FORELESNING 5
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TMA4275 LEVETIDSANALYSE

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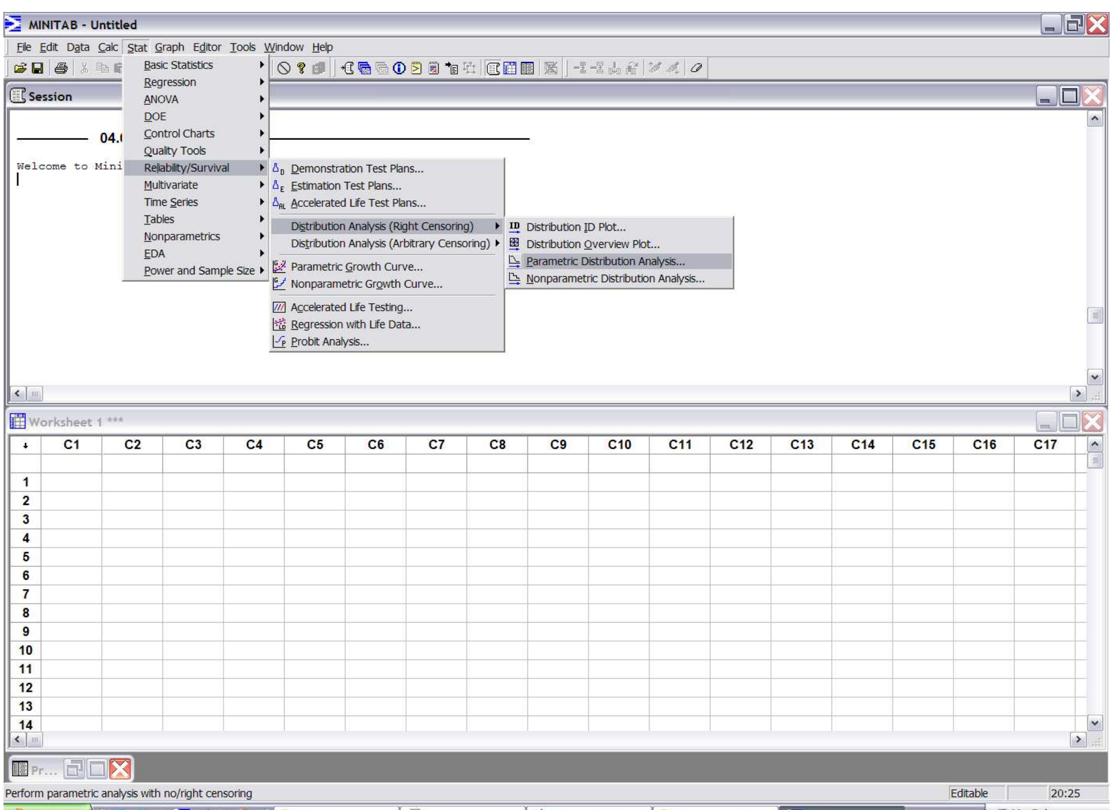
PARAMETRIC LIFETIME ANALYSIS IN MINITAB



DATA OPTIONS

RIGHT CENSORING:

Y_i	δ_i
Observed time	Cens. status 1: Lifetime 0: Censoring

ARBITRARY CENSORING:

Start variable A_i	End variable B_i	
1.7	1.7	Exact lifetime 1.7
2.0	*	Right censoring at time 2.0, i.e. lifetime is > 2.0
*	0.5	Left censoring at time 0.5, i.e. lifetime is < 0.5
1.0	1.5	Interval censoring: Lifetime between 1.0 and 1.5

LIKELIHOOD CONTRIBUTION

Obs. type	Start variable A_i	End variable B_i	Likelihood contribution
Exact lifetime	1.7	1.7	$f(1.7;\theta)$
Right censoring	2.0	*	$1 - F(2.0;\theta)$
Left censoring	*	0.5	$F(0.5;\theta)$
Interval censoring	1.0	1.5	$F(1.5;\theta) - F(1.0;\theta)$

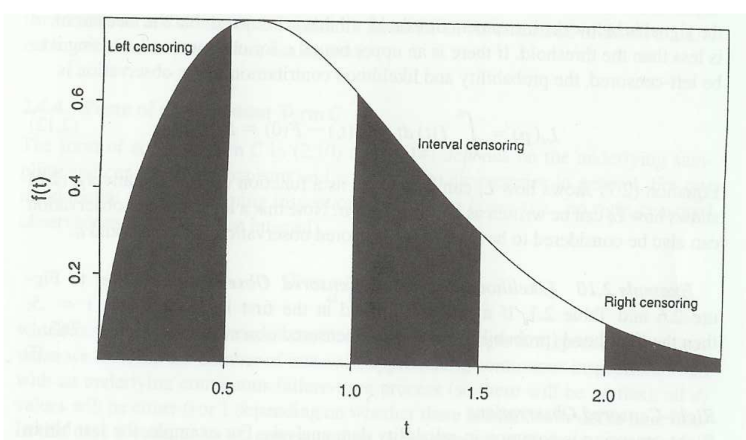
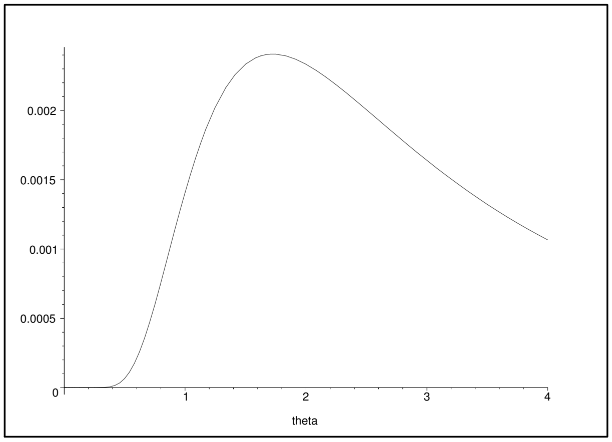


Figure 2.6. Likelihood contributions for different kinds of censoring.

LIKELIHOOD FOR MODEL $f(t;\theta) = (1/\theta)e^{-t/\theta}$

$$L(\theta) = (\frac{1}{\theta}e^{-1.7/\theta}) \cdot (e^{-2.0/\theta}) \cdot (1 - e^{-0.5/\theta}) \cdot (e^{-1.0/\theta} - e^{-1.5/\theta})$$



Maximum likelihood estimate: $\widehat{\theta} = 1.725$

EXAMPLE: RIGHT CENSORED DATA

Worksheet 1 ***

↓	C1	C2
	Y	D
1	0,6	0
2	0,8	1
3	2,1	1
4	3,2	1
5	3,3	0
6	4,4	1
7	8,6	1

Distribution Analysis: Y

Variable: Y

Censoring Information	Count
Uncensored value	5
Right censored value	2

Censoring value: D = 0

Estimation Method: Maximum Likelihood

Distribution: Exponential

Parameter Estimates

Parameter	Estimate	Standard Error	95,0% Normal CI Lower	Upper
Mean	4,6	2,05718	1,91465	11,0516

Log-Likelihood = -12,630

Goodness-of-Fit
Anderson-Darling (adjusted) = 3,767

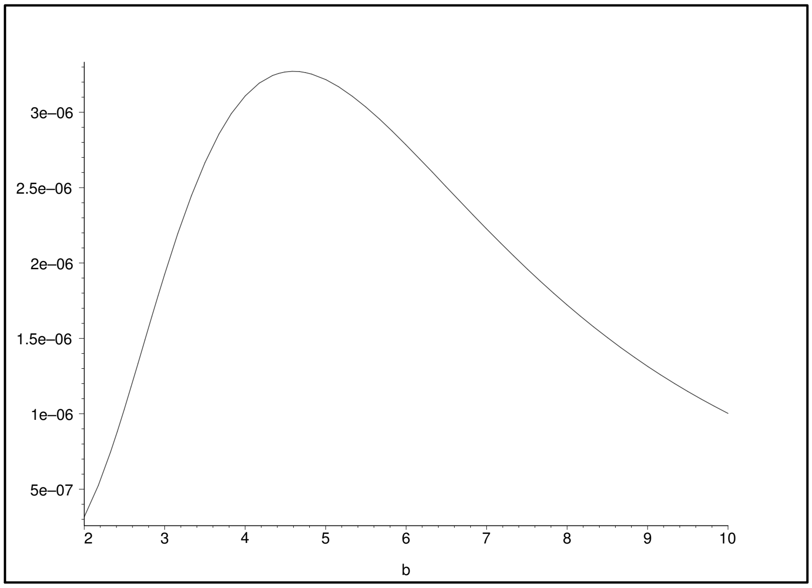
Characteristics of Distribution

	Estimate	Standard Error	95,0% Normal CI Lower	Upper
Mean(MTTF)	4,6	2,05718	1,91465	11,0516
Standard Deviation	4,6	2,05718	1,91465	11,0516
Median	3,18848	1,42593	1,32713	7,66041
First Quartile(Q1)	1,32334	0,591815	0,550810	3,17936
Third Quartile(Q3)	6,37695	2,85186	2,65427	15,3208
Interquartile Range(IQR)	5,05362	2,26005	2,10346	12,1415

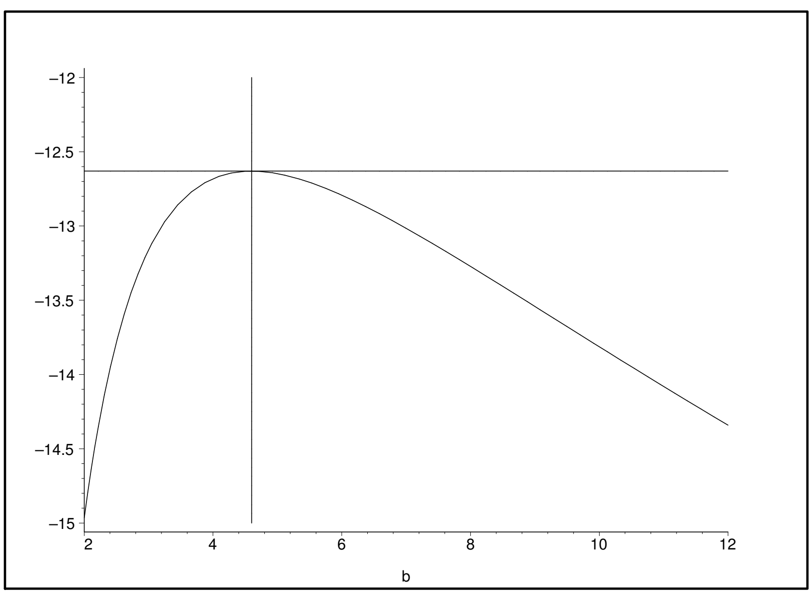
Table of Percentiles

Percent	Percentile	Standard Error	95,0% Normal CI Lower	Upper
1	0,0462315	0,0206754	0,0192429	0,111073
2	0,0929325	0,0415607	0,0386811	0,223273
3	0,140112	0,0626601	0,0583187	0,336624
4	0,187781	0,0839783	0,0781597	0,451150
5	0,235949	0,105520	0,0982086	0,566875
6	0,284627	0,127289	0,118470	0,683825
7	0,333825	0,149291	0,138947	0,802025
8	0,383555	0,171531	0,159646	0,921504
9	0,433829	0,194014	0,180572	1,04229
10	0,484658	0,216746	0,201728	1,16441
20	1,02646	0,459047	0,427241	2,46610
30	1,64070	0,733745	0,682907	3,94184
40	2,34980	1,05086	0,978051	5,64546
50	3,18848	1,42593	1,32713	7,66041
60	4,21494	1,88498	1,75437	10,1265
70	5,53827	2,47679	2,30518	13,3059
80	7,40341	3,31091	3,08151	17,7869
90	10,5919	4,73684	4,40864	25,4473
91	11,0765	4,95358	4,61037	26,6117
92	11,6184	5,19588	4,83588	27,9134
93	12,2326	5,47058	5,09155	29,3892
94	12,9417	5,78770	5,38669	31,0928
95	13,7804	6,16277	5,73577	33,1078
96	14,8068	6,62182	6,16301	35,5739
97	16,1302	7,21363	6,71382	38,7532
98	17,9953	8,04775	7,49015	43,2343
99	21,1838	9,47368	8,81728	50,8947

LIKELIHOOD FUNCTION FOR MODEL $f(t;\theta) = (1/\theta)e^{-1/\theta}$

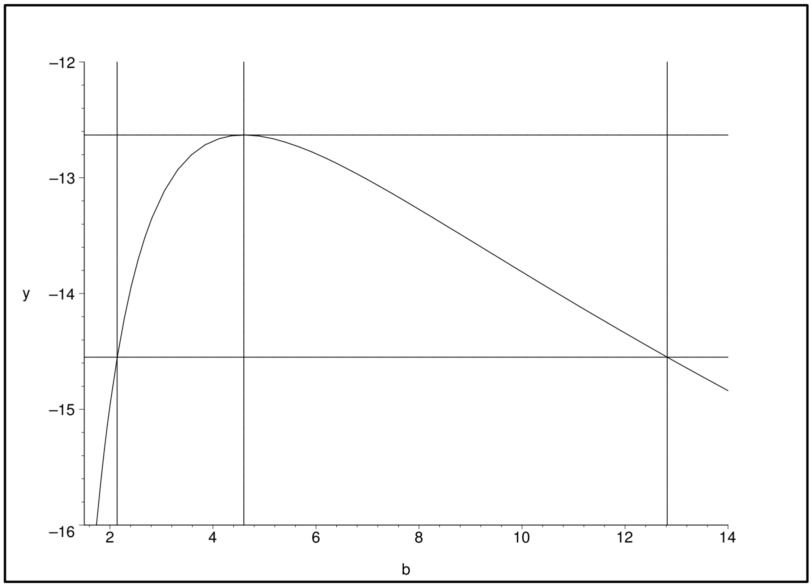


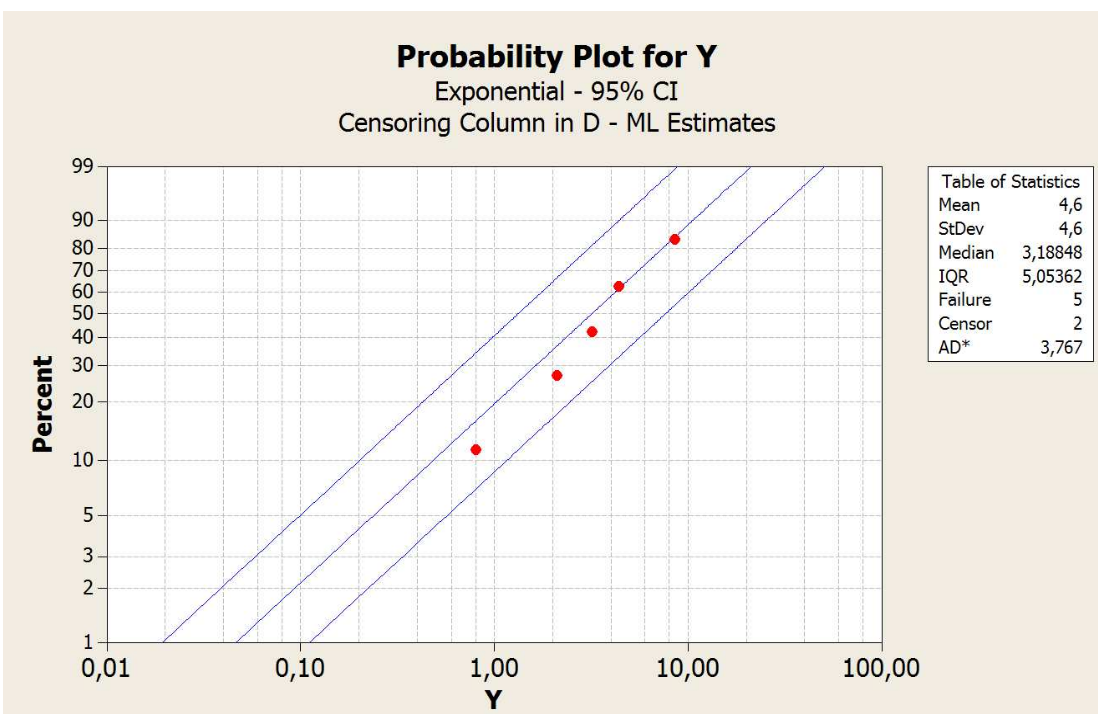
LOG-LIKELIHOOD FUNCTION



Maximum likelihood estimate: $\widehat{\theta} = 4.6$

LOG-LIKELIHOOD FUNCTION





Method – Probability Plot for Uncensored/Right Censored Data

main topic

Probability plots are based on a scheme that plots the observed failure times (or a transformation of the failure times) on the x-axis vs. the estimated cumulative probabilities (p) on the y-axis. The cumulative probabilities are transformed. Transformations of both the x and y data are needed to ensure that the plotted y values are a linear function of the plotted x values. To help assess the linearity of the plotted data, a fitted line is also drawn on the probability plots. The table below shows how the actual x and y plot points are constructed:

Distribution	x coordinate	y coordinate
Weibull	ln(failure time)	ln(-ln(1 - p))
Extreme value	failure time	ln(-ln(1 - p))
Exponential	failure time	$-\ln(1-p)$
Normal	failure time	$\Phi^{-1}(p)$
Lognormal basee	ln(failure time)	$\Phi^{-1}(p)$
Lognormal base10	log10(failure time)	$\Phi^{-1}(p)$
Logistic	failure time	$\ln\left(\frac{p}{1-p}\right)$
Loglogistic	ln(failure time)	$\ln\left(\frac{p}{1-p}\right)$

where

$\Phi^{-1}(p)$ = value of standard normal distribution, Z, such that prob $(Z <= \Phi^{-1}(p) = p)$

ln (x) = natural log of x

log10(x) = log base 10 of x

Minitab estimates the cumulative probabilities (p) using the Default, Modified Kaplan-Meier, Herd-Johnson, or Kaplan-Meier method. The Default method is the normal score for uncensored data; the modified Kaplan-Meier method for censored data. Each of these methods generates nonparametric estimates of F(t), the cumulative distribution function for the random variable T, which is time to failure.

Note If the largest observation is uncensored, the Kaplan-Meier method results in p=1 for the largest uncensored observation. In this case, the Kaplan-Meier estimate for the largest observation results in a number that cannot be used in the plot. This problem is corrected by recalculating the largest p as 90% of the distance between the prior p and 1.

This table displays the differences among the methods. For a sample of n observations, let x(1), x(2),...,x(n) be the order statistics, or the data ordered from smallest to largest. Then i is the rank of the ith ordered observation x(i). The formulas are as follows:

For uncensored data...

This method . . .	Uses this equation
Normal score (default)	$\frac{i - 3/8}{n + 1/4}$
Modified Kaplan-Meier	$\frac{i - 1/2}{n}$
Herd-Johnson	$\frac{i}{n + 1}$
Kaplan-Meier	$\frac{i}{n}$

For censored data...

This method...	Uses this equation
Modified Kaplan-Meier (default)	$p_i = 1 - \frac{(1 - p'_i) + (1 - p'_{i-1})}{2}$ where p'_i is the p_i from the Kaplan-Meier estimate and $p'_0 = 0$.
Herd-Johnson estimate	$p_i = 1 - \prod_{j=1:i} \frac{(n - j + 1)}{(n - j + 2)}^{\delta_j}$
Kaplan-Meier product limit estimate	$p_i = 1 - \prod_{j=1:i} \frac{(n - j)}{(n - j + 1)}^{\delta_j}$

where

δ_j = 0 if the jth observation is censored

1 if the jth observation is uncensored

If there are tied failure times in the data, either all points (default), the average (median), or the maximum of the tied points is plotted. If the tie involves failures and suspensions, the failures are considered to occur before the suspensions.

Row	C1	C2
1	0,35	1
2	0,50	0
3	0,75	0
4	1,00	1
5	1,30	1
6	1,80	1
7	3,00	0
8	3,15	0
9	4,85	0
10	5,50	1
11	5,50	0
12	6,25	0

Variable: C1

Censoring Information Count Uncensored value
5 Right censored value
7 Censoring value: C2 = 0

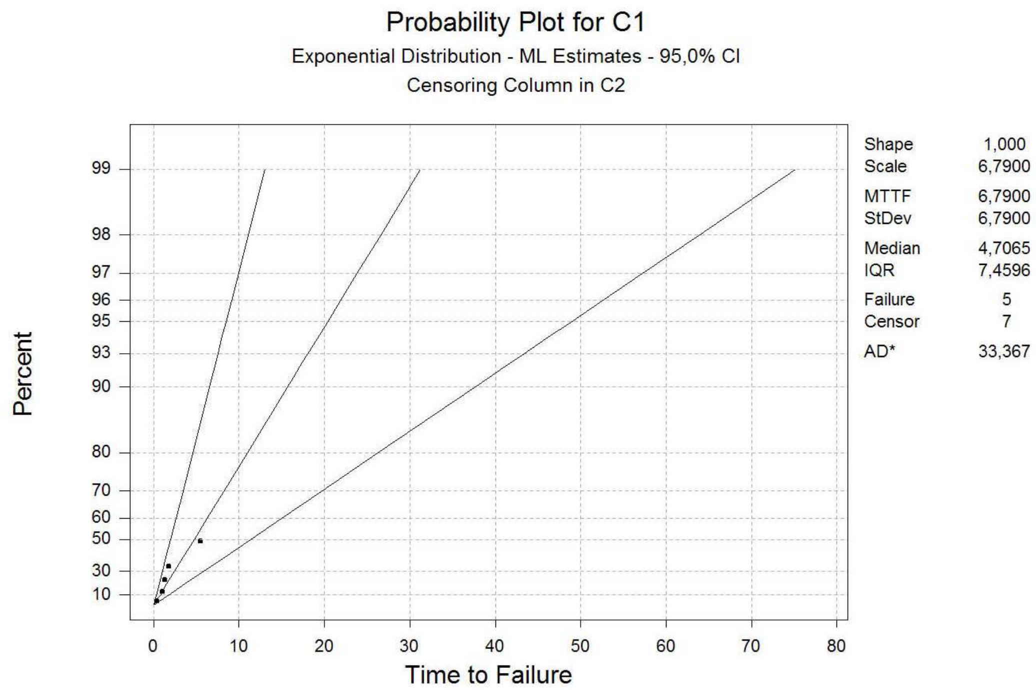
Estimation Method: Maximum Likelihood Distribution: Exponential

Parameter Estimates

Parameter	Estimate	Standard Error	95,0% Normal CI Lower	95,0% Normal CI Upper
Shape	1,00000			

Parameter	Estimate	Standard Error	95,0% Normal CI Lower	95,0% Normal CI Upper
Scale	6,790	3,037	2,826	16,313

Log-Likelihood = -14,577

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Variable:  C1

Censoring Information                Count Uncensored value
5 Right censored value 7 Censoring value:  C2 = 0

Estimation Method:  Maximum Likelihood
Distribution:  Weibull

Parameter Estimates
                         Standard        95,0% Normal CI
Parameter   Estimate        Error        Lower        Upper
Shape         0,9780       0,3694       0,4665       2,0504
Scale   6,880
3,517       2,526      18,740

Log-Likelihood = -14,576
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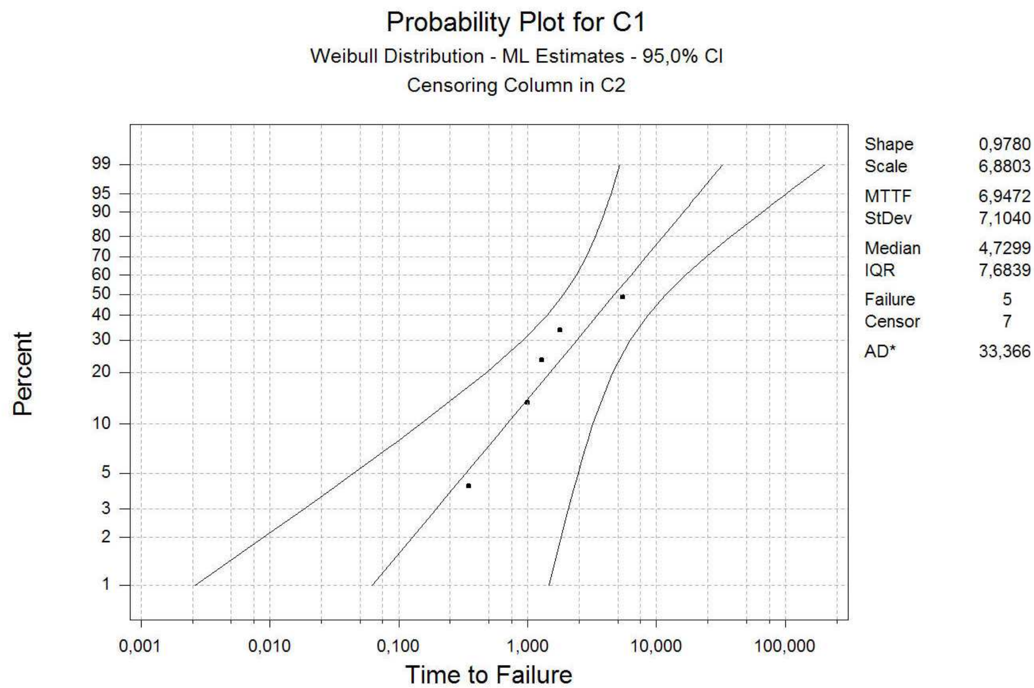

Probability Plot for C1
Weibull Distribution - ML Estimates - 95,0% CI
Censoring Column in C2
Percent
99
95
90
80
70
60
50
40
30
20
10
5
3
2
1
0,001
0,010
0,100
1,000
10,000
100,000
Time to Failure
Shape 0,9780
Scale 6,8803
MTTF 6,9472
StDev 7,1040
Median 4,7299
IQR 7,6839
Failure 5
Censor 7
AD* 33,366