

## Extra exercises in STK4080 2021

### Exercise E2.1

Consider the latent failure time approach to competing risks on p. 6-8/28 in Slides 9 2021.

Assume that  $k = 2$  and  $T_1, T_2$  are independent and exponentially distributed with

$$f_{T_j}(t) = \lambda_j e^{-\lambda_j t}, \quad t \geq 0$$

Let  $T = \min(T_1, T_2)$ ,  $H = \operatorname{argmin}_j(T_j)$ .

- a) Calculate the cumulative incidence functions

$$P(T \leq t, H = j) \quad \text{for } j = 1, 2$$

[Answer:  $\frac{\lambda_j}{\lambda_1 + \lambda_2} (1 - e^{-(\lambda_1 + \lambda_2)t})$ ]

- b) Show that the cause-specific hazard rates are

$$\alpha_j(t) = \lambda_j \quad \text{for } j = 1, 2$$

- c) The expressions in a) and b) indicate that cause specific hazards are not influenced by the corresponding one for the other risk, while this is the case for the cumulative incidence function. (This turns out to be a general property of competing risks which one should be aware of.) Discuss the intuitive content of this observation by considering the definitions of the two concepts.
- d) (Optional) Use the recipe for simulation and analysis in R given on p. 15-18/28 in Slides 9 to illustrate the above results. Choose reasonable values for  $\lambda_1, \lambda_2$  and include also a censoring time  $C$ .