FORELESNING 10

Våren 2004

1. april

TMA4275 LEVETIDSANALYSE

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Insulate.MTW ***

↓	C1	C2	C3	C4	C5-T	C6	C7	C8	C9	C10	C11
	Temp	ArrTemp	Plant	FailureT	Censor	Design	NewTemp	ArrNewT	NewPlant		
1	170	26,1865	1	343	F	80	80	32,8600	1		
2	170	26,1865	1	869	F	100	80	32,8600	2		
3	170	26,1865	1	244	C		100	31,0988	1		
4	170	26,1865	1	716	F		100	31,0988	2		
5	170	26,1865	1	531	F						
6	170	26,1865	1	738	F						
7	170	26,1865	1	461	F						
8	170	26,1865	1	221	F						
9	170	26,1865	1	665	F						
10	170	26,1865	1	384	C						
11	170	26,1865	2	394	C						
12	170	26,1865	2	369	F						
13	170	26,1865	2	366	F						
14	170	26,1865	2	507	F						
15	170	26,1865	2	461	F						
16	170	26,1865	2	431	F						
17	170	26,1865	2	479	F						
18	170	26,1865	2	106	F						
19	170	26,1865	2	545	F						
20	170	26,1865	2	536	F						
21	150	27,4242	1	2134	C						
22	150	27,4242	1	2746	F						
23	150	27,4242	1	2859	F						
24	150	27,4242	1	1826	C						

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Example of Accelerated Life Testing

main topic   interpreting results   session command   see also

Suppose you want to investigate the deterioration of an insulation used for electric motors. The motors normally run between 80 and 100° C. To save time and money, you decide to use accelerated life testing.

First you gather failure times for the insulation at abnormally high temperatures – 110, 130, 150, and 170° C – to speed up the deterioration. With failure time information at these temperatures, you can then extrapolate to 80 and 100° C. It is known that an Arrhenius relationship exists between temperature and failure time.

To see how well the model fits, you will draw a probability plot based on the standardized residuals.

1 Open the worksheet INSULATE.MTW.
2 Choose **Stat > Reliability/Survival > Accelerated Life Testing**.
3 In **Variables/Start variables**, enter **FailureT**. In **Accelerating variable**, enter **Temp**.
4 From **Relationship**, choose **Arrhenius**.
5 Click **Censor**. In **Use censoring columns**, enter **Censor**, then click **OK**.
6 Click **Graphs**. In **Enter design value to include on plot**, enter **80**. Click **OK**.
7 Click **Estimate**. In **Enter new predictor values**, enter **Design**, then click **OK** in each dialog box.

Session window output

Regression with Life Data: FailureT versus Temp

```
Response Variable:  FailureT

Censoring Information              Count
Uncensored value                      66
Right censored value                  14
Censoring value:  Censor = C

Estimation Method:  Maximum Likelihood
Distribution:  Weibull
Transformation on accelerating variable:  Arrhenius
```

Regression Table

Predictor	Coef	Standard Error	Z	P	95.0% Normal CI Lower	95.0% Normal CI Upper
Intercept	-15.1874	0.9862	-15.40	0.000	-17.1203	-13.2546
Temp	0.83072	0.03504	23.71	0.000	0.76204	0.89940
Shape	2.8246	0.2570			2.3633	3.3760

Log-Likelihood = -564.693

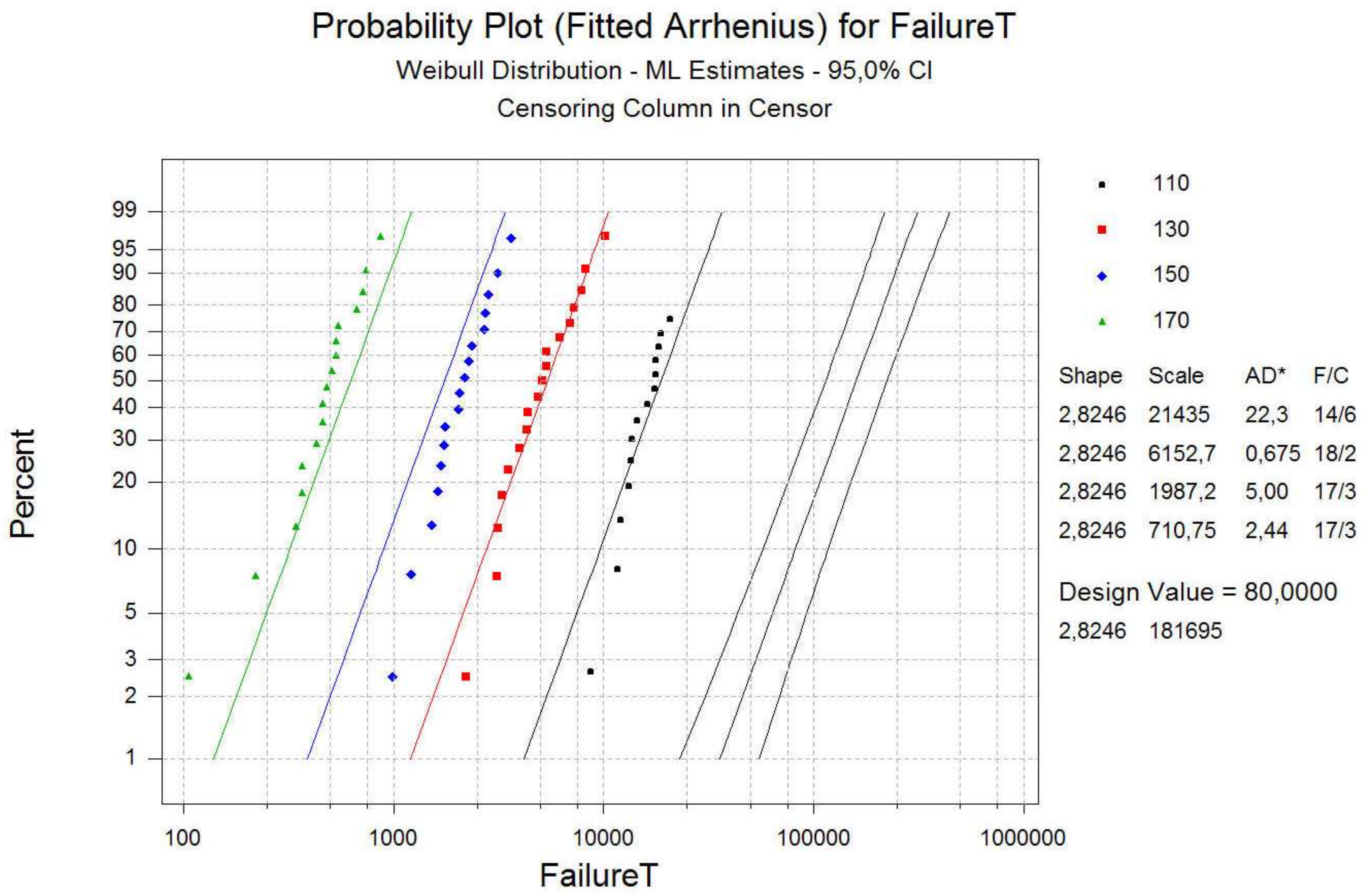
Anderson-Darling (adjusted) Goodness-of-Fit

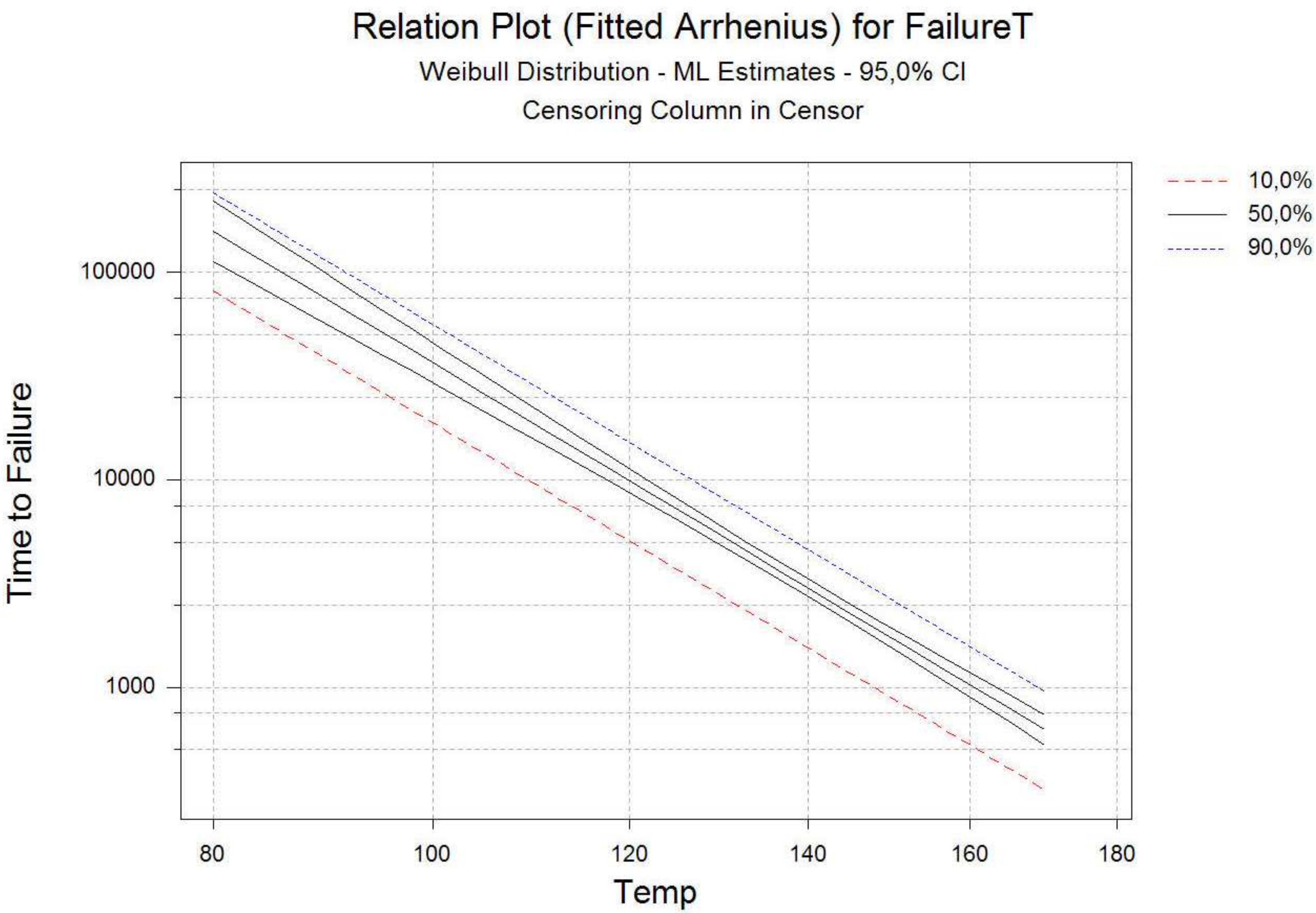
At each accelerating level

Level	Fitted Model
110	*
130	*
150	*
170	*

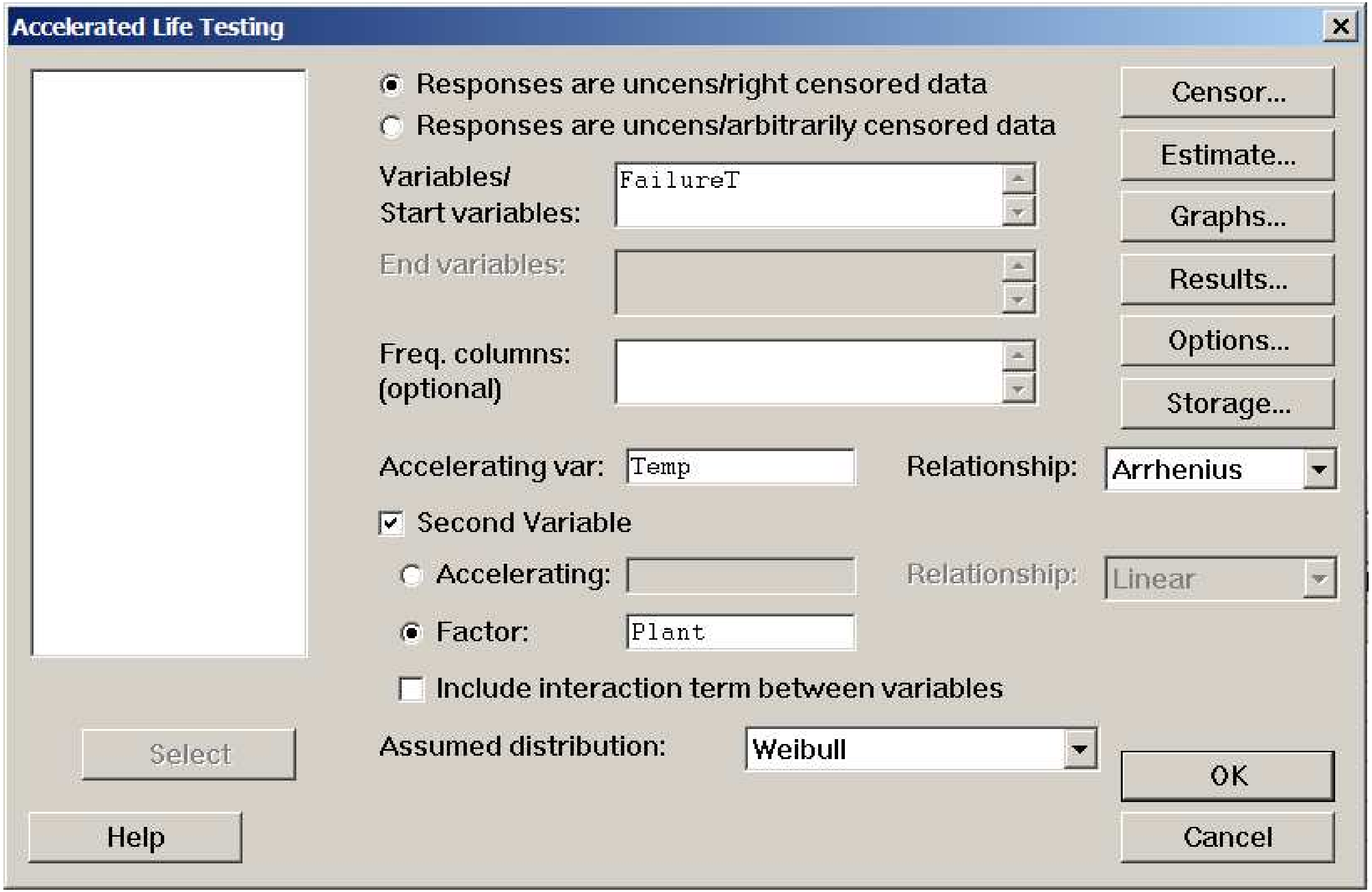
Table of Percentiles

Percent	Temp	Percentile	Standard Error	95.0% Normal CI Lower	95.0% Normal CI Upper
50	80.0000	159584.5	27446.85	113918.2	223557.0
50	100.0000	36948.57	4216.511	29543.36	46209.94


Probability Plot (Fitted Arrhenius) for FailureT
Weibull Distribution - ML Estimates - 95,0% CI
Censoring Column in Censor
Percent
99
95
90
80
70
60
50
40
30
20
10
5
3
2
1
100
1000
10000
100000
1000000
FailureT
110
130
150
170
Shape Scale AD* F/C
2,8246 21435 22,3 14/6
2,8246 6152,7 0,675 18/2
2,8246 1987,2 5,00 17/3
2,8246 710,75 2,44 17/3
Design Value = 80,0000
2,8246 181695


Relation Plot (Fitted Arrhenius) for FailureT
Weibull Distribution - ML Estimates - 95,0% CI
Censoring Column in Censor
Time to Failure
100000
10000
1000
80
100
120
140
160
180
Temp
10,0%
50,0%
90,0%

MED FAKTOR "PLANT" I TILLEGG:



EKSAMEN MAI 2003, OPPGAVE 3:

En komponent antas under normalstress å ha overlevelsesfunksjon (reliability function) $R_0(t)$ og hasardfunksjon (sviktintensitet) $z_0(t)$.

Man ønsker å estimere påliteligheten av denne komponenttypen ved hjelp av akselerert levetidstesting. Dette gjøres ved å utsette komponenter for stress s, $0 \leq s < \infty$, og måle levetiden (eventuelt sensurerte levetider). Normalstress svarer til $s = 0$.

To modeller betraktes:

Modell 1: Proporsjonal hasardmodell. Under stress s har komponenten hasardfunksjon

$$z_s^{PH}(t) = z_0(t)g(s)$$

for en funksjon $g(s)$ med g(0)=1.

Modell 2: Akselerert levetidsmodell. Under stress s har komponenten overlevelsesfunksjon

$$R_s^{AL}(t) = R_0(\phi(s)t)$$

for en funksjon $\phi(s)$ med $\phi(0) = 1$.

(a) Forklar kort hva som er hensikten med akselerert levetid-stesting. Hva er ideen bak de to modellene? Hva uttrykker de to funksjonene $g(s)$ og $\phi(s)$?

(b) La $R_s^{PH}(\cdot)$ være overlevelsesfunksjonen for en komponent under stress s i Modell 1. Vis at

$$R_s^{PH}(t) = R_0(t)^{g(s)}$$

La videre $z_s^{AL}(\cdot)$ være hasardfunksjonen for en komponent under stress s i Modell 2. Uttrykk $z_s^{AL}(\cdot)$ ved funksjonene $z_0(\cdot)$ og $\phi(\cdot)$.

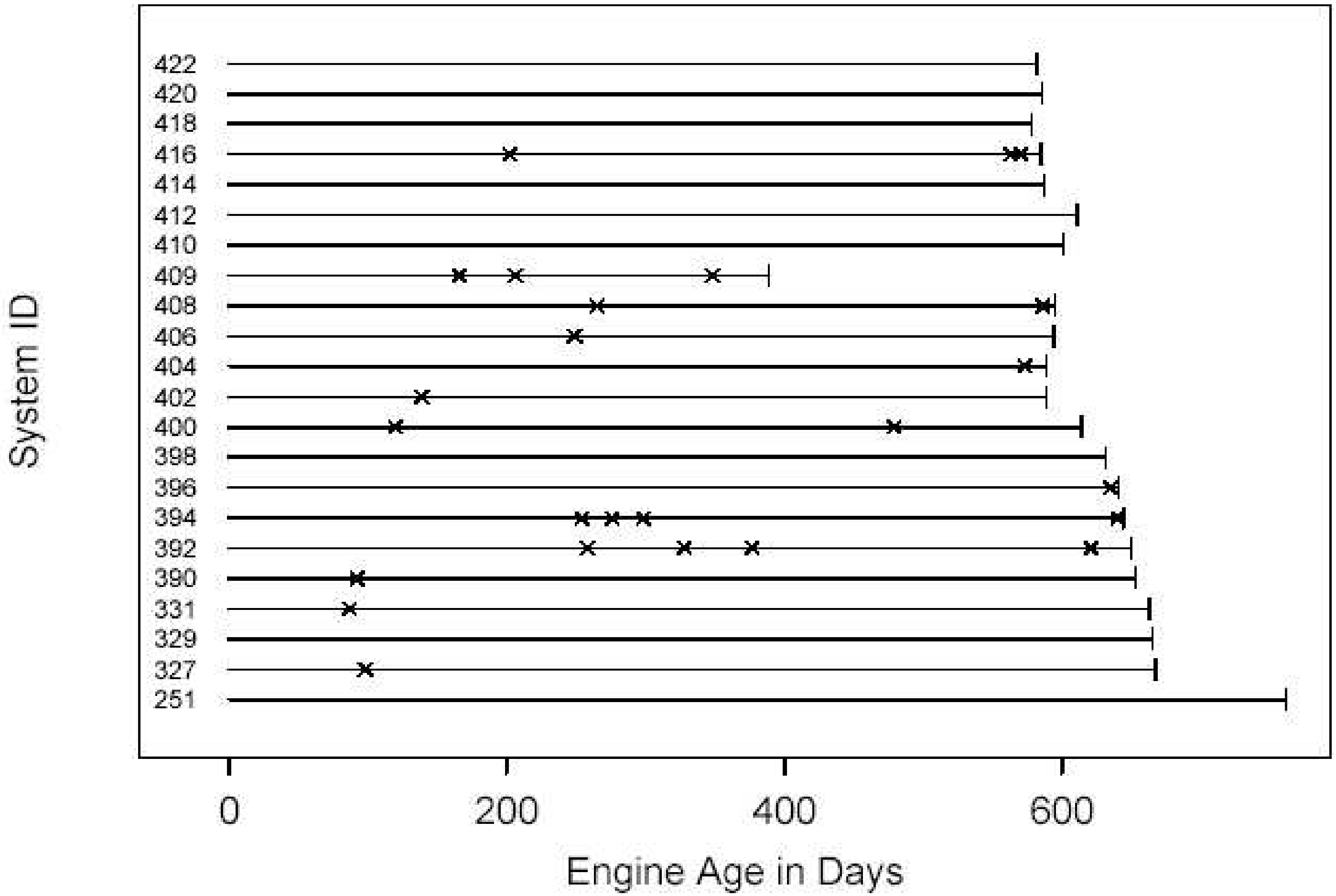
(c) Anta at komponentens levetid under normalstress er Weibull(α, θ), definert ved

$$R_0(t) = e^{-(t/\theta)^{\alpha}}$$

Vis at levetiden under stress $s > 0$ også er Weibull-fordelt under begge modellene. Hva blir parametrene i de tilsvarende Weibull-fordelingene?

I hvilken forstand kan vi si at Modell 1 og Modell 2 er ekvivalente ved Weibull-fordelte levetider?

Valve Seat Replacement Times Event Plot
(Nelson and Doganaksoy 1989)

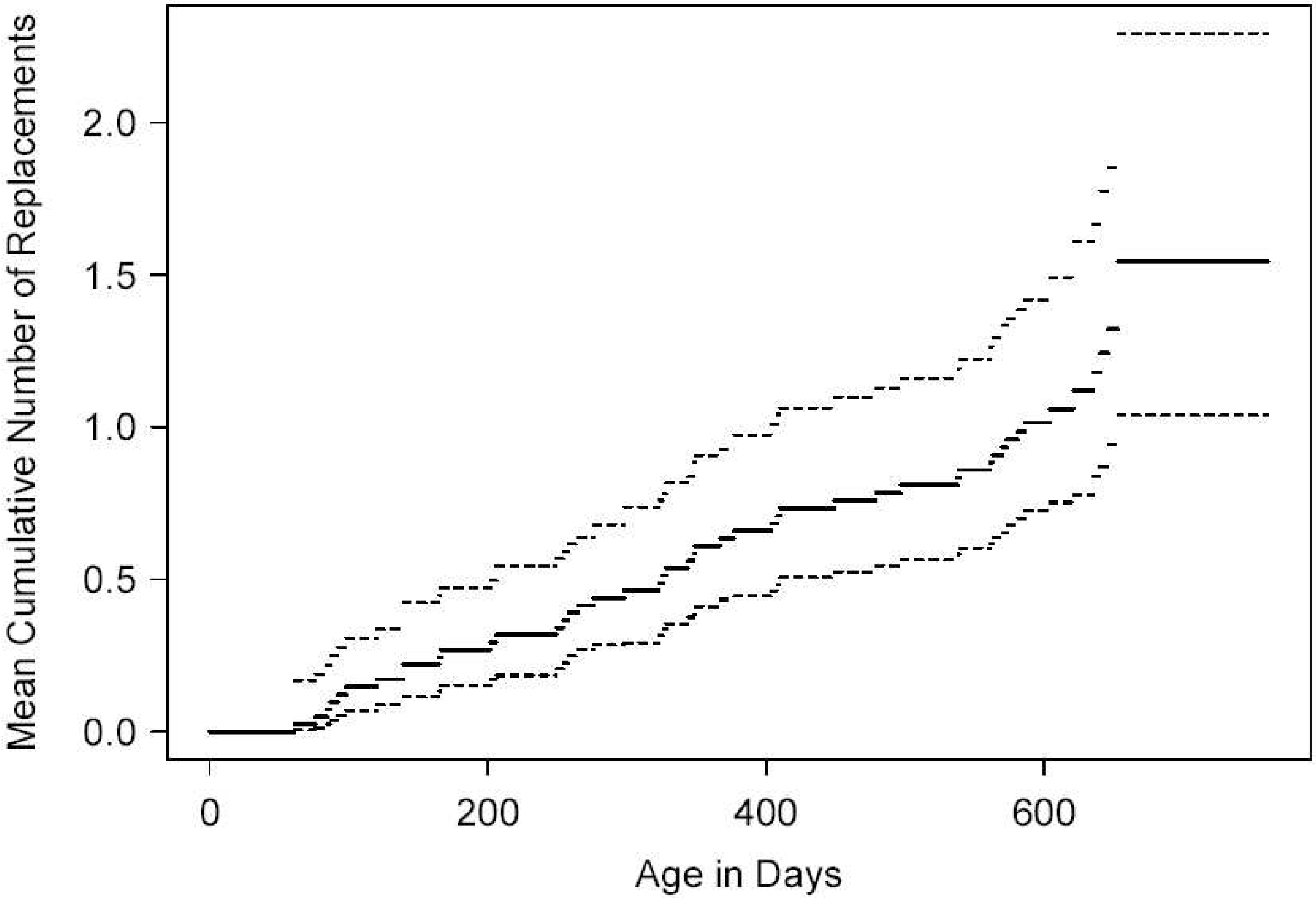


Valve Seat Replacement Times
(Nelson and Doganaksoy 1989)

Data collected from valve seats from a fleet of 41 diesel engines (days of operation)

- Each engine has 16 valves
- Does the replacement rate increase with age?
- How many replacement valves will be needed in the future?
- Can valve life in these systems be modeled as a renewal process?

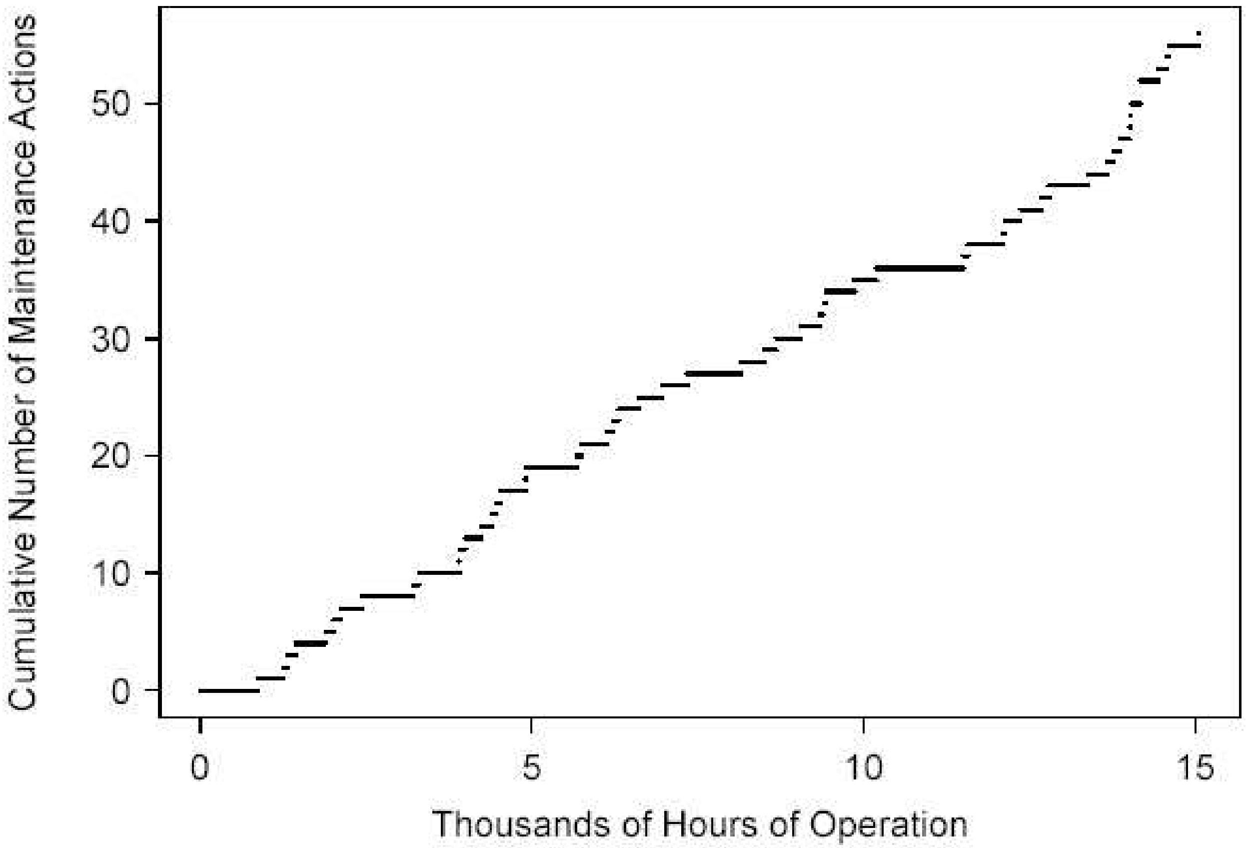
Estimate of Number of Valve Seat $\mu(t)$



Times of Unscheduled Maintenance Actions for a USS Grampus Diesel Engine

- Unscheduled maintenance actions caused by failure of imminent failure.
- Unscheduled maintenance actions are inconvenient and expensive.
- Data available for 16,000 operating hours.
- Data from Lee (1980).
- Is the system deteriorating (i.e., are failures occurring more rapidly as the system ages)?
- Can the occurrence of unscheduled maintenance actions be modeled by an HPP?

Cumulative Number of Unscheduled Maintenance Actions Versus Operating Hours for a USS Grampus Diesel Engine
Lee (1980)



The Likelihood for the NHPP - Single Unit

- With **interval** recurrence data.

  Suppose that the unit has been observed for a period $(0, t_a]$ and the data are the number of recurrences $d_1, \ldots, d_m$ in the nonoverlapping intervals $(t_0, t_1]$, $(t_1, t_2]$, $\ldots$, $(t_{m-1}, t_m]$ (with $t_0 = 0$, $t_m = t_a$).

$$
\begin{aligned}
L(\boldsymbol{\theta}) &= \Pr\left[N(t_0, t_1) = d_1, \ldots, N(t_{m-1}, t_m) = d_m\right] \\
&= \prod_{j=1}^{m} \Pr\left[N(t_{j-1}, t_j) = d_j\right] \\
&= \prod_{j=1}^{m} \frac{\left[\mu(t_{j-1}, t_j; \boldsymbol{\theta})\right]^{d_j}}{d_j!} \exp\left[-\mu(t_{j-1}, t_j; \boldsymbol{\theta})\right] \\
&= \prod_{j=1}^{m} \frac{\left[\mu(t_{j-1}, t_j; \boldsymbol{\theta})\right]^{d_j}}{d_j!} \times \exp\left[-\mu(t_0, t_a; \boldsymbol{\theta})\right]
\end{aligned}
$$

The Likelihood for the NHPP (Continued)

- If the number of intervals m increases and there are **exact** recurrences at $t_1 \leq \ldots \leq t_r$ (here $r = \sum_{j=1}^{m} d_j$, $t_0 \leq t_1$, $t_r \leq t_a$), then using a limiting argument it follows that the likelihood in terms of the density approximation is

$$L(\boldsymbol{\theta}) = \prod_{j=1}^{r} \nu(t_j; \boldsymbol{\theta}) \times \exp\left[-\mu(0, t_a; \boldsymbol{\theta})\right]$$

- For simplicity, above we assumed that the intervals are contiguous. Obvious changes to the formula above give the likelihood when there are gaps among the intervals.

- In both cases (the interval data or exact recurrences data) the same methods used in Chapters 7, 8 can be used to obtain the ML estimate $\widehat{\boldsymbol{\theta}}$ and confidence regions for $\boldsymbol{\theta}$ or functions of $\boldsymbol{\theta}$.

CONDITIONAL ROCOF BY MINIMAL REPAIR (NHPP) AND PERFECT REPAIR (RENEWAL PROCESS)