is an approximate confidence interval for $\log S(t)$. Finally we exponentiate the lower and upper limit of (E.18) to see that

$$\widehat{S}(t)^{\exp\left\{\pm z_{1-\alpha/2}\,\widehat{\tau}(t)/\left(\widehat{S}(t)\log\widehat{S}(t)\right)\right\}}$$

is an approximate $100(1-\alpha)\,\%$ confidence interval for $S(t)$. This shows (3.30) in the ABG-book.

Exercise 3.8

By the approximate normality of the Kaplan-Meier estimator, we have that

$$\frac{\widehat{S}(t) - S(t)}{\widehat{\tau}(t)}$$

is approximately standard normally distributed.

a) Let ξ_p be the p-th fractile of the survival distribution. Then $S(\xi_p) = 1 - p$.

   In order to test the null hypothesis

   $$\mathrm{H}_0 : \xi_p = \xi_p^0$$

   versus the alternative hypothesis

   $$\mathrm{H}_A : \xi_p \neq \xi_p^0$$

   we may use the test statistic

   $$Z = \frac{\widehat{S}\left(\xi_p^0\right) - S\left(\xi_p^0\right)}{\widehat{\tau}(\xi_p^0)} = \frac{\widehat{S}\left(\xi_p^0\right) - (1-p)}{\widehat{\tau}\left(\xi_p^0\right)}$$

   which is approximately $N(0,1)$ distributed under H_0.

   Therefore, rejecting H_0 when $|Z| > z_{1-\alpha/2}$, i.e. when

   $$\left|\frac{\widehat{S}\left(\xi_p^0\right) - (1-p)}{\widehat{\tau}\left(\xi_p^0\right)}\right| > z_{1-\alpha/2} \tag{E.19}$$

   gives us a test with significance level approximately equal to α.

b) According to a general result, we get a $100(1-\alpha)\%$ confidence interval for ξ_p as all ξ_p^0-values that are not rejected by the (E.19).

   This confidence interval is given by all values of ξ_p^0 that satisfy the inequality

   $$\left|\frac{\widehat{S}\left(\xi_p^0\right) - (1-p)}{\widehat{\tau}\left(\xi_p^0\right)}\right| \leq z_{1-\alpha/2}$$

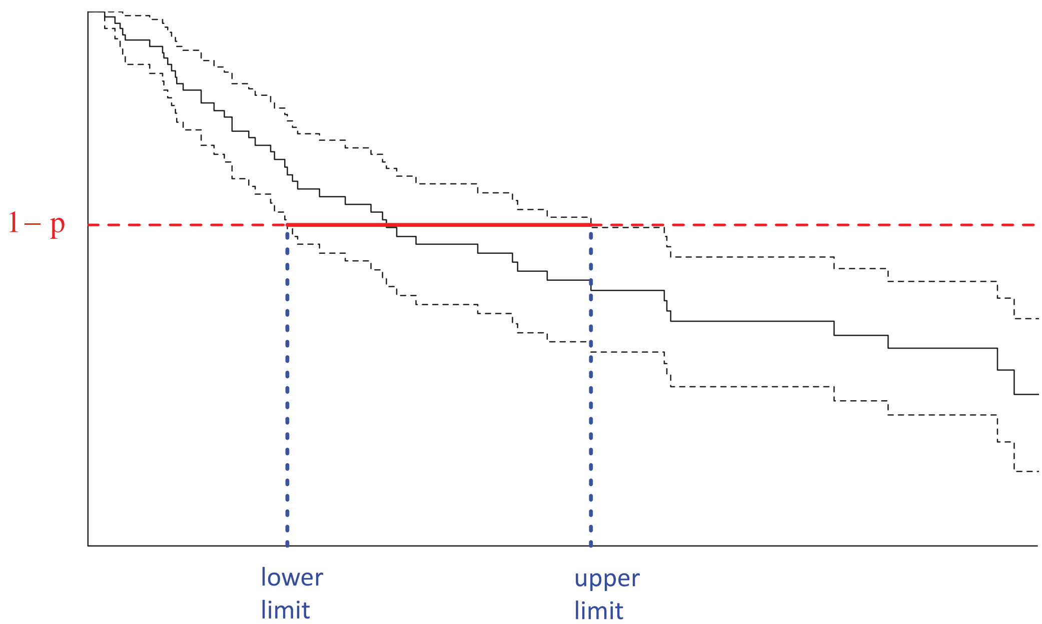
Or equivalently, all values of t that satisfy the inequality

$$\left|\widehat{S}(t) - (1-p)\right| \leq z_{1-\alpha/2}\,\widehat{\tau}\big(t\big) \tag{E.20}$$

Now the the standard $100(1-\alpha)\%$ confidence interval for $S(t)$ is given by

$$\widehat{S}(t) \pm z_{1-\alpha/2}\,\widehat{\tau}(t) \tag{3.29}$$

Thus by (E.20), the $100(1-\alpha)\%$ confidence interval for ξ_p is obtained as all values of t where $\widehat{S}(t)$ is closer to the horizontal line at $1-p$ than the upper or lower confidence limits given by (3.29). At the figure below, this corresponds to the part of the red line that is fully drawn. If we read of the values of t corresponding to the



Exercise 3.10

For $h = 1, 2$, we have that $N_h(t)$ is a counting process with intensity process $\lambda_h(t) = \alpha_h(t)Y_h(t)$. Under the null hypothesis

$$\mathrm{H}_0 : \alpha_1(t) = \alpha_2(t)$$

the aggregated counting process $N_{\bullet}(t) = N_1(t) + N_2(t)$ has intensity process $\lambda_{\bullet}(t) = \alpha(t)Y_{\bullet}(t)$, where $Y_{\bullet}(t) = Y_1(t) + Y_2(t)$ and $\alpha(t)$ is the common value of $\alpha_1(t)$ and $\alpha_2(t)$ under H_0.

We know that under H_0 the test statistic $Z_1(t_0)$ has predictable variation process

$$\langle Z_1\rangle(t_0) = \int_0^{t_0} \frac{L^2(t)Y_{\bullet}(t)}{Y_1(t)Y_2(t)}\alpha(t)\,\mathrm{d}t \tag{3.54}$$