FORELESNING 11

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TMA4275 LEVETIDSANALYSE

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CONDITIONAL ROCOF BY MINIMAL REPAIR (NHPP) AND PERFECT REPAIR (RENEWAL PROCESS)
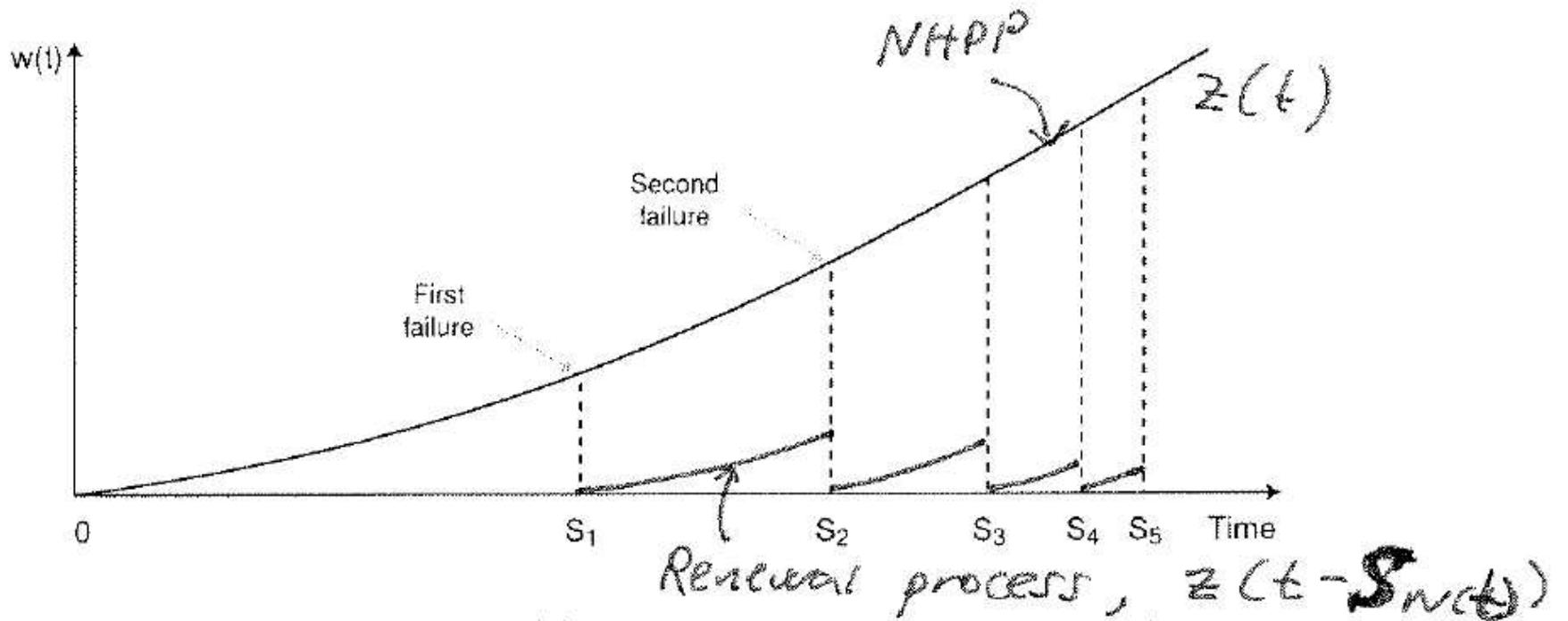
TYPICAL DATA − RECURRENT EVENTS

$0 \quad\quad S_{11} \quad\quad\quad S_{21} \quad \cdots \quad S_{N_1 1} \quad\quad \tau_1$

$\vdots$

$0 \quad\quad S_{1j} \quad\quad\quad\quad S_{2j} \quad\quad \cdots \quad\quad S_{N_j j} \quad\quad\quad \tau_j$

$\vdots$

$0 \quad\quad S_{1m} \quad\quad\quad S_{2m} \quad \cdots \quad S_{N_m m} \quad \tau_m$
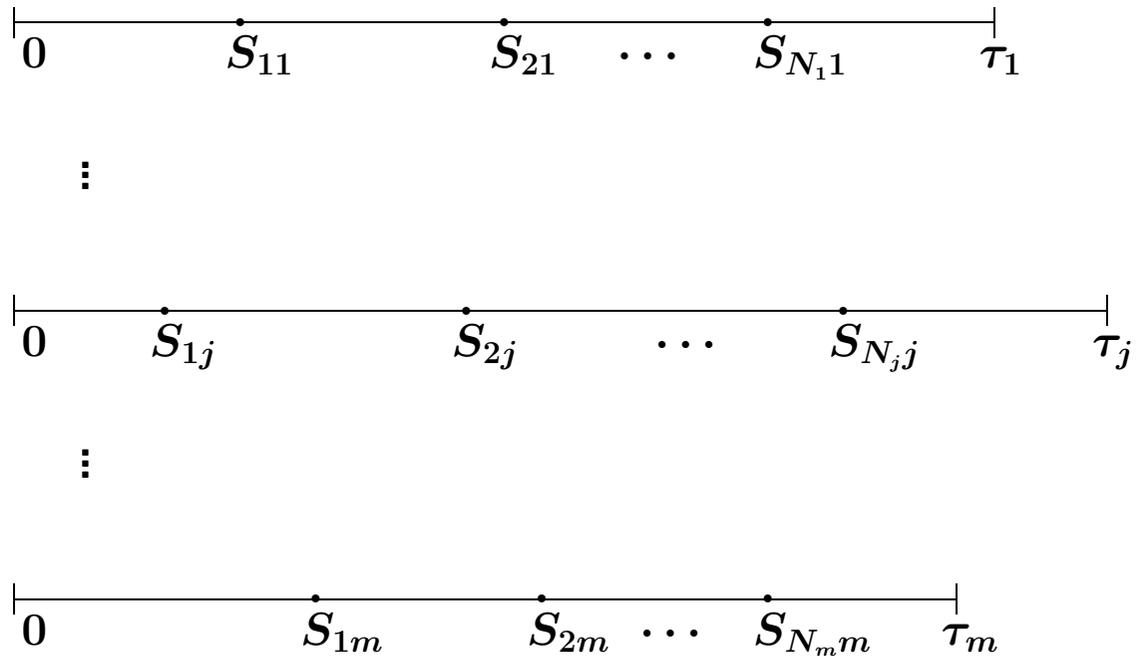
SIMPLE EXAMPLE WITH THREE SYSTEMS

Sys. 1:

$0 \qquad S_{11} = 5 \qquad S_{21} = 12 \qquad S_{31} = 17 \qquad \tau_1 = 20$

Sys. 2:

$0 \qquad\qquad S_{12} = 9 \qquad\qquad S_{22} = 23 \qquad \tau_2 = 30$

Sys. 3:

$0 \qquad S_{13} = 4 \qquad \tau_3 = 10$

Proj:

$0 \qquad 4 \ \ 5 \qquad 9 \qquad 12 \qquad 17 \qquad 23 \qquad\qquad t$

Y(t):

$Y(t) = 3 \qquad\qquad Y(t) = 2 \qquad\qquad Y(t) = 1$

4
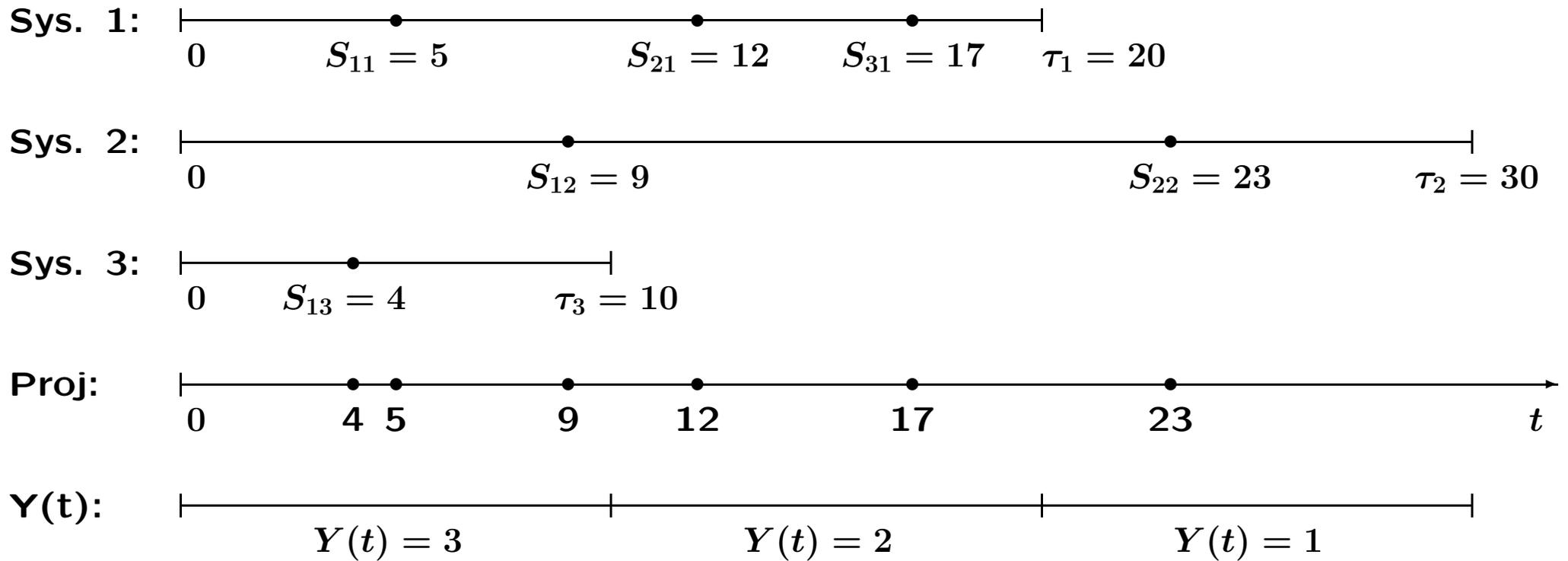
COMPUTATIONS FOR THE NELSON-AALEN ESTIMATOR

t	$1/Y(t)$	$1/Y(t)^2$	$\hat{W}(t)$	$\widehat{Var\hat{W}}(t)$	$\widehat{SD\hat{W}}(t)$
4	1/3	1/9	1/3	1/9	0.3333
5	1/3	1/9	2/3	2/9	0.4714
9	1/3	1/9	1	1/3	0.5774
12	1/2	1/4	3/2	7/12	0.7638
17	1/2	1/4	2	5/6	0.9129
23	1	1	3	11/6	1.3540

ESTIMATED $W(t)$ with 95% confidence limits (Nelson-Aalen)
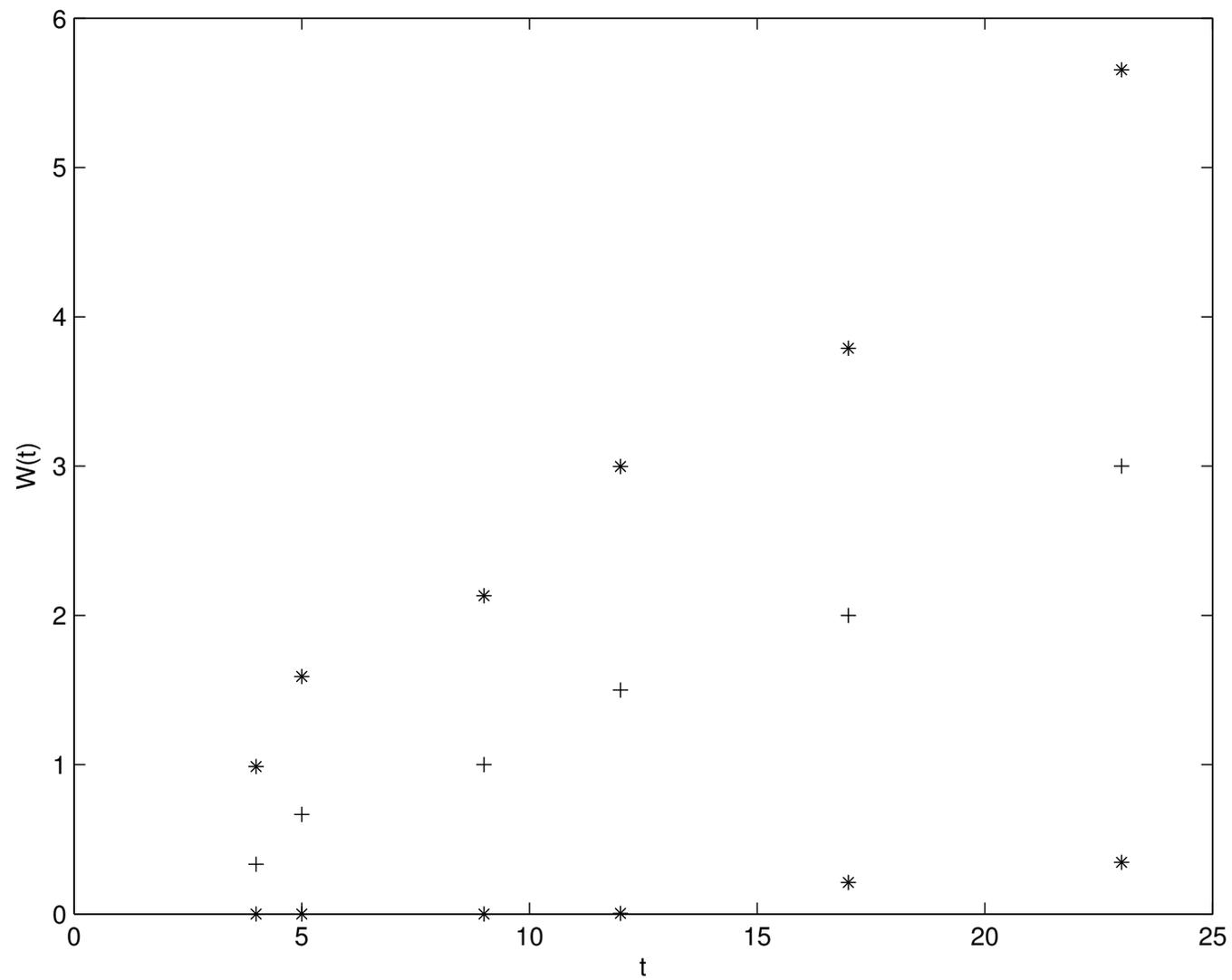
Valve Seat Replacement Times
(Nelson and Doganaksoy 1989)

Data collected from valve seats from a fleet of 41 diesel engines (days of operation)

- Each engine has 16 valves

- Does the replacement rate increase with age?

- How many replacement valves will be needed in the future?

- Can valve life in these systems be modeled as a renewal process?

VALVESEAT DATA

	C1	C2	C3	C4	C5	C6	C7	C8	C
	ID	Time							
1	1	761							
2	2	759							
3	3	98							
4	3	667							
5	4	326							
6	4	653							
7	4	653							
8	4	667							
9	5	665							
10	6	84							
11	6	667							
12	7	87							
13	7	663							
14	8	646							
15	8	653							
16	9	92							
17	9	653							
18	10	651							
19	11	258							
20	11	328							
21	11	377							
22	11	621							
23	11	650							
24	12	61							
25	12	539							
26	12	648							

TMA4275valveseat.MTW ***

Nonparametric Growth Curve

- Data are exact failure/retirement times
- Data are interval failure/retirement times

Variables/ Start   `Time`

End

System Information
- System ID `ID`
- Number of systems:

☐ By variable:

Retirement..
Cost-Freq...
Graphs...
Options...
Storage...

Select

Help

OK

Cancel

Nonparametric Growth Curve - Retirement

- Retirement time at largest time for system
  - Failure truncated systems
  - Time truncated systems
- Retirement time defined by retirement columns

Retirement

Retirement

Select

Help   OK   Cancel

VALVESEAT DATA

Event Plot for Time
System Column in ID

VALVESEAT DATA

Mean Cumulative Function for Time
95% CI
System Column in ID
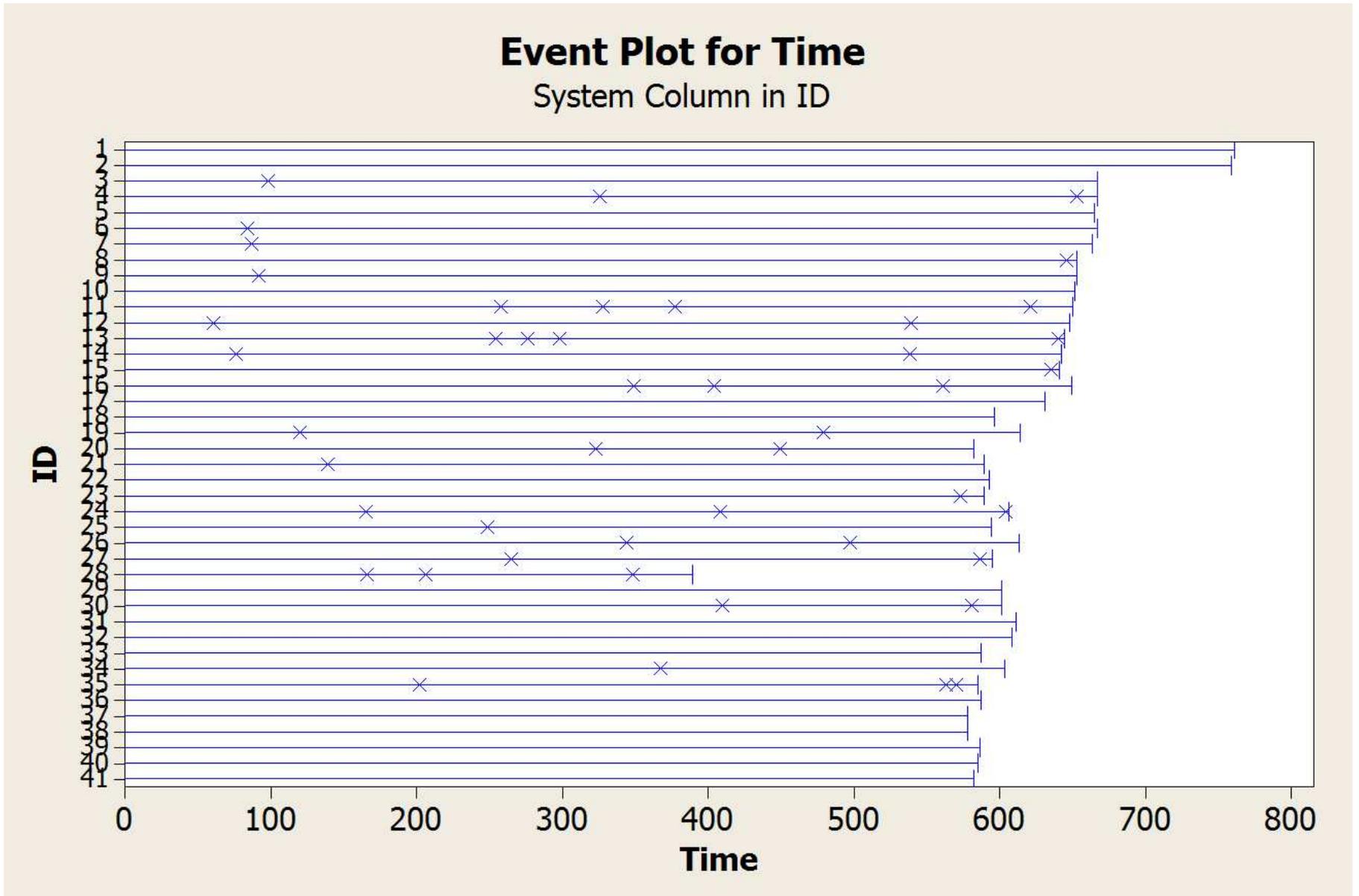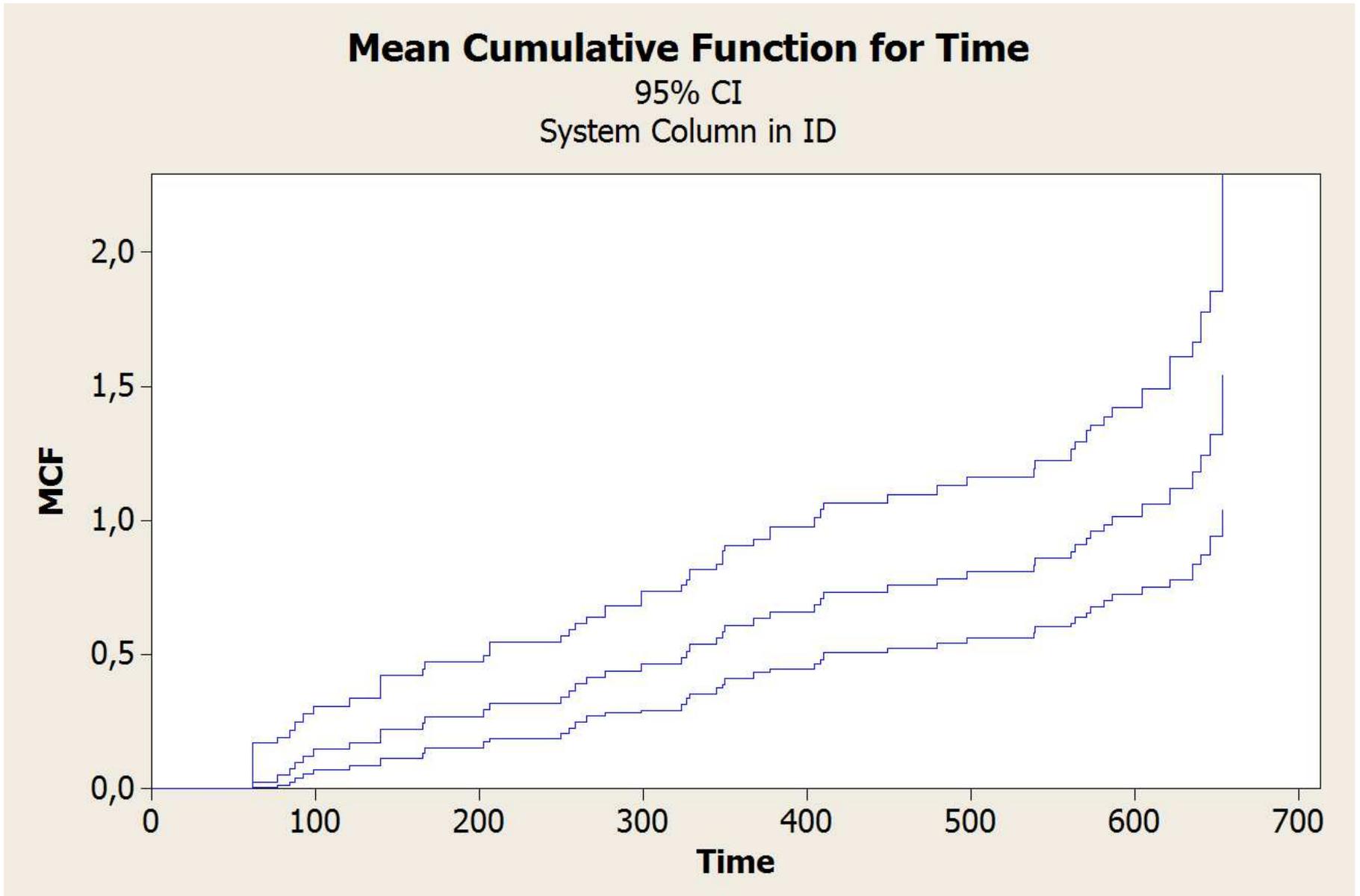
VALVESEAT DATA

Nonparametric Growth Curve: Time

System: ID

Nonparametric Estimates

Table of Mean Cumulative Function

Time	Mean Cumulative Function	Standard Error	95% Normal CI Lower	95% Normal CI Upper	System
61	0,02439	0,024091	0,00352	0,16903	12
76	0,04878	0,033641	0,01262	0,18848	14
84	0,07317	0,040670	0,02462	0,21750	6
87	0,09756	0,046340	0,03846	0,24750	7
92	0,12195	0,051105	0,05364	0,27726	9
98	0,14634	0,055199	0,06987	0,30650	3
120	0,17073	0,058764	0,08696	0,33519	19
139	0,19512	0,061891	0,10479	0,36333	21
139	0,21951	0,073270	0,11411	0,42226	21
165	0,24390	0,075417	0,13305	0,44711	24
166	0,26829	0,077317	0,15251	0,47196	28
202	0,29268	0,078988	0,17246	0,49672	35
206	0,31707	0,087527	0,18458	0,54467	28
249	0,34146	0,088680	0,20525	0,56807	25
254	0,36585	0,089656	0,22631	0,59143	13
258	0,39024	0,090461	0,24775	0,61468	11
265	0,41463	0,091101	0,26955	0,63780	27
276	0,43902	0,097858	0,28363	0,67955	13
298	0,46341	0,109607	0,29150	0,73671	13
323	0,48780	0,109740	0,31387	0,75812	20
326	0,51220	0,109740	0,33656	0,77949	4
328	0,53659	0,114907	0,35266	0,81643	11
344	0,56098	0,114654	0,37581	0,83737	26
348	0,58537	0,124250	0,38615	0,88737	28
349	0,60976	0,123782	0,40960	0,90772	16
367	0,63415	0,123194	0,43334	0,92801	34
377	0,65854	0,131842	0,44480	0,97498	11
404	0,68354	0,135939	0,46289	1,00936	16

Times of Unscheduled Maintenance Actions
## for a USS Grampus Diesel Engine

- Unscheduled maintenance actions caused by failure of imminent failure.

- Unscheduled maintenance actions are inconvenient and expensive.

- Data available for 16,000 operating hours.

- Data from Lee (1980).

- Is the system deteriorating (i.e., are failures occurring more rapidly as the system ages)?

- Can the occurrence of unscheduled maintenance actions be modeled by an HPP?

Cumulative Number of Unscheduled Maintenance
Actions Versus Operating Hours
for a USS Grampus Diesel Engine
Lee (1980)
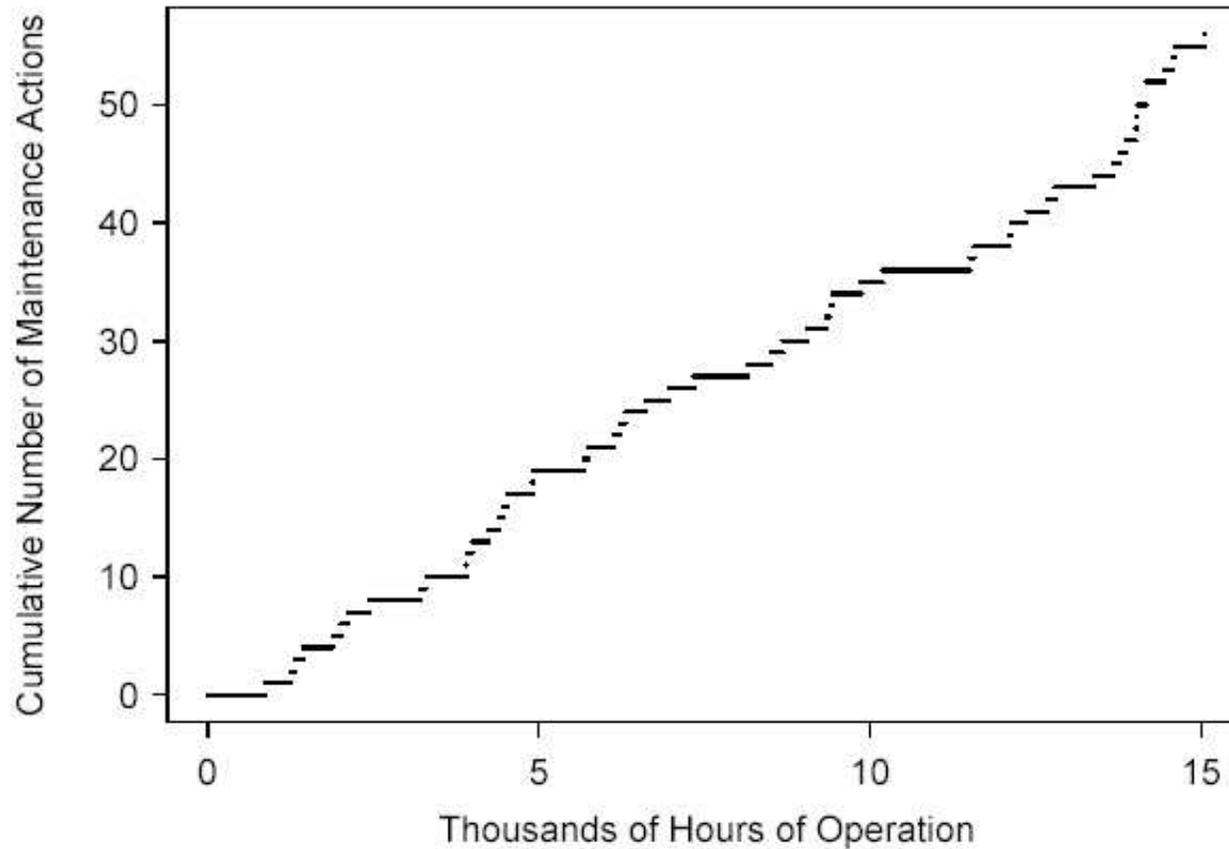
Grampus- data: Plot of (T_i, T_{i+1}) to investigate whether times between failures can be assumed independent. The figure does not indicate a correlation between successive times.

USS Grampus Diesel Engine
Plot of Times Between Unscheduled Maintenance
Actions Versus Lagged Times Between Unscheduled
Maintenance Actions
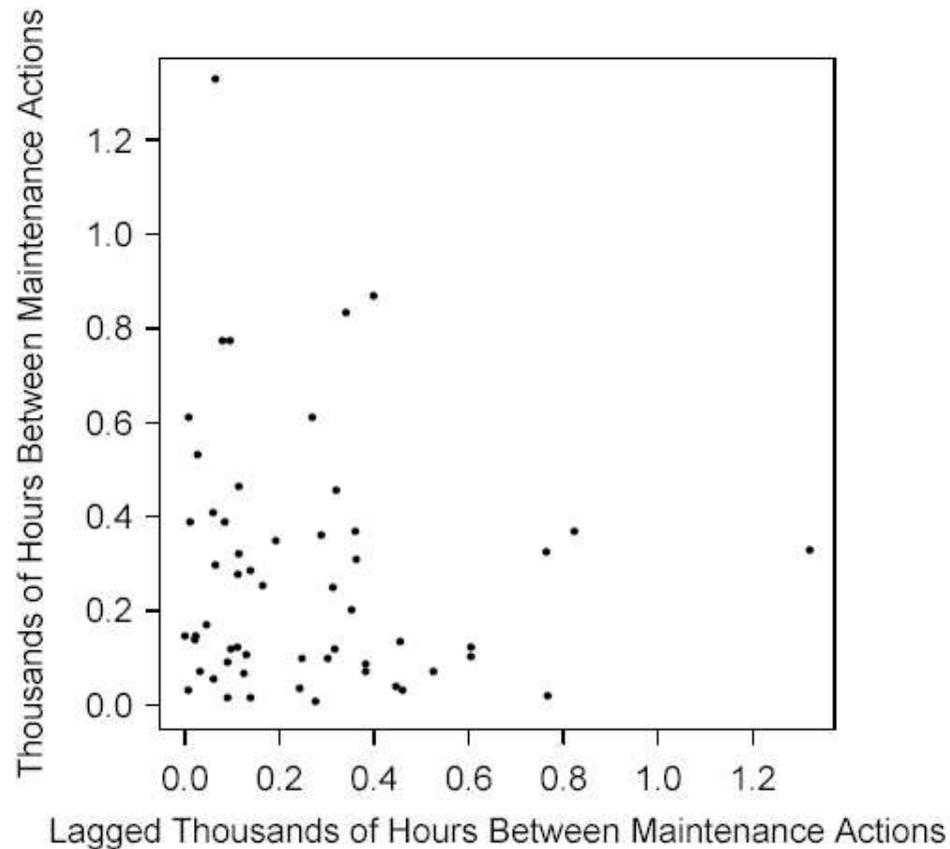
The Likelihood for the NHPP - Single Unit

- With **interval** recurrence data.

  Suppose that the unit has been observed for a period $(0, t_a]$ and the data are the number of recurrences $d_1, \ldots, d_m$ in the nonoverlapping intervals $(t_0, t_1]$, $(t_1, t_2]$, $\ldots$, $(t_{m-1}, t_m]$ (with $t_0 = 0$, $t_m = t_a$).

$$
\begin{aligned}
L(\boldsymbol{\theta}) &= \Pr\left[N(t_0, t_1) = d_1, \ldots, N(t_{m-1}, t_m) = d_m\right] \\
&= \prod_{j=1}^{m} \Pr\left[N(t_{j-1}, t_j) = d_j\right] \\
&= \prod_{j=1}^{m} \frac{\left[\mu(t_{j-1}, t_j; \boldsymbol{\theta})\right]^{d_j}}{d_j!} \exp\left[-\mu(t_{j-1}, t_j; \boldsymbol{\theta})\right] \\
&= \prod_{j=1}^{m} \frac{\left[\mu(t_{j-1}, t_j; \boldsymbol{\theta})\right]^{d_j}}{d_j!} \times \exp\left[-\mu(t_0, t_a; \boldsymbol{\theta})\right]
\end{aligned}
$$

# The Likelihood for the NHPP (Continued)

- If the number of intervals m increases and there are **exact** recurrences at $t_1 \leq \ldots \leq t_r$ (here $r = \sum_{j=1}^{m} d_j$, $t_0 \leq t_1$, $t_r \leq t_a$), then using a limiting argument it follows that the likelihood in terms of the density approximation is

$$L(\boldsymbol{\theta}) = \prod_{j=1}^{r} \nu(t_j; \boldsymbol{\theta}) \times \exp\left[-\mu(0, t_a; \boldsymbol{\theta})\right]$$

- For simplicity, above we assumed that the intervals are contiguous. Obvious changes to the formula above give the likelihood when there are gaps among the intervals.

- In both cases (the interval data or exact recurrences data) the same methods used in Chapters 7, 8 can be used to obtain the ML estimate $\hat{\boldsymbol{\theta}}$ and confidence regions for $\boldsymbol{\theta}$ or functions of $\boldsymbol{\theta}$.

PROFILE LIKELIHOOD FOR BETA ("SIMPLE EXAMPLE")
$\hat{\beta} = 1.20, \; \hat{\lambda} = 0.0538.$
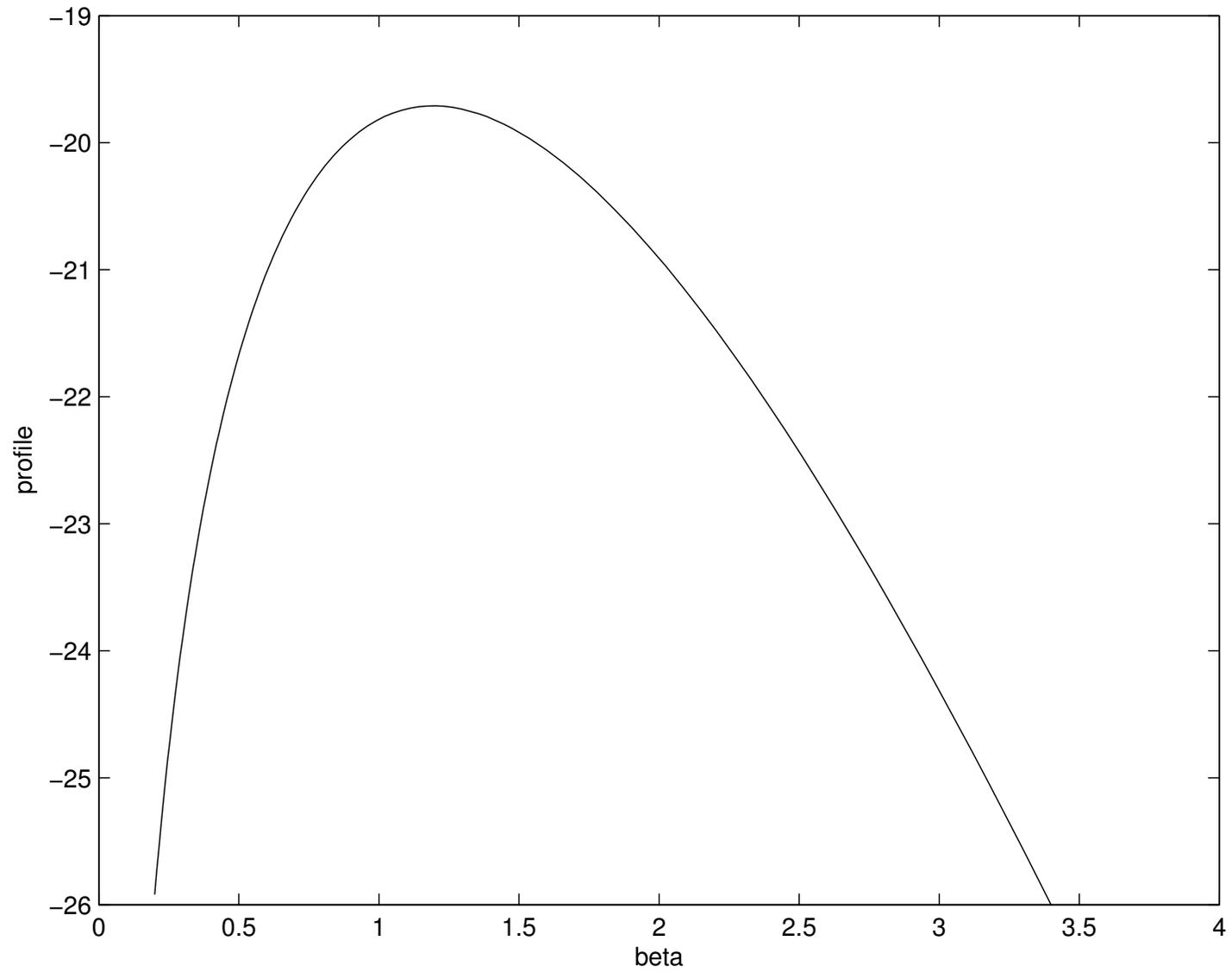
CONNECTION BETWEEN LAMBDA OG BETA ("SIMPLE EXAMPLE")
$\hat{\beta} = 1.20, \ \hat{\lambda} = 0.0538.$