STO101 2008V. Løsningsskisse

1a. La levetida være X. $X \sim N(100, 20)$.

$P(X \geq 120) = P\left(\frac{X-100}{20} \geq \frac{120-100}{20}\right) = P(Z \geq 1) = 0{,}1587$ (tabell D.3) $(Z \sim N(0,1))$

b. $P(X \geq 120 \mid X \geq 100) = \frac{P(X \geq 120 \cap X \geq 100)}{P(X \geq 100)} = \frac{P(X \geq 120)}{P(X \geq 100)} = \frac{0{,}1587}{0{,}5} = 0{,}3174$

c. La $X_1, \dots, X_{10}$ være levealdrene.

P(den som lever kortest lever i ≥ 90 døgn) = P(alle lever i ≥ 90 døgn)

$= P(X_1 \geq 90 \cap X_2 \geq 90 \cap \dots \cap X_{10} \geq 90) = (P(X_1 \geq 90))^{10} = \left(P\left(\frac{X_i - 100}{20} \geq \frac{90-100}{20}\right)\right)^{10}$

$= (P(Z \geq -0{,}5))^{10} = 0{,}6915^{10} = 0{,}025.$

d. La levetida til B være Y. $Y \sim N(115, 25)$.

$Y - X \sim N(115-100, \sqrt{20^2+25^2}) = N(15, 32{,}02)$.

$P(Y \geq X) = P(Y - X \geq 0) = P\left(\frac{Y-X-15}{32{,}02} \geq \frac{0-15}{32{,}02}\right) = P(Z \geq -0{,}47) = 0{,}68.$

2a. Alle 15 kjøringene må være uavhengig av hverandre. X har parametre $n=10$, $p=0{,}9$, Y har parametre $n=5$, $p=0{,}9$, og $X+Y$ har parametre $n=15$, $p=0{,}9$.

b. $P(X+Y \leq 13) = 1 - P(X+Y=14) - P(X+Y=15) = 1 - \binom{15}{14} \cdot 0{,}9^{14} \cdot 0{,}1 - 0{,}9^{15}$

$= 1 - 0{,}343 - 0{,}206 = 0{,}45.$

c. $P(X=9 \mid X+Y=11) = \frac{P(X=9 \cap X+Y=11)}{P(X+Y=11)} = \frac{P(X=9 \cap Y=2)}{P(X+Y=11)} = \frac{P(X=9) \cdot P(Y=2)}{P(X+Y=11)}$

$= \frac{\binom{10}{9} \cdot 0{,}9^9 \cdot 0{,}1 \cdot \binom{5}{2} \cdot 0{,}9^2 \cdot 0{,}1^3}{\binom{15}{11} \cdot 0{,}9^{11} \cdot 0{,}1^4} = \frac{\binom{10}{9}\binom{5}{2}}{\binom{15}{11}} = \frac{10 \cdot 10}{\frac{15 \cdot 14 \cdot 13 \cdot 12}{1 \cdot 2 \cdot 3 \cdot 4}} = \frac{20}{273} = 0{,}073$

d. $P(X=x \mid X+Y=11) = \frac{P(X=x \cap X+Y=11)}{P(X+Y=11)} = \frac{P(X=x \cap Y=11-x)}{P(X+Y=11)} = \frac{P(X=x) \cdot P(Y=11-x)}{P(X+Y=11)}$

$= \frac{\binom{10}{x} \cdot 0{,}9^x \cdot 0{,}1^{10-x} \cdot \binom{5}{11-x} \cdot 0{,}9^{11-x} \cdot 0{,}1^{5-(11-x)}}{\binom{15}{11} \cdot 0{,}9^{11} \cdot 0{,}1^4} = \frac{\binom{10}{x}\binom{5}{11-x} \cdot 0{,}9^{x+11-x} \cdot 0{,}1^{10-x+5-11+x}}{\binom{15}{11} \cdot 0{,}9^{11} \cdot 0{,}1^4}$

$= \frac{\binom{10}{x}\binom{5}{11-x}}{\binom{15}{11}}$, dvs. hypergeometrisk fordeling med parametre

$N=15$, $M=10$, $n=11$.