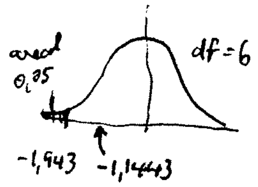
EKSAMEN ST0201 2007V – LØSNINGSSKISSE

1a. $H_0: \mu_x \geq \mu_Y$, $H_1: \mu_x < \mu_Y$. Bruker uparet T-test.

$$s_p^2 = \frac{3{,}0875+2{,}0675}{4+4-2} = 0{,}8592, \quad t = \frac{11{,}175-11{,}925}{\sqrt{0{,}8592}\sqrt{1/4+1/4}} = -1{,}1443.$$



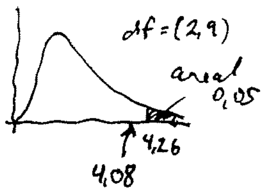
Forkaster H_0 hvis T er <u>liten</u>, T-fordelt med $8-2=6$ frihetsgrader hvis H_0 er sann. $-t_{0,05} = -1{,}943$. Forkaster <u>ikke</u> H_0.

b. $t_{0{,}025} = 2{,}447$ (df=6). 95%-konf.int. har grenser $11{,}175-11{,}925 \pm 2{,}447\cdot\sqrt{0{,}8592}\sqrt{1/4+1/4}$ $= -0{,}75 \pm 1{,}604$, dvs. 95%-konf.int. $[-2{,}35,\ 0{,}85]$.

c. Enveis variansanalyse. Gjennomsnitt av alle 12 obs.:
$(11{,}175 + 11{,}925 + 12{,}8)/3 = 11{,}967$.

$$SS_G = 4\cdot(11{,}175-11{,}967)^2 + 4\cdot(11{,}925-11{,}967)^2 + 4\cdot(12{,}8-11{,}967)^2 = 5{,}292$$

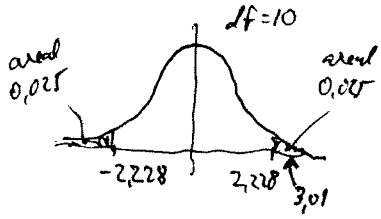
$$SS_E = 3{,}0875 + 2{,}0675 + 0{,}68 = 5{,}835$$



$f = \frac{SS_G/(3-1)}{SS_E/(12-3)} = 4{,}08$. Forkaster $H_0: \mu_x = \mu_Y = \mu_Z$ hvis F er <u>stor</u>, F-ford. med 2 og 9 fr.gr. hvis H_0 er sann.
$f_{0{,}05} = 4{,}26$. Forkaster <u>ikke</u> H_0.

d. Lineær regresjon. $\hat\beta = \frac{6{,}5}{8} = 0{,}8125$,
$\hat\alpha = 11{,}967 - 0{,}8125\cdot 2 = 10{,}342$. Skal teste $H_0: \beta = 0$ mot $H_1: \beta \neq 0$.
Trenger SS_E. $SS_T = SS_R + SS_E$, $1 = \frac{SS_R}{SS_T} + \frac{SS_E}{SS_T} = r^2 + \frac{SS_E}{SS_T}$,
dvs. $SS_E = (1-r^2)SS_T$. $r^2 = \frac{6{,}5^2}{8\cdot 11{,}127} = 0{,}475$, $SS_T = 11{,}127$,
dvs. $SS_E = (1-0{,}475)\cdot 11{,}127 = 5{,}846$.

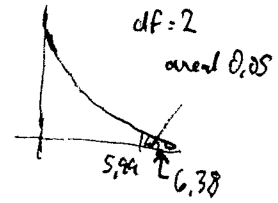
Testobs.: $t = \frac{\hat\beta}{\sqrt{\frac{SS_E/(n-2)}{\Sigma(x_i-\bar{x})^2}}} = \frac{0{,}8125}{\sqrt{\frac{5{,}846/10}{8}}} = 3{,}01.$



Forkaster H_0 hvis T er <u>liten eller stor</u>, T-fordelt med 10 fr.gr. hvis H_0 er sann. $t_{0{,}025} = 2{,}228$. Forkaster H_0.

2a. Modelltest.

	0	1	2	Sum
Obs. antall, X	27	22	6	55
Hypotesesanns.	0,64	0,26	0,10	1
Hypoteseforv., E	35,26	14,10	5,64	55
$(X-E)^2/E$	1,93	4,42	0,02	6,38



Kritisk verdi: $\chi_{0{,}05} = 5{,}99$ (df=2). Forkaster H_0.

b. $L = \prod_{i=1}^{n} \frac{c^{x_i}}{1+c+c^2} = (1+c+c^2)^{-n} c^{\Sigma x_i}$, $\ln L = -n\ln(1+c+c^2) + (\Sigma x_i)\ln c$,

$\frac{d\ln L}{dc} = -n\frac{1+2c}{1+c+c^2} + \frac{\Sigma x_i}{c}$. $\frac{d\ln L}{dc} = 0$: $\frac{1+2c}{1+c+c^2} = \frac{\bar{x}}{c}$, $c + 2c^2 = \bar{x} + \bar{x}c + \bar{x}c^2$,

$(2-\bar{x})c^2 + (1-\bar{x})c - \bar{x} = 0$, $c = \frac{-1+\bar{x} \pm \sqrt{(1-\bar{x})^2 + 4(2-\bar{x})\bar{x}}}{2(2-\bar{x})} = \frac{-1+\bar{x} \pm \sqrt{1+6\bar{x}-3\bar{x}^2}}{2(2-\bar{x})}$.

Sannsynlighetsmaks. estimat: $\hat{c} = \frac{-1+\bar{x}+\sqrt{1+6\bar{x}-3\bar{x}^2}}{2(2-\bar{x})}$

Her: $\bar{x} = \frac{27\cdot 0 + 22\cdot 1 + 6\cdot 2}{55} = 0{,}618$. Gir $\hat{c} = 0{,}54$ (velger positivt fortegn for å få positivt estimat).