

Extra exercises in STK4080/9080 2021

Exercise E1.1

Let $N(t)$ be a nonhomogeneous Poisson process (NHPP) with intensity function $\alpha(t)$, which means that, when defining $A(t) = \int_0^t \alpha(s) ds$

- $N(t) - N(s) \sim \text{Poisson}(A(t) - A(s))$ when $s < t$
- $N(t) - N(s)$ is independent of \mathcal{F}_s when $s < t$

Suppose n NHPP-processes $N_1(t), \dots, N_n(t)$ as above have been observed, on time intervals, respectively, $[0, \tau_i]$ for $i = 1, \dots, n$, where the processes share the same $\alpha(t)$.

- a) What is the 'at risk' indicator $Y_i(t)$ for the i th process?
Write down the intensity function $\lambda_i(t)$ for the i th process.
- b) Show that a multiplicative intensity model for $\alpha(t)$ results from this.
Write down an expression for the Nelson-Aalen estimator for $A(t)$.
- c) Suppose $n = 3$, $\tau_1 = 20$, $\tau_2 = 30$, $\tau_3 = 10$, and that the observed events for the three processes are given as

i=1: 5, 12, 17

i=2: 9, 23

i=3: 4

Calculate the Nelson-Aalen estimator for $A(t)$ and draw a graph (by hand!).

Calculate also the variance of the estimator and find pointwise 95% confidence bounds for the Nelson-Aalen curve.

How can you do this in R?

- d) Formulate a result on the asymptotic behavior of the estimator as the number n of processes tends to infinity. Consider in particular the case when $\tau_i \equiv \tau$ for all i .

Exercise E1.2

This exercise is based on the book ASAUR. In addition to the R-package 'survival' you will here need the package 'asaaur' which contains several data sets from this book.

Example 1.2 p. 6 in ASAUR introduces the data "gastricXelox".

- a) For the above data, use R to determine how many patients, N , had the event (death or progression). Then compute the total observation time for all patients, R , and consider the ratio

$$\frac{N}{R} = \frac{\text{“occurrences”}}{\text{“exposure”}}$$

which in Exercise 1.2 p. 11 in ASAUR is called *the event rate per person-week*. What is the interpretation of this quantity?

- b) Run the R-code on page 30 in ASAUR where the above data are analyzed by the Kaplan-Meier estimator. Give an interpretation of what you see from the plot.
- c) Do the corresponding analysis using the Nelson-Aalen estimator. Give an interpretation of what you see from the plot. What is the connection with the quantity calculated in question (a)?