensored value	22
Right censored value	4

Censoring value: C = 0

Estimation Method: Maximum Likelihood
Distribution: Weibull

Regression Table

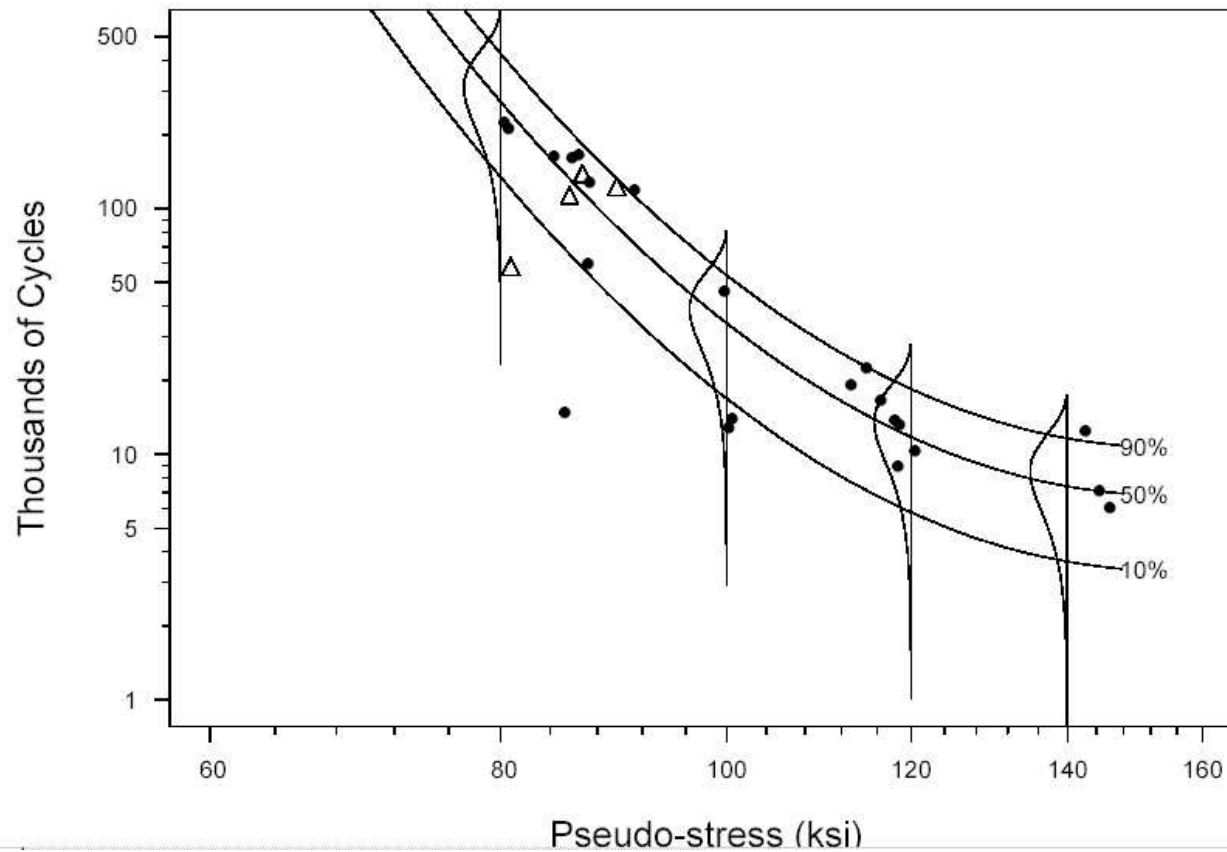
Predictor	Coef	Standard Error	Z	P	95,0% Normal CI	
					Lower	Upper
Intercept	217,61	62,13	3,50	0,000	95,83	339,39
x	-85,52	26,55	-3,22	0,001	-137,55	-33,49
x*x	8,483	2,831	3,00	0,003	2,934	14,032
Shape	2,6685	0,4777			1,8789	3,7900

Log-Likelihood = -93,382

Anderson-Darling (adjusted) Goodness-of-Fit

Standardized Residuals = 0,9283; Cox-Snell Residuals = 0,9283

**Log-Quadratic Weibull Regression Model with
Constant ($\beta = 1/\sigma$) Fit to the Fatigue Data**
 $\log[\hat{t}_p(x)] = \hat{\mu}(x) + \Phi_{\text{sev}}^{-1}(p)\hat{\sigma}, x = \log(\text{pseudo-stress})$



Regression with Life Data: Y versus x

Response Variable: Y

Table of Percentiles

Percent	s	x	Percentile	Standard Error	95,0% Normal CI	
					Lower	Upper
10	80	4,3820	133,3747	34,0579	80,8565	220,0048
10	100	4,6052	16,7928	3,4263	11,2577	25,0494
10	120	4,7875	5,7830	1,2364	3,8034	8,7929
10	140	4,9416	3,6458	0,8760	2,2766	5,8386
50	80	4,3820	270,1879	56,0580	179,9121	405,7621
50	100	4,6052	34,0186	4,3027	26,5494	43,5891
50	120	4,7875	11,7151	1,5950	8,9713	15,2980
50	140	4,9416	7,3856	1,2828	5,2547	10,3807
90	80	4,3820	423,6933	90,4646	278,8097	643,8659
90	100	4,6052	53,3461	6,8162	41,5281	68,5272
90	120	4,7875	18,3709	2,4567	14,1351	23,8760
90	140	4,9416	11,5817	1,9813	8,2824	16,1952

ESTIMERT KOVARIANSMATRISSE FOR $(\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2, \hat{\sigma})$

3860,37	-1649,17	175,82	-0,80
-1649,17	704,70	-75,15	0,33
175,82	-75,15	8,02	-0,03
-0,80	0,33	-0,03	0,23

