THE UNIVERSITY OF TRONDHEIM
College of Arts and Science

English

Examination in : MA 001 ELEMENTARY MATHEMATICS WITH APPLICATIONS

Date : January 6, 1993

Duration : 6 hours
Points (credits) : 5

Permitted aids:
Calculator, text book

Grades to be announced on: January 20, 1993

This 2-page examination paper consists of 6 problems.

Problem 1.

The radioactive isotope thorium-234 disintegrates in accordance with the differential equation

$$(*) \qquad \frac{dy}{dt} = -ay$$

where a is a positive constant and $y = y(t)$ is the amount of thorium-234 at the time t.

a) Solve the differential equation $(*)$ and find the half life of thorium-234 (in days) given that 100 mg of the isotope after 7 days are reduced to 82,04 mg.

b) In a container holding 100 mg thorium-234 at the time $t = 0$ 1 mg of the isotope is added each day. Let $y(t)$ be the amount of thorium-234 in the container at the time t. Calculate $y(t)$ and find the limit of $y(t)$ as $t \to \infty$.

Problem 2.

a) Write $z = \frac{1}{2} + \frac{1}{2}\sqrt{3}i$ in the form $re^{iv}, (r \geq 0, v \in [0, 2\pi))$ and find $\sqrt{z}$. Then write $\sqrt{z}$ in the form $x + yi, (x, y \in \mathbf{R})$.

b) Solve the quadratic equation

$$z^2 + 2z + \frac{1}{2} - \frac{1}{2}\sqrt{3}i = 0.$$

Indicate the solutions in the complex plane.

Problem 3.

The population size of a country is increasing by 3% per year. By how many % per year must the gross national product (GNP) increase if the income per capita (=GNP/size of population) is to double in 20 years?

Problem 4.

Solve the system of differential equations

$$x' = 2x - 2y$$
$$y' = 2x - 3y.$$

Problem 5.

A box without lid is to be made with a given total area A of the bottom and the sides. Let x and y denote the edges of the rectangular bottom.

a) Show that the volume V of the box is given by

$$V = \frac{Axy - x^2y^2}{2(x+y)}$$

b) Find x and y such that the volume becomes as large as possible and calculate this volume. (You may assume that such a maximum for V exists.)

Problem 6.

In a model for the spreading of rumours in a society of B individuals one assumes that the number $x = x(t)$ of individuals knowing a certain rumour at the time t satisfies the differential equation

$$(*) \qquad \frac{dx}{dt} = ax(B - x)$$

where a is a positive constant.

a) Let N_0 be the number of individuals knowing the rumour at the time $t = 0$. Find the solution of $(*)$ in terms of a, B, N_0, and t.

b) A rumour about the impending financial collapse of a large airline started to spread among a group of stockbrokers. At the time $t = 0$ 10% of the stockbrokers knew the rumour,and 2 hours later 25% knew it. After how long time did 75% of the stockbrokers know the rumour when we assume that it spread in accordance with $(*)$?

NB. The students will find the examination results (grades) on special notice boards. Unfortunately, telephone inquiries about grades cannot be answered.