FORELESNING 8
Våren 2004
11. mars

TMA4275 LEVETIDSANALYSE

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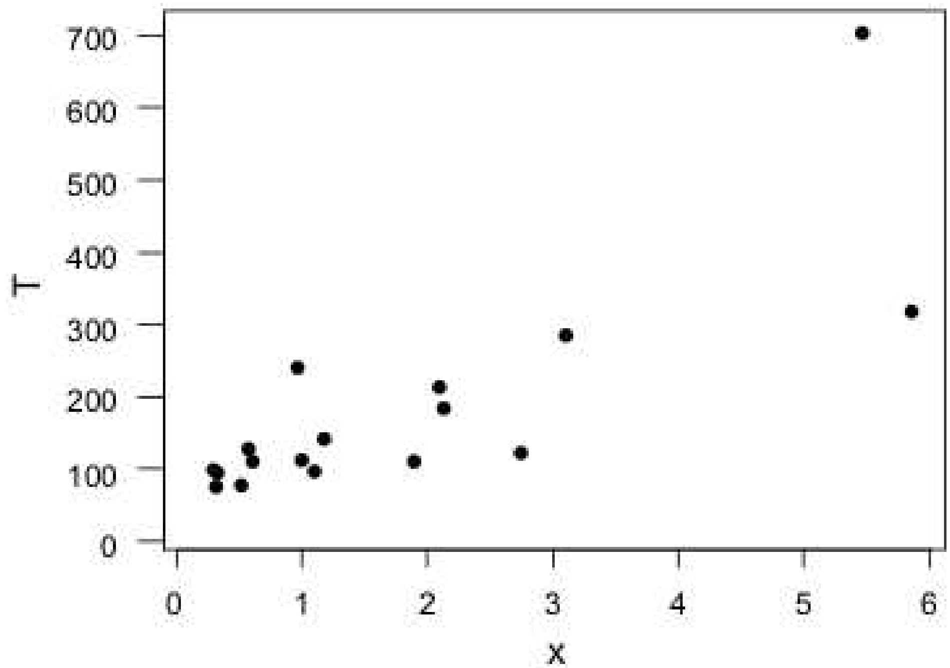
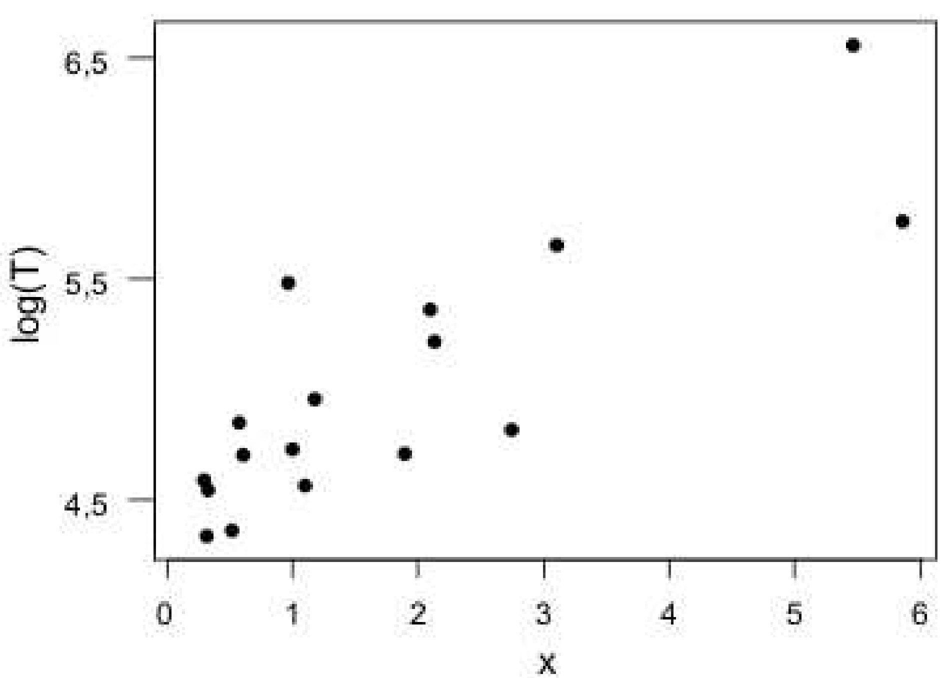
COMPUTER PROGRAM EXECUTION TIME vs SYSTEM LOAD

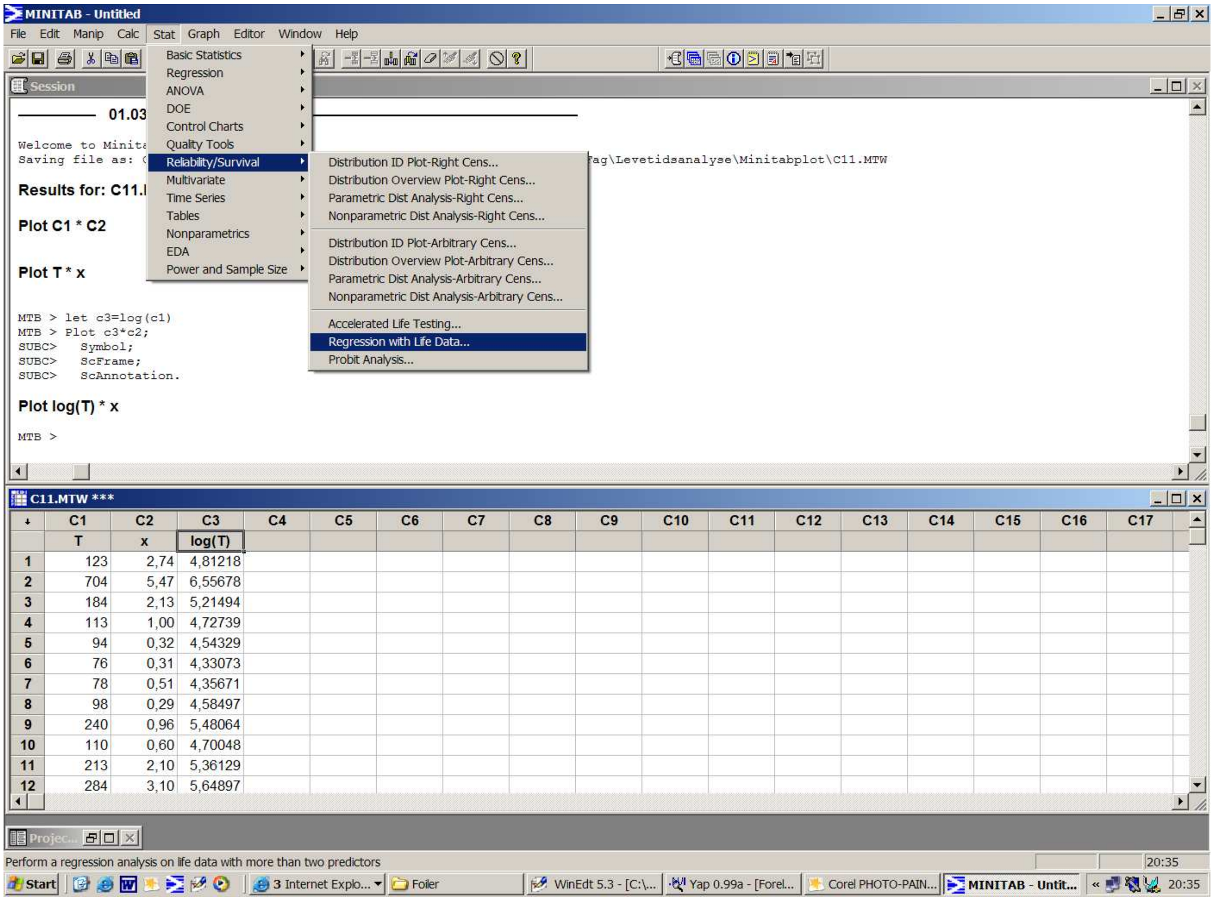
Data: 17 observasjoner av (T,x)

- **Tid for å gjennomføre en dataintensiv oppgave**
- **Info fra Unix "uptime"-kommando**
- **Prediksjon ønskes for framtidig fler-stegs oppgave**

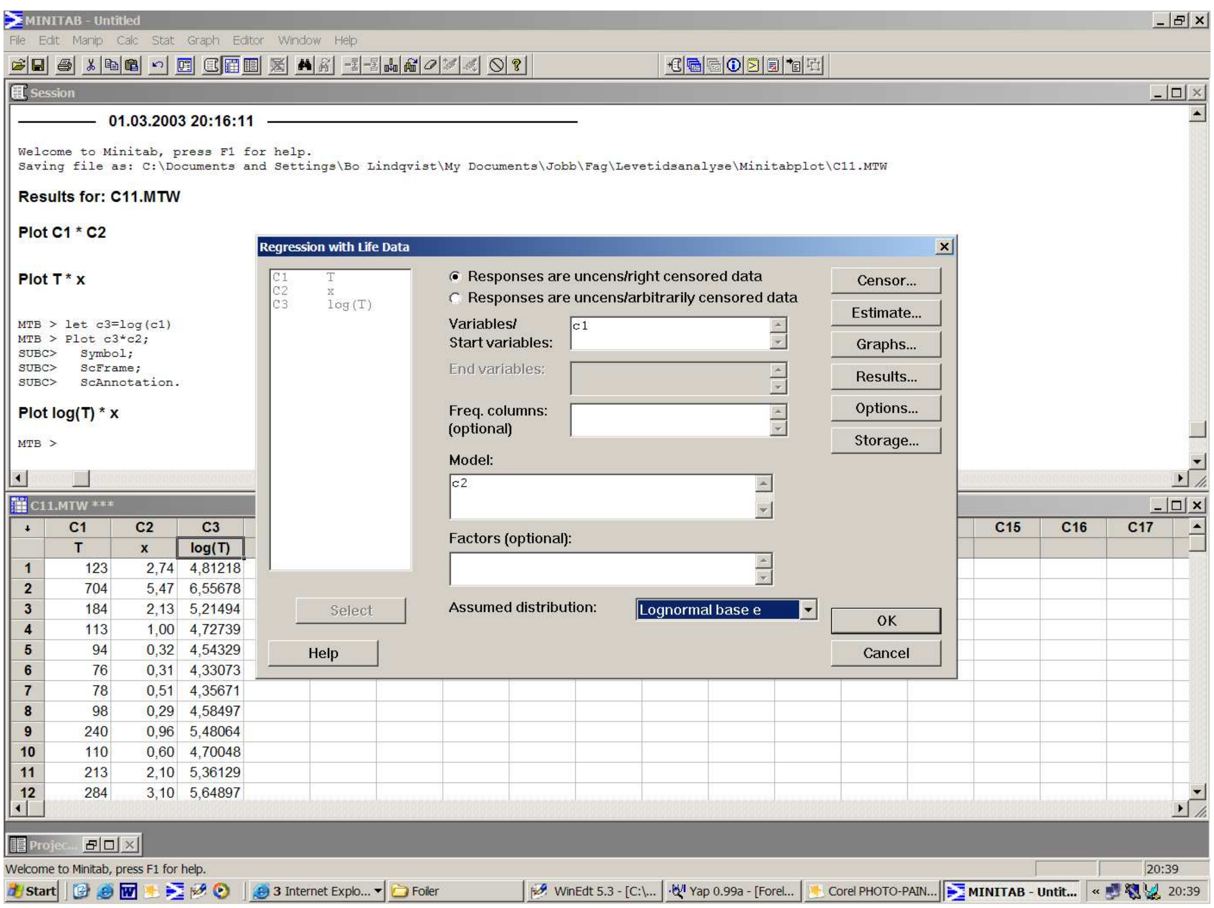
Seconds (T)	Load (x)	Seconds (T)	Load (x)
123	2,74	110	,60
704	5,47	213	2,10
184	2,13	284	3,10
113	1,00	317	5,86
94	,32	142	1,18
76	,31	127	,57
78	,51	96	1,10
98	,29	111	1,89
240	,96		


T
700
600
500
400
300
200
100
0
0
1
2
3
4
5
6
x


log(T)
6,5
5,5
4,5
0
1
2
3
4
5
6
x



	C1	C2	C3
	T	x	log(T)
1	123	2,74	4,81218
2	704	5,47	6,55678
3	184	2,13	5,21494
4	113	1,00	4,72739
5	94	0,32	4,54329
6	76	0,31	4,33073
7	78	0,51	4,35671
8	98	0,29	4,58497
9	240	0,96	5,48064
10	110	0,60	4,70048
11	213	2,10	5,36129
12	284	3,10	5,64897



Regression with Life Data: T versus x

```
Response Variable:  T

Censoring Information              Count
Uncensored value                     17

Estimation Method:  Maximum Likelihood
Distribution:  Lognormal base e

Regression Table
                      Standard                           95,0% Normal CI
Predictor      Coef      Error        Z      P       Lower       Upper
Intercept    4,4936     0,1112    40,39  0,000      4,2756      4,7116
x           0,29075    0,04595     6,33  0,000     0,20069     0,38080
Scale       0,31247    0,05359                     0,22327     0,43730

Log-Likelihood = -89,498

Anderson-Darling (adjusted) Goodness-of-Fit

Standardized Residuals = 0,8356; Cox-Snell Residuals = 0,8170
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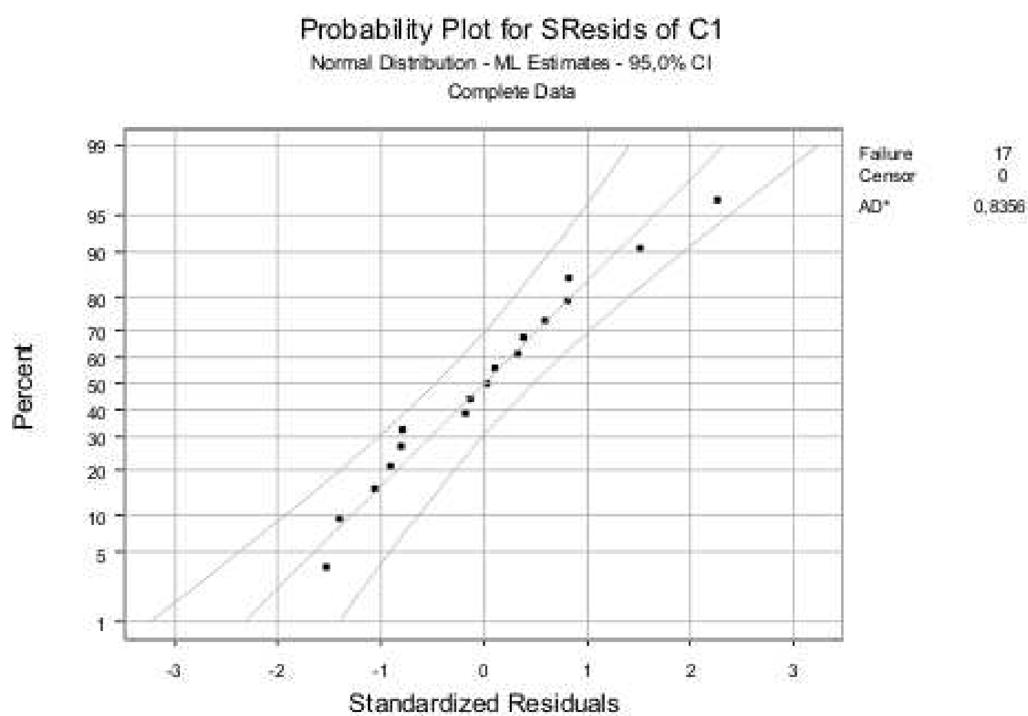

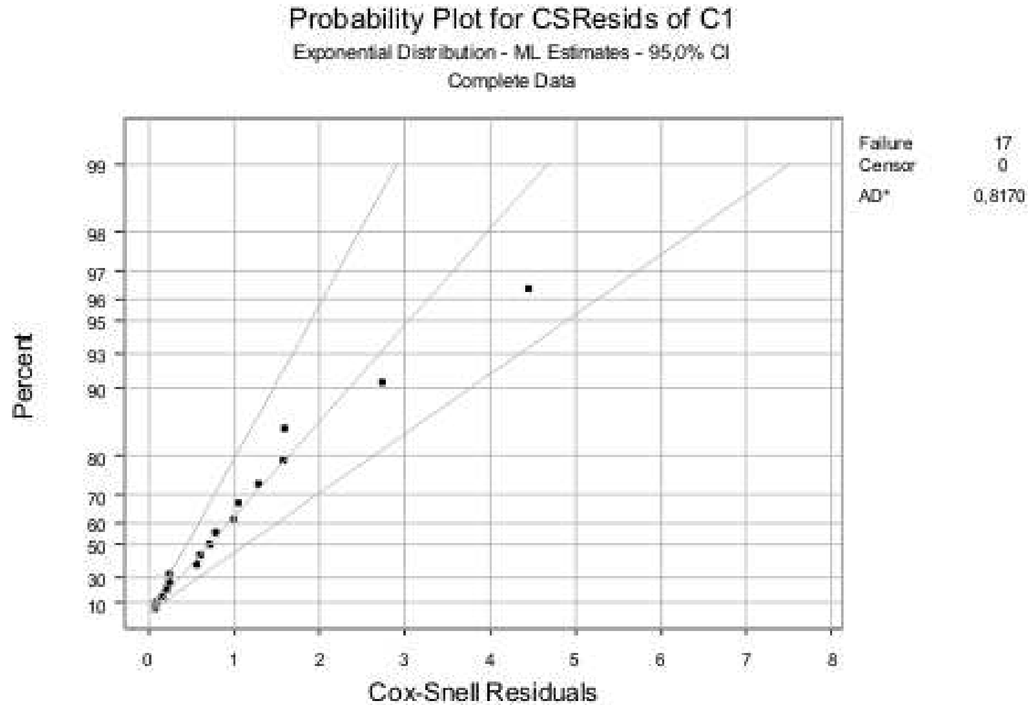

Regression with Life Data: C1 versus C2

```
Response Variable:  C1

Censoring Information                 Count
Uncensored value                         17

Estimation Method:  Maximum Likelihood
Distribution:  Weibull

Regression Table
                      Standard                            95,0% Normal CI
Predictor       Coef     Error       Z      P        Lower        Upper
Intercept     4,6182    0,1219   37,88  0,000       4,3792       4,8572
C2           0,31118   0,04939    6,30  0,000      0,21437      0,40799
Shape         3,0604    0,5245                      2,1873       4,2820

Log-Likelihood = -91,504

Anderson-Darling (adjusted) Goodness-of-Fit
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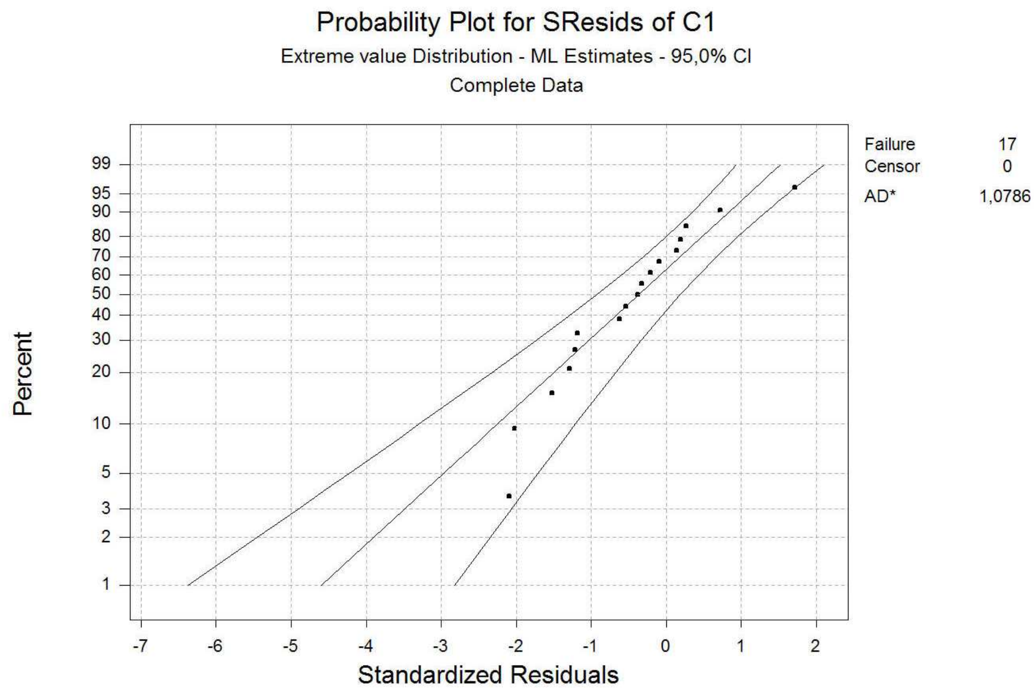
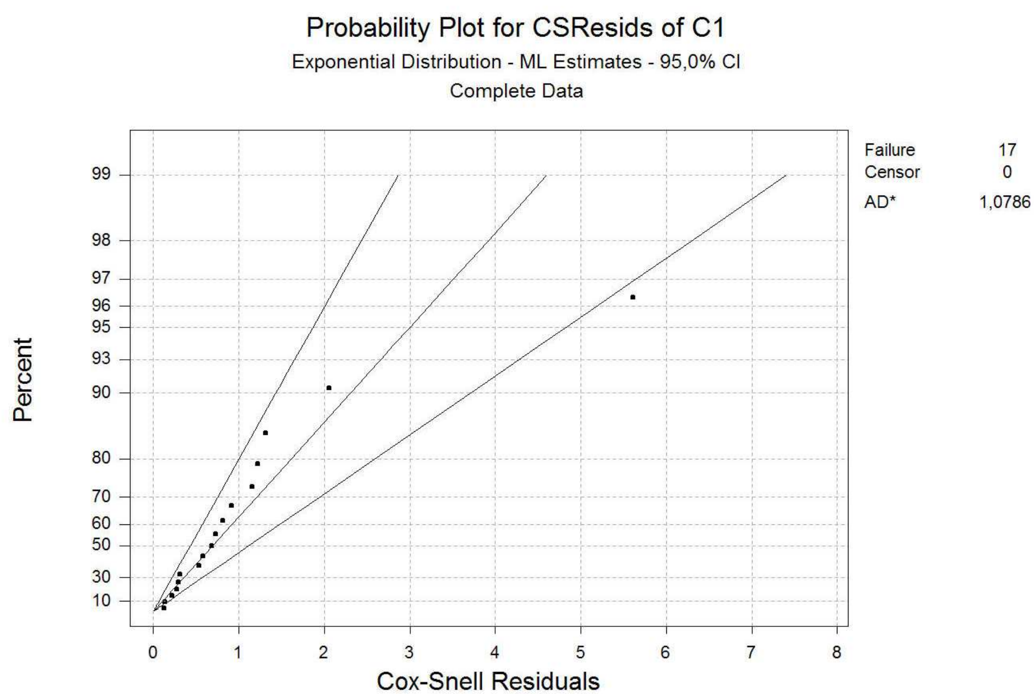

Probability Plot for SResids of C1
Extreme value Distribution - ML Estimates - 95,0% CI
Complete Data
Failure 17
Censor 0
AD* 1,0786
Percent
Standardized Residuals


Probability Plot for CSResids of C1
Exponential Distribution - ML Estimates - 95,0% CI
Complete Data
Failure 17
Censor 0
AD* 1,0786
Percent
Cox-Snell Residuals

Ordinary residuals

$y_i - x'_i\hat{\beta}$

where

yi is the ith response value

x'_i is the vector of predictor values associated with the ith response value

$\hat{\beta}$ represents the estimated regression coefficients

Standardized residuals

$$\frac{y_i - x'_i\hat{\beta}}{\hat{\sigma}}$$

where $\hat{\sigma}$ is the estimated scale parameter.

Cox-Snell residuals

$-\ln(\hat{R}(y_i))$

where

$\hat{R}(y_i)$ is the estimated survival (reliability) probability for the response value yi

ln(x) is the natural log of x

Lognormal Distribution Simple Regression Model with Constant Shape Parameter $\beta = 1/\sigma$

- The lognormal simple regression model is

$$\Pr(T \le t) = F(t; \mu, \sigma) = F(t; \beta_0, \beta_1, \sigma) = \Phi_{\text{nor}}\left[\frac{\log(t) - \mu}{\sigma}\right]$$

where $\mu = \mu(x) = \beta_0 + \beta_1 x$ and σ does not depend on x.

- The failure-time log quantile function

$$\log[t_p(x)] = \mu(x) + \Phi_{\text{nor}}^{-1}(p)\sigma$$

is linear in x.

Notice that

$$\frac{t_p(x)}{t_p(0)} = \exp(\beta_1 x)$$

implies that this regression model is a scale accelerated failure time (SAFT) model with $\mathcal{AF}(x) = \exp(-\beta_1 x)$.

Likelihood for Lognormal Distribution Simple Regression Model with Right Censored Data

The likelihood for n independent observations has the form

$$\begin{aligned} L(\beta_0, \beta_1, \sigma) &= \prod_{i=1}^{n} L_i(\beta_0, \beta_1, \sigma; \text{data}_i) \\ &= \prod_{i=1}^{n} \left\{ \frac{1}{\sigma t_i} \phi_{\text{nor}} \left[\frac{\log(t_i) - \mu_i}{\sigma} \right] \right\}^{\delta_i} \left\{ 1 - \Phi_{\text{nor}} \left[\frac{\log(t_i) - \mu_i}{\sigma} \right] \right\}^{1-\delta_i} \end{aligned}$$

where $\text{data}_i = (x_i, t_i, \delta_i)$, $\mu_i = \beta_0 + \beta_1 x_i$,

$$\delta_i = \begin{cases} 1 & \text{exact observation} \\ 0 & \text{right censored observation} \end{cases}$$

$\phi_{\text{nor}}(z)$ is the standardized normal pdf and $\Phi_{\text{nor}}(z)$ is the corresponding normal cdf.

The parameters are $\boldsymbol{\theta} = (\beta_0, \beta_1, \sigma)$.

Estimated Parameter Variance-Covariance Matrix

Local (observed information) estimate

$$\begin{aligned} \widehat{\Sigma}_{\widehat{\boldsymbol{\theta}}} &= \begin{bmatrix} \widehat{\text{Var}}(\widehat{\beta}_0) & \widehat{\text{Cov}}(\widehat{\beta}_0, \widehat{\beta}_1) & \widehat{\text{Cov}}(\widehat{\beta}_0, \widehat{\sigma}) \\ \widehat{\text{Cov}}(\widehat{\beta}_1, \widehat{\beta}_0) & \widehat{\text{Var}}(\widehat{\beta}_1) & \widehat{\text{Cov}}(\widehat{\beta}_1, \widehat{\sigma}) \\ \widehat{\text{Cov}}(\widehat{\sigma}, \widehat{\beta}_0) & \widehat{\text{Cov}}(\widehat{\sigma}, \widehat{\beta}_1) & \widehat{\text{Var}}(\widehat{\sigma}) \end{bmatrix} \\ &= \begin{bmatrix} -\frac{\partial^2 \mathcal{L}(\beta_0,\beta_1,\sigma)}{\partial \beta_0^2} & -\frac{\partial^2 \mathcal{L}(\beta_0,\beta_1,\sigma)}{\partial \beta_0 \partial \beta_1} & -\frac{\partial^2 \mathcal{L}(\beta_0,\beta_1,\sigma)}{\partial \beta_0 \partial \sigma} \\ -\frac{\partial^2 \mathcal{L}(\beta_0,\beta_1,\sigma)}{\partial \beta_1 \partial \beta_0} & -\frac{\partial^2 \mathcal{L}(\beta_0,\beta_1,\sigma)}{\partial \beta_1^2} & -\frac{\partial^2 \mathcal{L}(\beta_0,\beta_1,\sigma)}{\partial \beta_1 \partial \sigma} \\ -\frac{\partial^2 \mathcal{L}(\beta_0,\beta_1,\sigma)}{\partial \sigma \partial \beta_0} & -\frac{\partial^2 \mathcal{L}(\beta_0,\beta_1,\sigma)}{\partial \sigma \partial \beta_1} & -\frac{\partial^2 \mathcal{L}(\beta_0,\beta_1,\sigma)}{\partial \sigma^2} \end{bmatrix}^{-1} \end{aligned}$$

Partial derivatives are evaluated at $\widehat{\beta}_0, \widehat{\beta}_1, \widehat{\sigma}$.

Standard Errors and Confidence Intervals for Parameters

- Lognormal ML estimates for the computer time experiment were $\widehat{\boldsymbol{\theta}} = (\widehat{\beta}_0, \widehat{\beta}_1, \widehat{\sigma}) = (4.49, .290, .312)$ and an estimate of the variance-covariance matrix for $\widehat{\boldsymbol{\theta}}$ is

$$\widehat{\Sigma}_{\widehat{\boldsymbol{\theta}}} = \begin{bmatrix} .012 & -.0037 & 0 \\ -.0037 & .0021 & 0 \\ 0 & 0 & .0029 \end{bmatrix}.$$

- Normal-approximation confidence interval for the computer execution time regression slope is

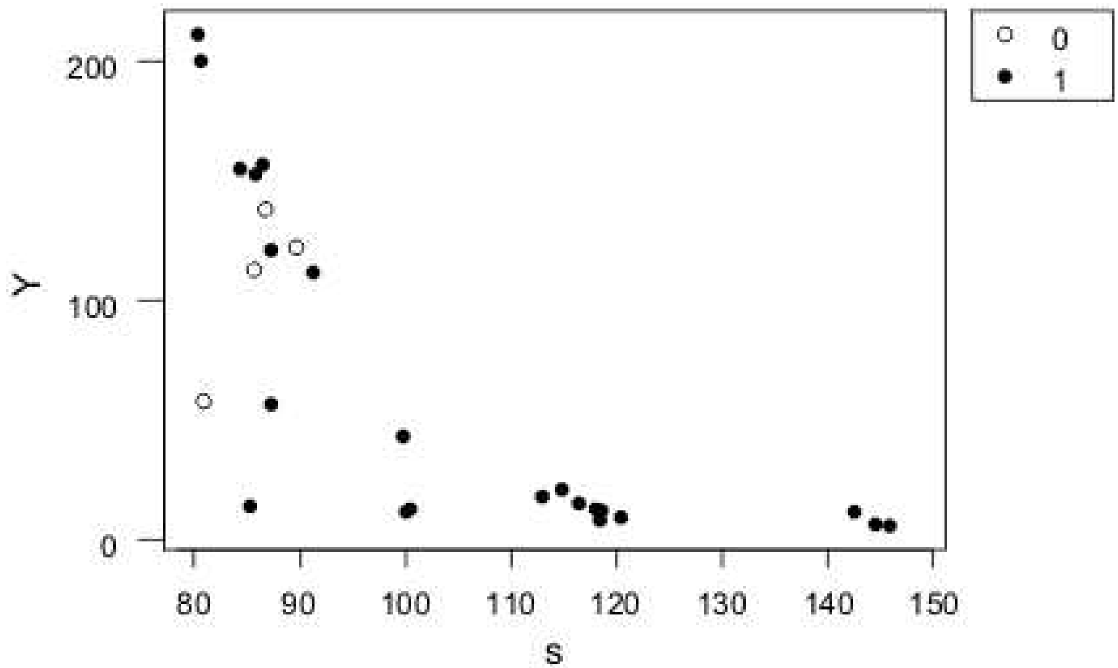
$$[\underset{\sim}{\beta_1}, \quad \widetilde{\beta}_1] = \widehat{\beta}_1 \pm z_{(.975)} \widehat{\mathrm{se}}_{\widehat{\beta}_1} = .290 \pm 1.96(.046) = [.20, \quad .38]$$

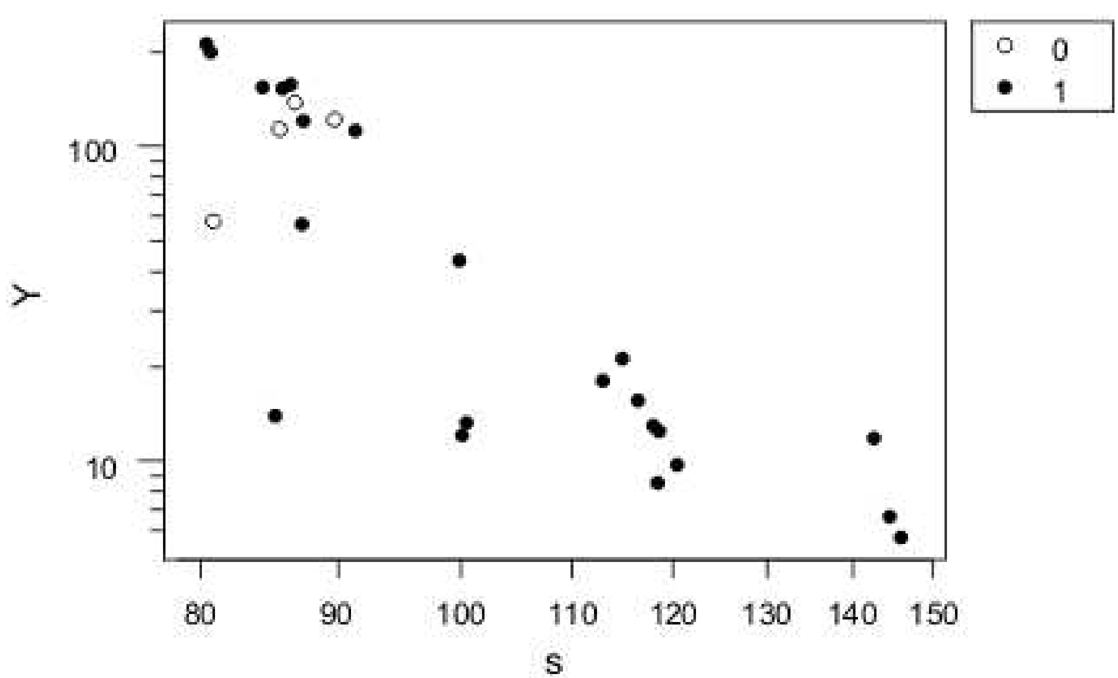
where $\widehat{\mathrm{se}}_{\widehat{\beta}_1} = \sqrt{.0021} = .046$.

Row i	Pseudo-stress s	k-Cycles Y	Status (1=failed, 0=censored) C
1	80,3	211,629	1
2	80,6	200,027	1
3	80,8	57,923	0
4	84,3	155,000	1
5	85,2	13,949	1
6	85,6	112,968	0
7	85,8	152,680	1
8	86,4	156,725	1
9	86,7	138,114	0
10	87,2	56,723	1
11	87,3	121,075	1
12	89,7	122,372	0
13	91,3	112,002	1
14	99,8	43,331	1
15	100,1	12,076	1
16	100,5	13,181	1
17	113,0	18,067	1
18	114,8	21,300	1
19	116,4	15,616	1
20	118,0	13,030	1
21	118,4	8,489	1
22	118,6	12,434	1
23	120,4	9,750	1
24	142,5	11,865	1
25	144,5	6,705	1
26	145,9	5,733	1

DATA DESCRIPTION:
Low-Cycle Fatigue Life of Nickel-Base Superalloy Specimens (in units of thousands of cycles to failure).

Data from Nelson (1990).


200
100
0
Y
80
90
100
110
120
130
140
150
s
0
1


100
10
Y
80
90
100
110
120
130
140
150
s
0
1

Regression with Life Data: Y versus x

Response Variable: Y

Censoring Information	Count
Uncensored value	22
Right censored value	4

Censoring value: C = 0

Estimation Method: Maximum Likelihood
Distribution: Weibull

Regression Table

Predictor	Coef	Standard Error	Z	P	95,0% Normal CI Lower	Upper
Intercept	31,432	2,008	15,65	0,000	27,496	35,368
x	-5,9600	0,4329	-13,77	0,000	-6,8085	-5,1116
Shape	2,2105	0,3894			1,5651	3,1221

Log-Likelihood = -97,155

Anderson-Darling (adjusted) Goodness-of-Fit

Standardized Residuals = 1,0768; Cox-Snell Residuals = 1,0768

Regression with Life Data: Y versus x

Response Variable: Y

Censoring Information	Count
Uncensored value	22
Right censored value	4

Censoring value: C = 0

Estimation Method: Maximum Likelihood
Distribution: Weibull

Regression Table

Predictor	Coef	Standard Error	Z	P	95,0% Normal CI Lower	Upper
Intercept	217,61	62,13	3,50	0,000	95,83	339,39
x	-85,52	26,55	-3,22	0,001	-137,55	-33,49
x*x	8,483	2,831	3,00	0,003	2,934	14,032
Shape	2,6685	0,4777			1,8789	3,7900

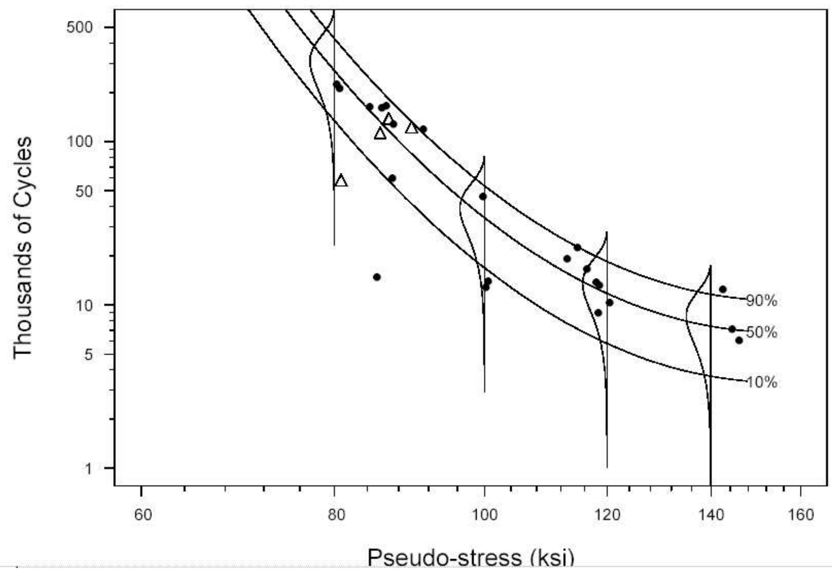
Log-Likelihood = -93,382

Anderson-Darling (adjusted) Goodness-of-Fit

Standardized Residuals = 0,9283; Cox-Snell Residuals = 0,9283

Log-Quadratic Weibull Regression Model with Constant ($\beta = 1/\sigma$) Fit to the Fatigue Data

$$\log[\widehat{t_p}(x)] = \widehat{\mu}(x) + \Phi^{-1}_{\text{sev}}(p)\widehat{\sigma},\ x = \log(\textbf{pseudo-stress})$$



Regression with Life Data: Y versus x

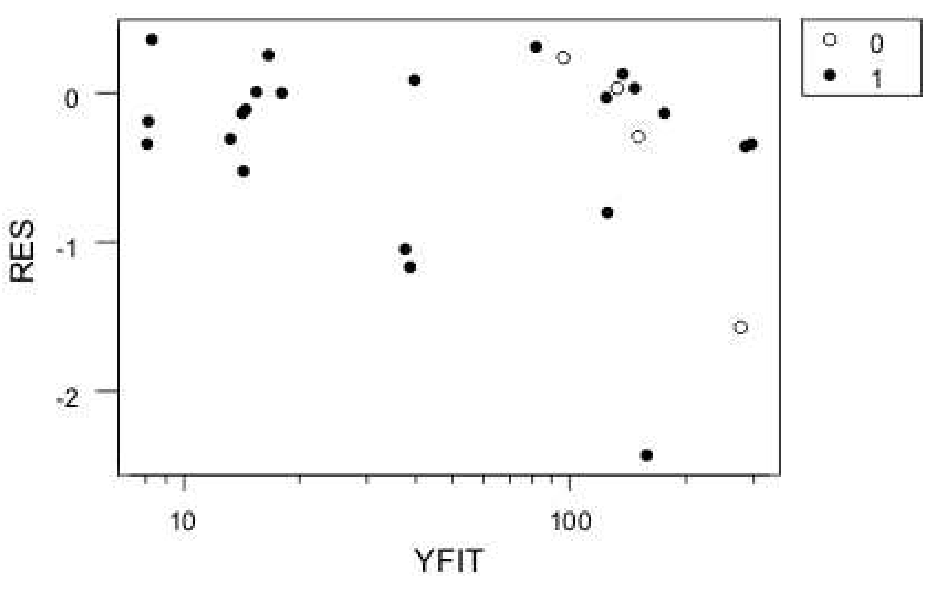
Response Variable: Y

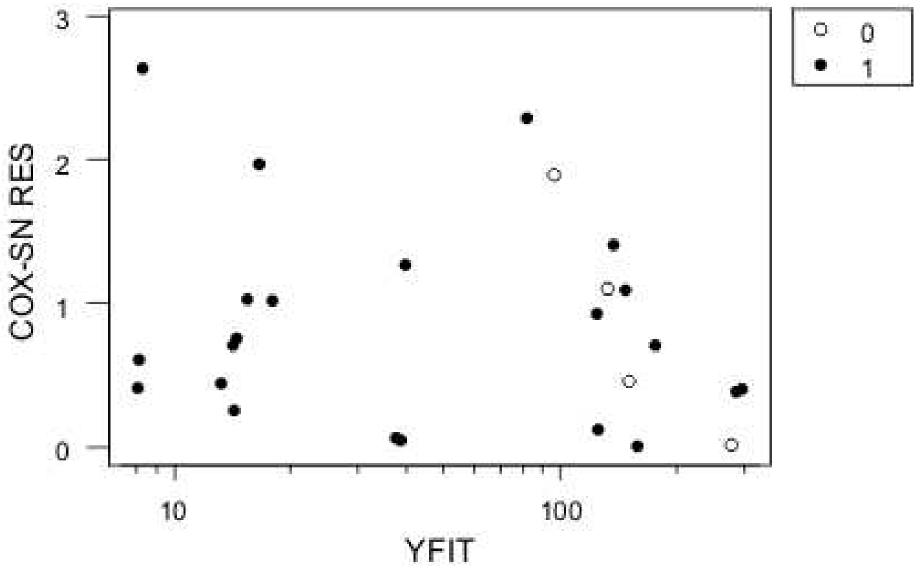
Table of Percentiles

Percent	s	x	Percentile	Standard Error	95,0% Normal CI Lower	Upper
10	80	4,3820	133,3747	34,0579	80,8565	220,0048
10	100	4,6052	16,7928	3,4263	11,2577	25,0494
10	120	4,7875	5,7830	1,2364	3,8034	8,7929
10	140	4,9416	3,6458	0,8760	2,2766	5,8386
50	80	4,3820	270,1879	56,0580	179,9121	405,7621
50	100	4,6052	34,0186	4,3027	26,5494	43,5891
50	120	4,7875	11,7151	1,5950	8,9713	15,2980
50	140	4,9416	7,3856	1,2828	5,2547	10,3807
90	80	4,3820	423,6933	90,4646	278,8097	643,8659
90	100	4,6052	53,3461	6,8162	41,5281	68,5272
90	120	4,7875	18,3709	2,4567	14,1351	23,8760
90	140	4,9416	11,5817	1,9813	8,2824	16,1952

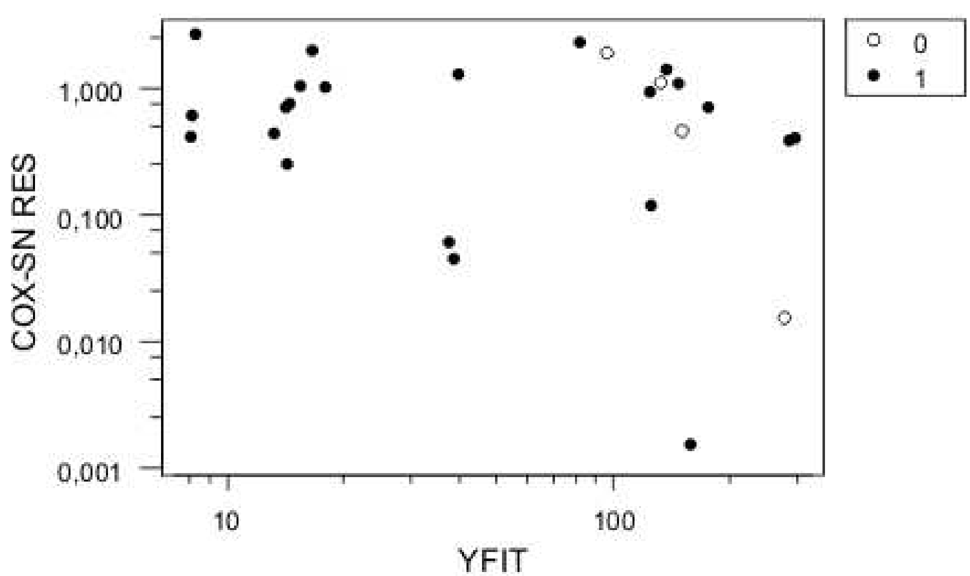
ESTIMERT KOVARIANSMATRISE FOR $(\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2, \hat{\sigma})$

3860,37	-1649,17	175,82	-0,80
-1649,17	704,70	-75,15	0,33
175,82	-75,15	8,02	-0,03
-0,80	0,33	-0,03	0,23




3
2
1
0
COX-SN RES
10
100
YFIT
0
1

27


1,000
0,100
0,010
0,001
COX-SN RES
10
100
YFIT
0
1