

# STK4080 SURVIVAL AND EVENT HISTORY ANALYSIS

## Slides 1: Introduction

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## Course content

*The course gives an introduction to the most important concepts and methods in survival and event history analysis. These methods have applications for instance in insurance, medicine and reliability.*

## Learning outcome

*The course gives the background for analyzing a wide specter of models for time to one event as well as models for complex event histories. The course gives a practical introduction to these methods as well as a theoretical understanding of them.*

## Lifetime

Time to occurrence of some event of interest for individuals in some population.

## Medical research:

- Time to death of a patient after start of certain treatment
- Time from entrance to discharge from a hospital
- Times between successive epileptic seizures for patient

## Reliability engineering:

- Time to failure of a component or a system
- Number of cycles to failure (fatigue testing)
- Times between successive failures of a machine

# WHY COLLECT AND ANALYZE LIFETIME/SURVIVAL/RELIABILITY DATA?

## **Medical research:**

- Compare different treatments with respect to survival or recurrence
- Predict the outcome of an intervention or the life expectancy after the invention
- Identify risk factors for diseases and assess their magnitude

## **Reliability engineering:**

- Assess reliability of a system/component/product
- Compare two or more products with respect to reliability
- Predict product reliability in the design phase
- Predict warranty claims for a product in the market

# SPECIAL ASPECTS OF LIFETIME ANALYSIS IN STATISTICS

- *Censored data* (how can we use data from individuals or units for which the event of interest has not occurred within the observation period?)
- subjects may not be followed from time 0 (in the study time scale), but only from a later entry time. This is called *delayed entry* or *left-truncation*.
- Definition of *starting time and failure time* may be difficult
- Definition of *time scale* (in reliability: operation time, calendar time or number of cycles?)
- Effect of *covariates* (demographic, medical, environmental)
- What if an individual or unit dies or fails of another cause than the one we would like to study? ("*competing risks*")
- *Recurrent events* – what if the system can fail several times; how to analyze recurring stages of a disease?

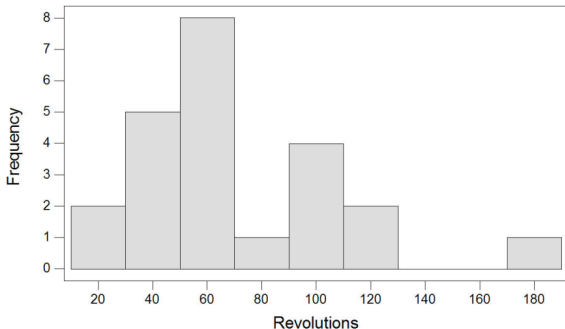
# COMPLETE DATA: *BALL BEARING FAILURE DATA*

**Data:** Millions of revolutions to fatigue failure for 23 units

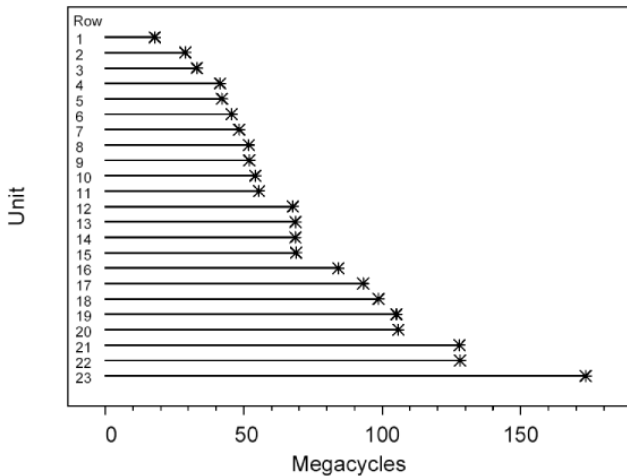
**Question:** How can we fit a parametric lifetime distribution to these data?

17,88	28,92	33,00	41,52	42,12	45,60	48,40	51,84
51,96	54,12	55,56	67,80	68,64	68,64	68,88	84,12
93,12	98,64	105,12	105,84	127,92	128,04	173,40	

Histogram of Revolutions



# BALL BEARING FAILURE DATA (EVENT PLOT)



- Integrated circuit failure times in hours
  - $n = 4156$  ICs tested for 1,370 hours at 80° C and 80% relative humidity
  - There were 28 failures
  - When the test ended at 1,370 hours, 4128 units were still running

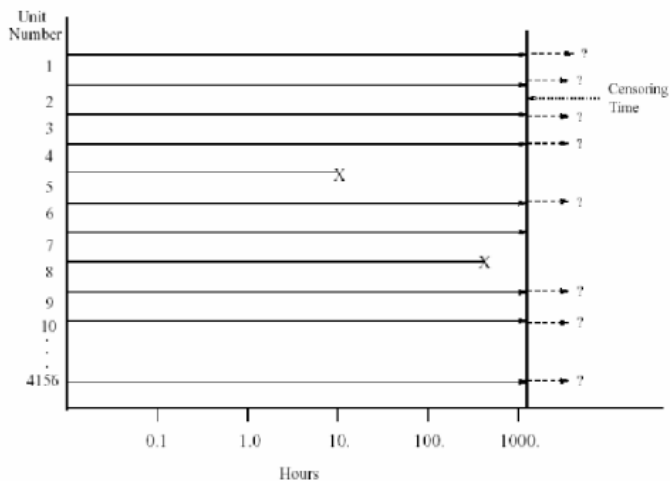
.10	.10	.15	.60	.80	.80
1.20	2.5	3.0	4.0	4.0	6.0
10.0	10.0	12.5	20.	20.	43.
43.	48.	48.	54.	74.	84.
94.	168.	263.	593.		

### Questions of interest:

- How to estimate the distribution of the failure time when there are censored observations?
- Probability of failure before 100 hours?
- Failure rate by 100 hours?
- Proportion failed after  $10^5$  hours?



# IC DATA (EVENT PLOT)



- *Multiple myeloma* is a malignant disease characterised by the accumulation of abnormal plasma cells, a type of white blood cell, in the bone marrow.
- Data (next slide) from Medical Center of the University of West Virginia, USA.
- **Aim:** To examine the association between certain explanatory variables or covariates and the survival time of patients in months from diagnosis until death from multiple myeloma).

**Table 1.3** *Survival times of patients in a study on multiple myeloma.*

Patient number	Survival time	Status	Age	Sex	Bun	Ca	Hb	Pcells	Protein
1	13	1	66	1	25	10	14.6	18	1
2	52	0	66	1	13	11	12.0	100	0
3	6	1	53	2	15	13	11.4	33	1
4	40	1	69	1	10	10	10.2	30	1
5	10	1	65	1	20	10	13.2	66	0
6	7	0	57	2	12	8	9.9	45	0
7	66	1	52	1	21	10	12.8	11	1
8	10	0	60	1	41	9	14.0	70	1
9	10	1	70	1	37	12	7.5	47	0
10	14	1	70	1	40	11	10.6	27	0
11	16	1	68	1	39	10	11.2	41	0
12	4	1	50	2	172	9	10.1	46	1
13	65	1	59	1	28	9	6.6	66	0
14	5	1	60	1	13	10	9.7	25	0
15	11	0	66	2	25	9	8.8	23	0
16	10	1	51	2	12	9	9.6	80	0
17	15	0	55	1	14	9	13.0	8	0
18	5	1	67	2	26	8	10.4	49	0
19	76	0	60	1	12	12	14.0	9	0
20	56	0	66	1	18	11	12.5	90	0

## Problem 2

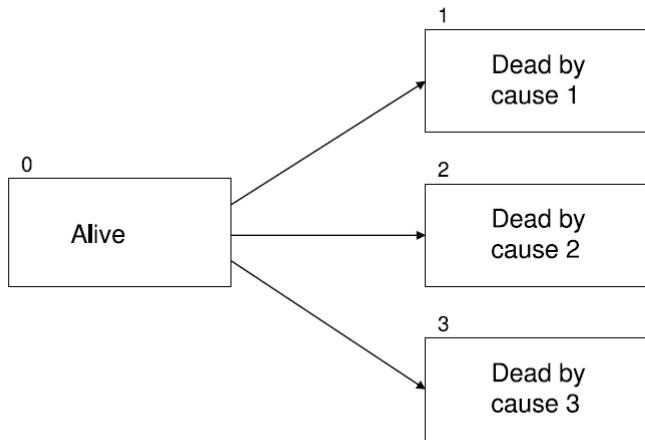
A clinical trial to evaluate the efficacy of chemotherapy for a specific cancer was conducted. After reaching a state of remission (disappearance of cancer) through treatment, the patients who entered the study were randomized into two groups. The first group received maintenance chemotherapy, the second (or control) group did not. For a preliminary analysis during the course of the trial the data were as follows:

Length of complete remission (in weeks).

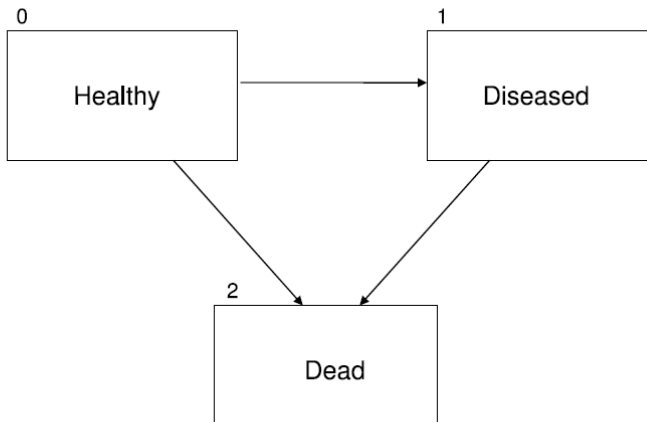
*Maintenance group:* 9, 13, 13<sup>+</sup>, 23, 24<sup>+</sup>, 34, 45<sup>+</sup>, 55, 161<sup>+</sup>

*Control group:* 5, 13, 13, 16<sup>+</sup>, 20, 21, 43, 45

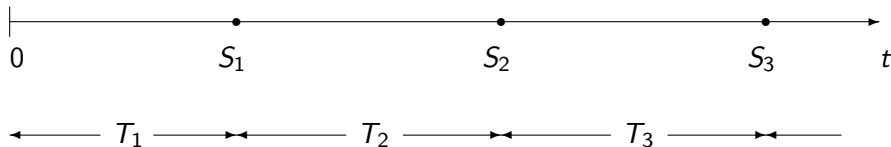
+ indicates censored observation.



**Fig. 1.8** A model for  $k = 3$  competing causes of death.



**Fig. 1.10** *An illness-death model without recovery.*



For each individual in the study we observe repeated occurrences of an event, for example

- 1 Relapse from disease (epileptic seizures, recurrence of tumors)
- 2 System is repaired and put into use again.
- 3 Machine part is replaced.

# EXAMPLE OF DATA FOR RECURRENT EVENTS

Aalen and Husebye (1991): Migratory motor complex (MMC) periods in 19 patients, 1-9 events per individual.

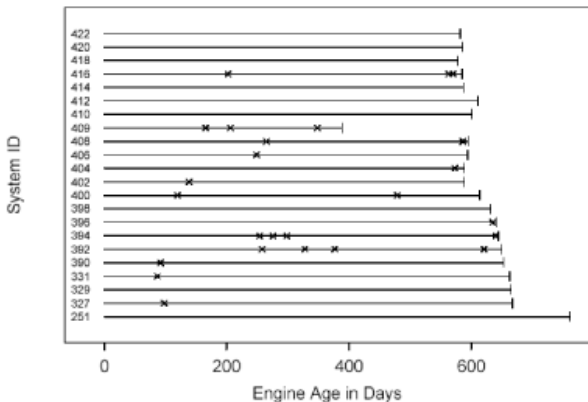
Individual	Observed periods (minutes)					
1	112 33	145 51	39 (54)	52	21	34
2	206	147	(30)			
3	284	59	186	(4)		
4	94	98	84	(87)		
5	67	(131)				
6	124 58	34 142	87 75	75 (23)	43	38
7	116	71	83	68	125	(111)
8	111	59	47	95	(110)	
9	98	161	154	55	(44)	
10	166	56	(122)			
11	63	90	63	103	51	(85)
12	47	86	68	144	(72)	
⋮			⋮			



*See Example 7.1 in ABG:*

The task is to estimate the average duration of the intervals and the variation within and between individuals

## Valve Seat Replacement Times Event Plot (Nelson and Doganaksoy 1989)



Data on previous slide are collected from valve seats from a fleet of 41 diesel engines. Each engine has 16 valves. (Time unit is days of operation).

## Questions of interest:

- Does the replacement rate increase with age?
- How many replacement valves will be needed in the future?
- Can valve life in these systems be modeled as a renewal process?

# ESTIMATED NUMBER OF VALVE SEAT REPLACEMENTS

- Middle curve is cumulative estimated number of replacements for one engine, as a function of age.
- Lower and upper curves are 95% confidence limits.

