Extra exercises in STK4080 2019

Exercise E1.1

Let $N(t)$ be a nonhomogeneous Poisson process (NHPP) with intensity function $\alpha(t)$, which means that, when defining $A(t) = \int_0^t \alpha(s)ds$

- $N(t) - N(s) \sim$ Poisson$(A(t) - A(s))$ when $s < t$
- $N(t) - N(s)$ is independent of $\mathcal{F}_s$ when $s < t$

Suppose n NHPP-processes $N_1(t), \ldots, N_n(t)$ as above have been observed, on time intervals, respectively, $[0, \tau_i]$ for $i = 1, \ldots, n$, where the processes share the same $\alpha(t)$.

a) What is the 'at risk' indicator $Y_i(t)$ for the ith process?

Write down the intensity function $\lambda_i(t)$ for the ith process.

b) Show that a multiplicative intensity model for $\alpha(t)$ results from this.

Write down an expression for the Nelson-Aalen estimator for $A(t)$.

c) Suppose $n = 3$, $\tau_1 = 20, \tau_2 = 30, \tau_3 = 10$, and that the observed events for the three processes are given as

i=1: 5, 12, 17

i=2: 9, 23

i=3: 4

Calculate the Nelson-Aalen estimator for $A(t)$ and draw a graph (by hand!).

Calculate also the variance of the estimator and find pointwise 95% confidence bounds for the Nelson-Aalen curve.

How can you do this in R?

d) Formulate a result on the asymptotic behavior of the estimator as the number n of processes tends to infinity. Consider in particular the case when $\tau_i \equiv \tau$ for all i.