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Department of Mathematical Sciences

English

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ST0101 Probability with applications

Thursday 22 May 2008
9:00–13:00
Permitted aids: Any written and printed material. One calculator.
Grades to be announced: 12 June 2008

The final examination consists of two parts:

1. The problems on the next page.
2. Appendix with a multiple choice questionnaire.

The Appendix is to be submitted with the form filled in together with the answer to part (1). Part (1) and (2) count equally in the evaluation of the final examination.

In addition to the final examination the mid-term examination counts 20% if it is advantageous to the candidate.

In the evaluation of part (1) (next page) each of the eight points counts equally.

In part (1) you should demonstrate how you arrive at your answers (e.g. by including intermediate answers or referral to theory). Answers based on calculator or table look-up only will not be accepted.

Problem 1

An insect species, A, has a survival time that is normally distributed with mean $\mu = 100$ days and standard deviation $\sigma = 20$ days.

a) Find the probability that an insect lives for 120 days or more.

b) Find the conditional probability that an insect lives for 120 days or more given that it was alive when 100 days had passed.

c) We have ten individuals of the species. Assume that their survival times are independent. Find the probability that the one surviving the shortest time survives for 90 days or more.

Another insect species, B, has a survival time that is normally distributed with mean $\mu = 115$ days and standard deviation $\sigma = 25$ days.

d) One individual of species A and one individual of species B are hatched at the same time. Find the probability that the individual of species B lives longer than the individual of species A.

Problem 2

A laboratory assistant runs an reaction 15 times: 10 times on Monday and 5 times on Tuesday. Each run is successful with probability 0.9. Let X be the number of times the reaction is successful on Monday and Y the number of times the reaction is successful on Tuesday.

a) Which assumptions must be done for X, Y and $X + Y$ to be binomially distributed? What are the parameters of the three binomial distributions?

Now we assume that X, Y and $X + Y$ are binomially distributed, and that X and Y are independent.

b) Find $P(X + Y \leq 13)$.

c) Find the conditional probability that the reaction is successful exactly 9 times on Monday given that it is successful 11 times in total on Monday and Tuesday.

d) Find an expression for $P(X = x \mid X + Y = 11)$, that is, the conditional probability that the reaction is successful exactly x times on Monday given that it is successful 11 times in total on Monday and Tuesday. Let Z be a random variable such that $P(Z = x) = P(X = x \mid X + Y = 11)$ for $x = 6, 7, 8, 9, 10$. What is the probability distribution of Z called?