Exam 2019

Solution 3d:

The contribution from the ith process to the conditional likelihood is by (3),

$$\left\{\prod_{k=1}^{N_i(\tau_i)} Z_i \alpha(T_{ik};\theta)\right\} e^{-\int_0^{\tau_i} Z_i \alpha(t;\theta)dt}$$

$$= \left\{\prod_{k=1}^{N_i(\tau_i)} \alpha(T_{ik};\theta)\right\} Z_i^{N_i(\tau_i)} e^{-Z_i \int_0^{\tau_i}\alpha(t;\theta)dt}$$

The unconditional contribution is the expected value of this with respect to the distribution of Z_i, i.e.,

$$\left\{\prod_{k=1}^{N_i(\tau_i)} \alpha(T_{ik};\theta)\right\} E\left[Z_i^{N_i(\tau_i)} e^{-Z_i \int_0^{\tau_i}\alpha(t;\theta)dt}\right]$$

$$= \left\{\prod_{k=1}^{N_i(\tau_i)} \alpha(T_{ik};\theta)\right\} (-1)^{N_i(\tau_i)} \mathcal{L}^{(N_i(\tau_i))}\left(\int_0^{\tau_i}\alpha(t;\theta)dt\right)$$

The full unconditional likelihood is the product of these for $i=1,\dots,n$.