On the Use of Diagonal Control vs. Decoupling for Ill-Conditioned Processes

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Abstract

In this paper we argue that ill-conditioned processes do not necessarily preclude the use of decoupling control. This applies also if the process is ill-conditioned in the cross-over frequency range of a diagonal controller. Therefore, the rule to never use a decoupler for ill-conditioned plants is misleading. The investigation is founded on a discussion of the performance sensitivity of ill-conditioned systems and observations made from analyzing a semi-realistic model of a distillation column.

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1 Introduction

Decouplers and diagonal controllers may be viewed as two extreme classes of controllers for multi-variable control. Diagonal controllers are acknowledged for their simplicity and robustness. One severe limitation is that they often sacrifice performance. A decoupler, on the other hand, may provide good performance – especially nominally – but the decoupling may be non-robust. Decouplers may be particularly non-robust when applied to ill-conditioned plants. The purpose of this paper is to show that this, however, does not necessarily discard the use of decouling control.

High-purity distillation columns using the reflux and vapor flow-rates for two point composition control are always ill-conditioned at low frequencies. As a consequence of this there has been a widespread discussion in the distillation column community whether decoupling is a viable approach to this control problem (Luyben 1970 [10], Arkun & Palazoblu 1984 [11], Skogestad & Morari 1987 [9]). In Skogestad & Morari (1987) [9] it was showed that large relative gain array or RGA - values in the cross-over region lead to severe robustness problems with respect to input uncertainty when a decoupler is used.

Initially, the RGA-matrix (Bristol 1966 [1]) was introduced as an aid to facilitate pairing for decentralized control. It has later been used as a measure for the ill-conditionedness of a process. The RGA-matrix was originally defined for steady state conditions and a physical interpretation can be given at steady-state. The RGA-matrix has later been extended to include dynamic conditions. An overview of the properties of the frequency-dependent RGA is given in Hovd & Skogestad (1992) [6].

This paper continues by first clarifying some issues on the use of diagonal control vs. decoupling for ill-conditioned processes. Thereafter, this is highlighted through a comprehensive investigation on a semi-realistic model of a distillation column. Finally, the findings are discussed and some conclusions are formulated.

2 Performance Sensitivity because of Ill-Conditioning

In Skogestad & Morari (1987) [9] the difficulties of ill-conditioning are related directly to the RGA matrix, $\{\lambda_{ij}\}$, of a process. A process is ill-conditioned if $\lambda_{ij} \gg 1$. This is useful because inherent control limitations related to ill-conditionedness of a process can be investigated by using the RGA-matrix.

To illustrate this, assume a decoupling controller $C(s) = G^{-1}(s)C_D(s)$, where $G(s)$ and $C_D(s)$ are the process model and a diagonal controller, respectively, and a diagonal input uncertainty matrix $\Delta_I = diag\{\Delta_1, \Delta_2\}$. The error matrix is, in the 2×2 case, given by

$$\begin{aligned}
G(s)\Delta_I G^{-1}(s) &= \frac{1}{det(G(s))}\begin{pmatrix} g_{11}g_{22}\Delta_1 - g_{12}g_{21}\Delta_2 & g_{11}g_{12}(\Delta_2 - \Delta_1) \\ g_{21}g_{22}(\Delta_1 - \Delta_2) & g_{22}g_{11}\Delta_2 - g_{12}g_{21}\Delta_1 \end{pmatrix} \\
&= \begin{pmatrix} \lambda_{11}\Delta_1 + \lambda_{12}\Delta_2 & -\lambda_{11}\frac{g_{12}}{g_{22}}(\Delta_2 - \Delta_1) \\ \lambda_{11}\frac{g_{21}}{g_{11}}(\Delta_1 - \Delta_2) & \lambda_{21}\Delta_1 + \lambda_{22}\Delta_2 \end{pmatrix} \qquad (1)
\end{aligned}$$

where λ_{ij} are elements of the RGA-matrix of $G(s)$.

As we see from (1) the diagonal error terms are directly related to the RGA elements of the process. It is important to note that large RGA-entries do not necessarily pose a control problem if they occur in a frequency range where either $\underline{\sigma}(GC) >> 1$ or $\overline{\sigma}(GC) << 1$. In the first case the uncertainty sensitivity is reduced by feedback. In the high-frequency range the uncertainty is reduced by the open-loop gain provided we make the mild assumption that the compensated process GC_D is strictly proper. There will, however, inevitably be a frequency range – the intermediate frequency range – where the feedback properties are sensitive to model errors. The consequence of this is that control limitations because of ill-conditionedness refers to the value of the RGA-elements in the intermediate frequency range and not large RGA-elements in general.

To further elaborate it is useful to discuss the sensitivity function using the decoupler defined above.

$$S_P = [I + (I + G\Delta G^{-1})C_D]^{-1} \qquad (2)$$

We observe from (2) that $\overline{\sigma}(S_P) \to 0$ as $\underline{\sigma}(C_D) \to \infty$. If the controller includes integral control the sensitivity function will go to 0 irrespective of the controller at steady state. Similarly, at high frequencies $\overline{\sigma}(S_P) \to I$ since $\overline{\sigma}(C_D) \to 0$ if the compensated plant GC_D is strictly proper. Hence, at the two extremes steady-state and infinity frequency the feedback properties will be independent of the controller provided it has integral action and the loop transfer function is strictly proper. These issues are treated in Freudenberg (1990) [4].

A process with large RGA-values in the cross-over region will be inherently difficult to control irrespectively of the controller used. However, ill-conditioning is a relative term. There are borderline cases where decoupling can be used. This may be the situation when the ill-conditioning declines over increasing frequency. A decoupler may be able to push the bandwidth into a higher frequency range than diagonal control. However, a decoupler will display larger difference between nominal and robust performance. In a later to come example we show that the robust performance may not be too different for the two types of controllers. The reason for this is that the diagonal controller exhibits worse nominal performance than the decoupler.

Based on the above we claim that there are situations where it is incorrect to discard the potential use of a decoupler based on ill-conditioning in the bandwidth area of a diagonal controller.

3 Example

The example is defined in Skogestad & Lundstrøm (1990) [8]. The nominal model is given by:

$$G(s) = \begin{pmatrix} \frac{87.8}{194s+1} & -\frac{87.8}{194s+1} + \frac{1.4}{15s+1} \\ \frac{108.2}{194s+1} g_L(s) & -\frac{108.2}{194s+1} - \frac{1.4}{15s+1} \end{pmatrix} \quad g_L(s) = \frac{1}{[1+(2.46/5)s]^5} \tag{3}$$

This is a low-order approximation to a model consisting of 82 states. It is a model of a high-purity distillation column which contains liquid flow dynamics represented by $g_L(s)$. This is usually denoted the LV-configuration where the control inputs are the reflux flow rate, u_1, and the boilup rate, u_2. The process outputs are the distillate concentration, y_1, and the bottom product composition, y_2. The real system, $G_P(s)$, is defined in the following form

$$G_P(s) = G(s)(I + w_I(s)\Delta), \qquad \overline{\sigma}(\Delta) < 1 \ \forall \omega \geq 0 \tag{4}$$

$$w_I(s) = 0.2(5s+1)/(0.5s+1) \tag{5}$$

where $w_I(s)\Delta$ defines the uncertainty associated with the nominal model. We observe that the uncertainty is 20% at steady-state and increases to 200% for large frequencies.

The matrix Δ may be either diagonal or full, and the corresponding input uncertainty is referred to as either diagonal or full, respectively. In physical terms the above model uncertainty is approximately equivalent to

$$G_P(s) = G(s) \begin{bmatrix} l_1 e^{-\theta_1 s} & 0 \\ 0 & l_2 e^{-\theta_2 s} \end{bmatrix} \tag{6}$$

where $l_i \in [0.8, 1.2]$ and $\theta_i \in [0, 1]$, $\quad i \in \{1, 2\}$.

3.1 System analysis

In Fig.1 the RGA-element, λ_{11}, of the plant is plotted. The plant is ill-conditioned at low frequencies, but the RGA-element drops with increasing frequency. Therefore, we cannot on the basis of the RGA plot alone decide on the ill-conditionedness of the control problem. It is also necessary to know if the bandwidth area is located in a frequency interval where the RGA-elements are small or at lower frequencies where the RGA-elements are large ($\omega > 0.05$).

Based on this it should be quite straightforward to implement decoupling control if the bandwidth is large. If the bandwidth is low, the use of decoupling will introduce problems.

3.2 Control structures

Three controllers will be used to investigate the example process. First, we use two conventional control structures, diagonal PID-control and decoupling. Thereafter, we design a μ-optimal controller. The purpose of this controller is to be reference controller in the sense that it gives a good indication of the best possible performance of a linear controller.

The PID controllers used in the diagonal controller are given in the following form:

$$C(s) = diag\{c_j(s)\} \qquad c_j(s) = k_j \frac{1 + T_{i_j} s}{T_{i_j} s} \frac{1 + T_{D_j} s}{1 + 0.1 T_{D_j} s} \qquad j \in \{1, 2\} \tag{7}$$

We choose a decoupler as:

$$C(s) = G^{-1}(s) G_{diag} C_D(s) \tag{8}$$

G_{diag} denotes the diagonal entries of G in (3) while C_D is a diagonal PID controller in the form (7).

The μ-optimal controller is designed using the DK-iteration (Doyle 1983 [2], Safonov 1983 [5]).

3.3 Performance criteria

Before evaluating the controllers a set of frequency domain performance criteria are specified. These are (i) steady-state offset less than A, (ii) closed-loop bandwidth higher than ω_B, and (iii) amplification of high-frequency noise less than a factor M.

The performance requirements can be encapsulated into the following weight.

$$w_P = \frac{1}{M}\frac{\tau_P s + 1}{\tau_P s + A/M} \tag{9}$$

The performance specification is satisfied if $\overline{\sigma}(w_P S_P) < 1 \;\; \forall \omega \geq 0$.

In the sequel the following performance weights defined in Skogestad & Lundstrøm (1990) [8] will be used. These are modifications of the above performance specifications.

$$w_{P_1} = \frac{1}{M_1}\frac{\tau_{P_1} s + 1}{\tau_{P_1} s} \tag{10}$$

where $M_1 = 2$ and $\tau_{P_1} = 10$, and

$$w_{P_2} = \frac{1}{M_2}\frac{(\tau_{P_2} s + 1)}{\tau_{P_2} s}\frac{(\tau_{P_2} s + 1)}{(\tau_{P_2} s + 1/\alpha)} \tag{11}$$

where $M_2 = 2$ and $\tau_{P_2} = 16.7$ and $\alpha = 4$.

We observe that the performance weight w_{P_2} contains an additional term compared to w_{P_1}. The reason for this is to penalize intermediate frequencies heavier. A plot of the inverse of the performance weights is presented in Fig. 2. As we see performance weight w_{P_2} penalizes low to intermediate frequencies more than the weight w_{P_1}.

We note that the performance measure implies a bandwidth of approximately $0.1 rad/min$. Viewing the RGA-plot in Fig. 1 we observe that the controller is not ill-conditioned for this frequency.

In the following the controller parameters are calculated by a computer search, minimizing the peak value of robust performance corresponding to the weights w_{P1} and w_{P2}, respectively.

It should be noted that in the following all the performance considerations are taken with respect to the H_∞-norm.

3.4 Results with diagonal input uncertainty

In this section we investigate the use of the controllers assuming diagonal input uncertainty to the process given by (4). The controller parameters are given in Table 1.

Fig. 3 shows the nominal performance, referring to w_{P_1}, for the controllers. From a maximum peak value viewpoint both the decoupler and the μ-optimal controller provide equally good performance. However, this criterion does not lend itself to this problem. This is due to the fact that it is impossible to achieve a controller with nominal performance peak value less than $1/M = 0.5$. The performance will approach $1/M = 0.5$ as $s \rightarrow \infty$ for any controller. This applies whether model uncertainty is present or not. Therefore, it is unachievable to obtain a controller with nominal performance peak value less than $1/M = 0.5$. As can be expected the decoupler provides significantly better nominal performance than the μ-optimal controller from low to intermediate frequencies.

The robust performance for the decoupler along with that of a diagonal PID controller given in Skogestad & Lundstrøm (1990) [8] is plotted in Fig. 4. First, we observe that the robust performance curve for the μ-optimal controller is very flat over a large frequency range and that it suddenly approaches $1/M = 0.5$ at higher frequencies. This indicates that the "μ-optimal" controller is close to the true μ-optimal controller. As we see both the PID controller as well as the decoupler provide close to μ-optimal performance.

Fig. 5 shows the nominal performance, referring to w_{P_2}, for the controllers. As expected the decoupler provides better nominal performance than the PID controller from low to intermediate frequencies.

In Fig. 6 we show the robust performance curves for the controllers for w_{P_2}. In this case the decoupler seems somewhat better than the PID controller. This illustrates the advantage of a multivariable controller over a diagonal one. Performance specification w_{P_2} requires more gain from low to intermediate frequencies. This should be performed in parallel with keeping the cross-over frequency of the largest singular value limited. This is more difficult to achieve with a diagonal controller. This is the classical loop-shaping problem (Doyle & Stein 1981 [3]). However, if the plant is ill-conditioned in the cross-over region this becomes impossible to achieve also with a multivariable controller (Skogestad & Morari 1987 [9]). In this case the plant is not ill-conditioned in the bandwidth area and the classical reasoning applies. The parallel controller provides approximately the same performance as the PID controller in terms of robust performance peak value. However, much better robust decoupling is achieved in the intermediate frequency range.

Fig. 5 shows the corresponding nominal performance curves. As expected the decoupler provides better nominal performance than the PID controller from low to intermediate frequencies.

The prior investigation assumes diagonal uncertainty in the control inputs as shown in (6). A similar study with a full block input uncertainty on this distillation example gave similar results to the above findings, cf. Gjøsæter (1995) [7]. In the latter case the condition number of the plant plays the same role as a sensitivity measure, as the RGA does for the diagonal input uncertainty case.

3.5 Time domain simulations

As seen from the RP μ-plots in Figs. 4 and 6 the three controllers do not display much difference in performance. Fig. 7 shows the nominal response for a set-point change in y_1 at $t = 0$. The reference signal is filtered by a time constant of 5 min. Similar plot is shown in Fig. 8 with 20% input gain model uncertainty. As we see the input gain uncertainty introduced here does not seriously affect the performance of the decoupler. Remark that also the PID and μ-optimal controllers are affected by the introduction of input uncertainty, but to lesser extent. As can be expected the control inputs are somewhat more aggresive for the decoupler than in the two other cases.

3.6 Reducing the bandwidth

In the previous problem definition we have only one minute time delay in the uncertainty weight. Therefore, we can have a closed loop bandwidth in a frequency range where the plant is not ill-conditioned. Now the time delay range is increased by choosing $\theta_i \in [0, 6]$ in the uncertainty weight, see (6). In this case we have to accept a drastic reduction in performance and $\tau_P = 55$ is chosen for the performance weight, see (10).

In Table. 2 the robust performance peak values along with the controller parameters for the PID controller and the decoupler are given. The robust performance peak value, μ_{RP}, is larger than one for the decoupler. Robust stability is a special case of robust performance, and is achieved automatically if $\mu_{RP} \leq 1$. The converse is not necessarily true, so robust stability has, hence, been checked for the decoupler.

Both the PID controller and the decoupler provide poor performance in this case. However, the relative difference in performance between the decoupler and the diagonal controller is not extreme. In broad terms, they provide approximately equally poor performance. In Fig. 9 we view the open-loop singular values of GC pertaining to the PID and decoupler, respectively. As we see the decoupler has a higher bandwidth than the PID controller. A further reduction of the bandwidth will be in favor of the PID controller. However, such an analysis is not too interesting because the performance is already extremely bad with the increased time delay.

4 Discussion

In this simulation study special types of decouplers and PID controllers are chosen. Therefore, it is speculative to draw general conclusive inferences with respect to the performance of diagonal control vs. decoupling. Also, the results seem to be somewhat dependent on how performance is defined. However, the results indicate that PID control and ideal decoupling yield approximately the same results in terms of robust performance – worst case performance for the example in question. Large closed-loop bandwidth is in favor of the decoupler. If the bandwidth is chosen sufficiently large the two controller structures provide approximately equally good performance. If the controllers are detuned they seem to provide equally poor performance.

μ-analysis is a worst case analysis. It minimizes the H_∞-norm with respect to the structured matrix Δ. A worst case analysis is particularly useful for ill-conditioned systems in the cross-over frequency range. This is due to the fact that such systems may provide large difference between nominal and robust performance. It is interesting to notice that the decoupler provides much better nominal performance. As a result of this it is reasonable to believe that the decoupler "on the average" provides better results. This is to say a worst case criterion is not the only reasonable criterion for controller performance.

The relative difference in terms of robust performance is larger for performance specification w_{P_2} than for w_{P_1}. This illustrates the potential advantage of a multivariable controller over a diagonal controller. Performance specification w_{P_2} requires more gain from low to intermediate frequencies. This is easier achieved with a multivariable controller without simultaneously increasing the cross-over frequency of the largest singular value too much. This is due to the interactions of the process.

The example shows that one should exhibit caution when interpreting inherent performance limitations from an RGA plot. In this case the plant has large RGA values below the bandwidth area. This does, however, not preclude the use of a decoupler. In this case a decoupler can be used with results comparable to that of PID control without resorting to excessive loop gain at lower frequencies. It is not the sensitivity of the controller seen in isolation that should be the focus of analysis rather the feedback properties – the sensitivity and complementary sensitivity functions.

5 Conclusions

It is shown that – from a performance viewpoint – exact decoupling may be possible also for a high-purity distillation column if the RGA-values are not too large in the cross-over region. Ideal decoupling plus PID control yields approximately the same robust performance (worst-case performance) as diagonal PID control for the process studied here.

This example study shows that care should be taken when discarding the possibility of decoupling design when viewing the RGA-values of the plant. The magnitude of the RGA-values should be related to the loop gain. Large RGA-values at lower frequencies are not sufficient to rule out the possibility of good decoupling design. The expected bandwidth – particularly nominally – may be larger for a decoupler than a diagonal controller. Therefore, it is in general incorrect to inspect the RGA-values only in the expected bandwidth frequency range of a diagonal controller. This is not to say that the use of a decoupling control is recommendable for high-purity distillation columns. There may be other problems that make decoupling control less attractive.

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Controller:	weight	μ_{RP}	k_1	k_2	T_{i_1}	T_{i_2}	T_{D_1}	T_{D_2}
PID	w_{P_1}	0.80	0.38	0.36	6.49	5.80	1.13	0.91
Ideal decoupler	w_{P_1}	0.78	0.84	0.62	26.7	29.2	0.52	0.48
μ-Optimal	w_{P_1}	0.71	–	–	–	–	–	–
PID	w_{P_2}	0.90	0.74	0.55	6.09	4.46	1.06	0.74
Ideal decoupler	w_{P_2}	0.81	0.94	0.70	13.8	14.5	0.53	0.45
μ-Optimal	w_{P_2}	0.72	–	–	–	–	–	

Table 1: PID controller parameters for the diagonal PID controller and the ideal decoupler plus PID control with diagonal input uncertainty.

Controller:	weight	μ_{RP}	k_1	k_2	T_{i_1}	T_{i_2}	T_{D_1}	T_{D_2}
PID	w_{P_1}	1.0	0.14	0.12	16.6	14.3	3.14	3.54
Ideal decoupler	w_{P_1}	1.12	0.18	0.13	75.6	51.5	6.49	6.57
μ-Optimal	w_{P_1}	0.9	–	–	–	–	–	–

Table 2: PID controller parameters for the diagonal PID controller and the ideal decoupler plus PID control.

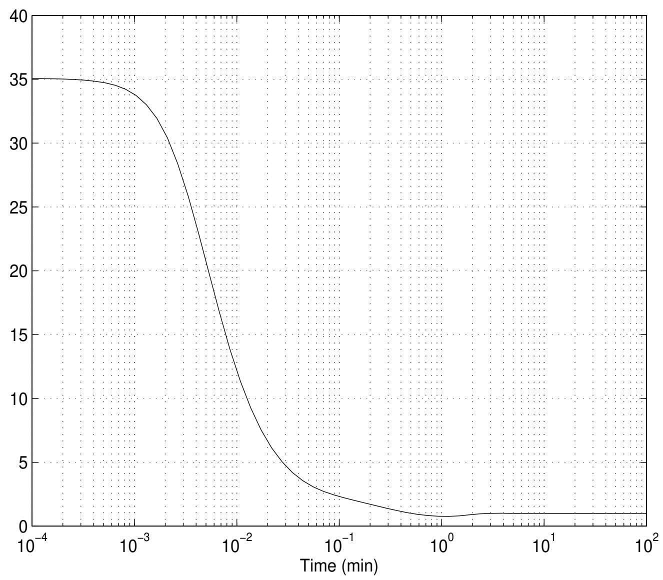

Figure 1: A plot of the magnitude of the $|\lambda_{11}|$ element (in dB) of the plant vs. frequency (in rad/min).

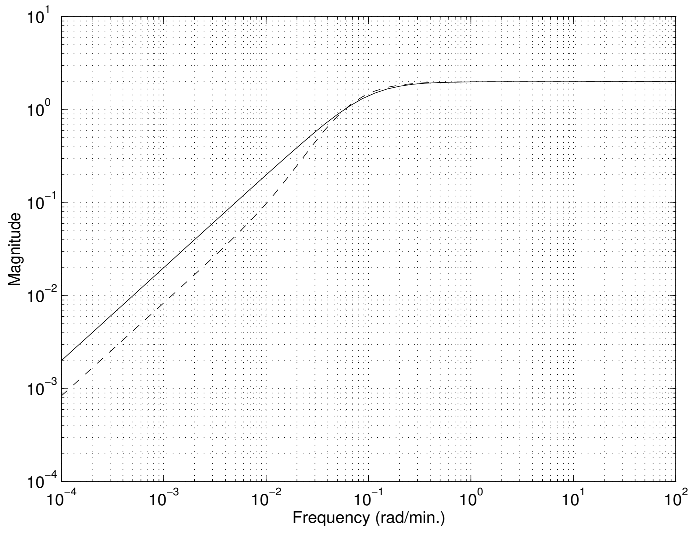

Figure 2: The performance weights $|w_{P_1}|^{-1}$ (solid) and $|w_{P_2}|^{-1}$ (dashed).

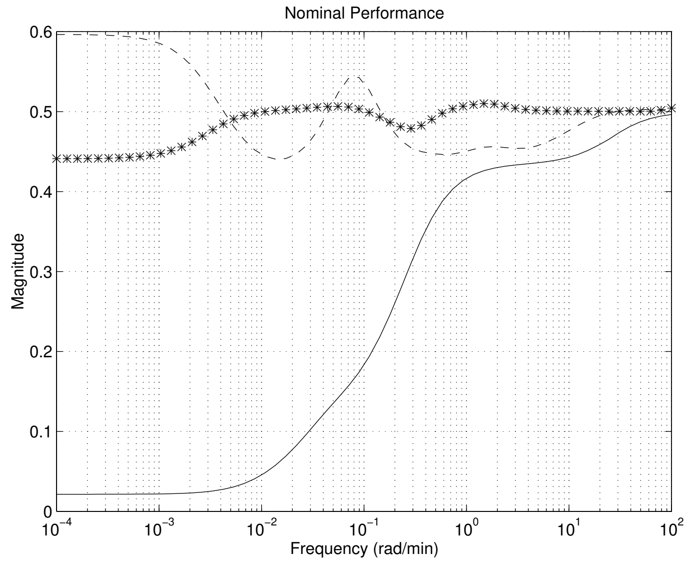

Figure 3: Nominal performance plots for the diagonal PID controller (dashed), the ideal decoupler (solid) and the μ-optimal controller (star) for performance weight w_{P_1}.

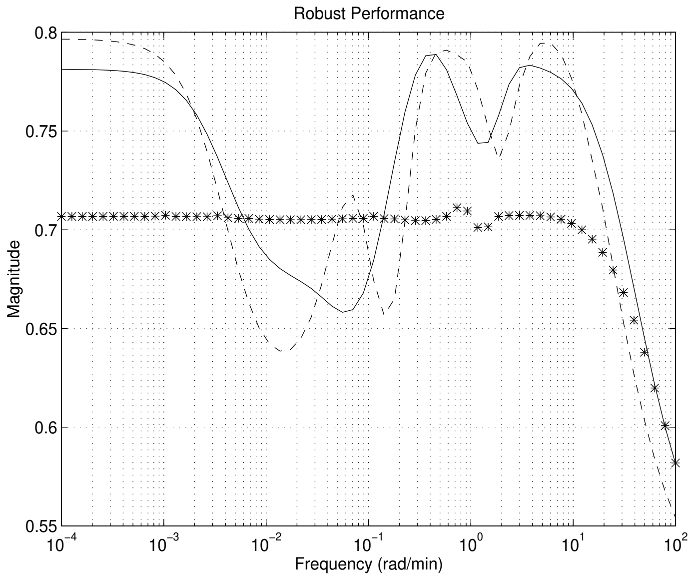

Figure 4: Robust performance plots for the decoupler (solid) and the diagonal PID controller (dashed) and the μ-optimal controller (star) for performance weight w_{P_1}.

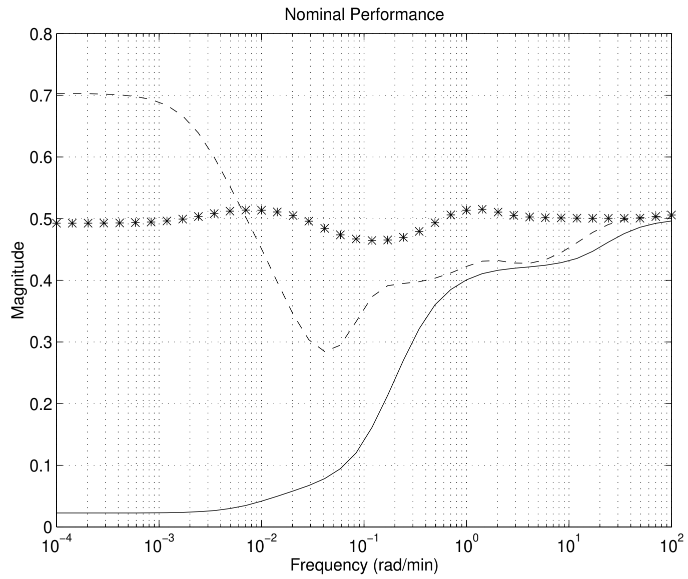

Figure 5: Nominal performance plots for the decoupler (solid), the diagonal PID controller (dashed) and the μ-optimal controller (star) for the weight w_{P_2}.

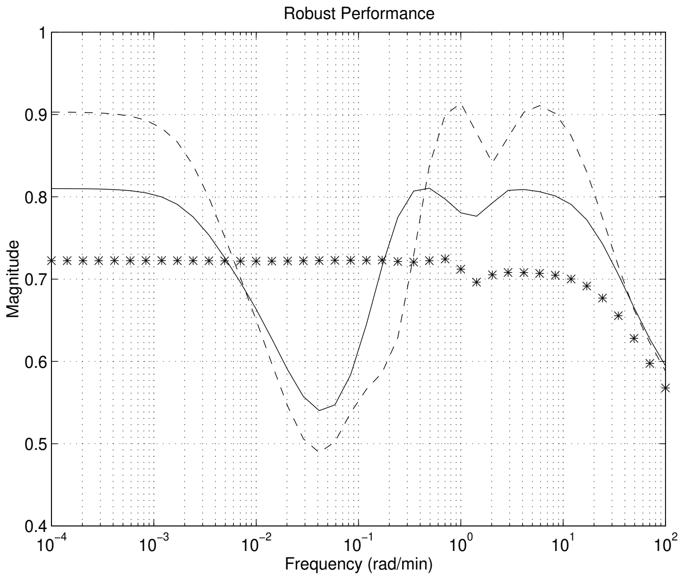

Figure 6: Robust performance plots for the ideal decoupler (solid), the diagonal PID controller (dashed) and the μ-optimal controller (star) for the weight w_{P_2}.

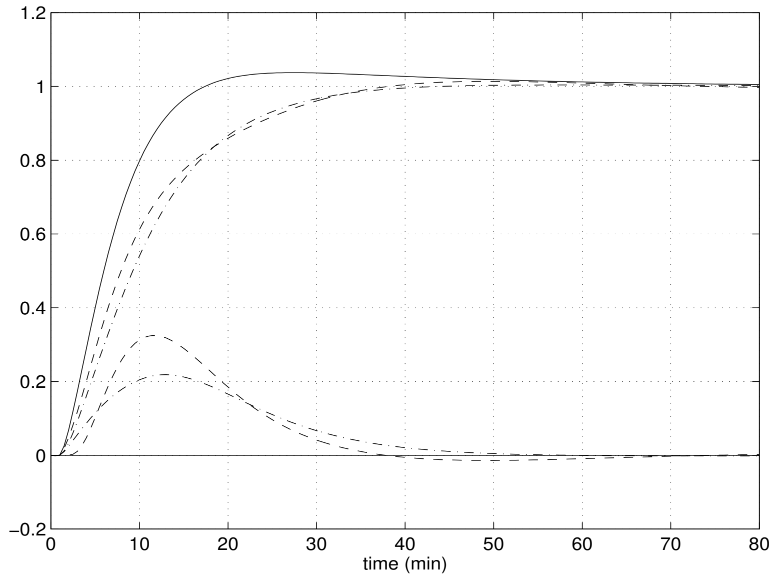

Figure 7: The diagonal PID controller (dashed), the ideal decoupler (solid) and the μ-optimal controller (dashed dotted) for the nominal case.

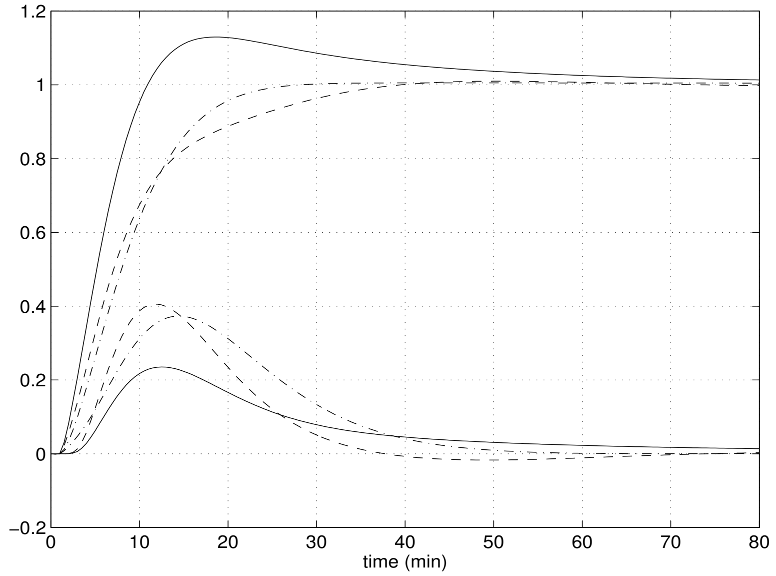

Figure 8: The diagonal PID controller (dashed), the ideal decoupler (solid) and the μ-optimal controller (dashed dotted) for the perturbed case ($\theta_1 = \theta_2 = 1$ and $k_1 = 1.2, k_2 = 0.8$)

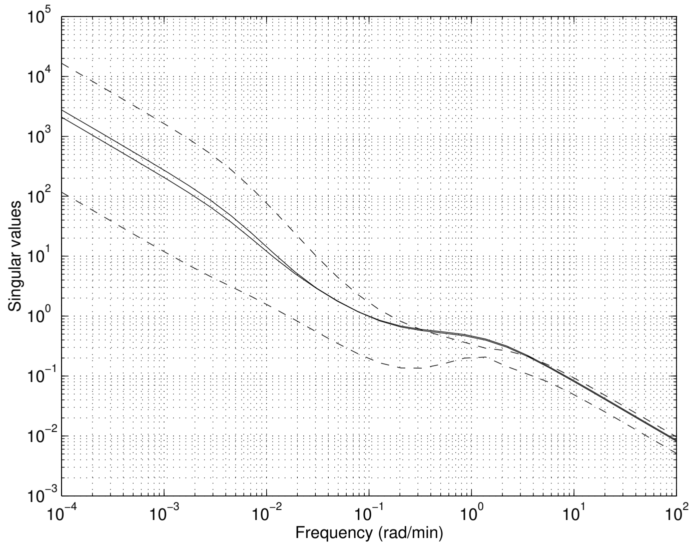

Figure 9: The open-loop singular values for GC of decoupler (solid) and the PID controller (dashed), respectively.