1.

Riemannsum: $\int_a^b f(x)dx \approx \sum_{i=0}^{n-1} h \cdot f(x_i)$

$$h = \frac{b-a}{n}, \quad x_i = a + h \cdot i$$

$$\int_\Gamma ds = \int_a^b ||\dot{\underline{x}}(t)|| \, dt$$

Ettersom $\dot{\underline{x}}(t)$ gir tangentvektor til kurven ved tidspunktet t, vil lengden $||\dot{\underline{x}}(t)|| \cdot dx$ for liten dx, gi et greit estimat for en dellengde av kurven. Totallengden kan dermed estimeres:

$$\sum_i ||\dot{\underline{x}}(t_i)|| \cdot dx$$

Lar vi $dx \to 0$

i.e. $\lim_{dx \to 0} \sum_i \|\dot{\mathbf{x}}(t)\| dx = \int_{t_0}^{t_1} \|\dot{\mathbf{x}}(t)\| dx$ $\square$

$$\mathbf{x}(t) = \begin{bmatrix} \ln(1+t^2) \\ 2\arctan(t) \end{bmatrix}$$

$$\Rightarrow \dot{\mathbf{x}}(t) = \begin{bmatrix} \frac{2t}{1+t^2} \\ \frac{2}{1+t^2} \end{bmatrix}$$

$$\Rightarrow \|\dot{\mathbf{x}}(t)\| = \frac{1}{1+t^2}\sqrt{4t^2+4}$$

$$= \frac{2}{\sqrt{1+t^2}}$$

$$\therefore \int_0^2 \frac{2}{\sqrt{1+t^2}} dt = 2\sinh^{-1}(t)\Big|_0^2$$

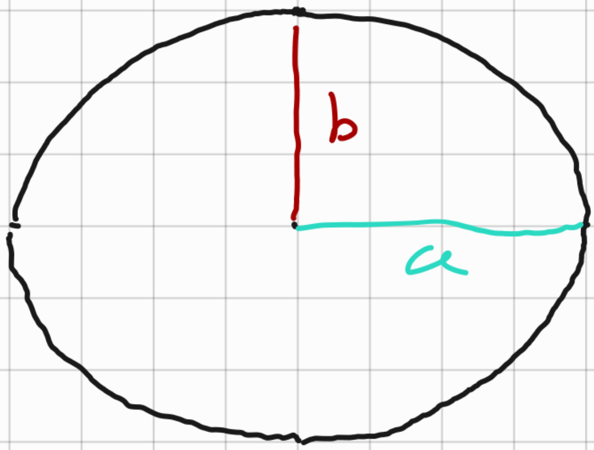
$$= 2\ln\left(\sqrt{t^2+1}+t\right)\Big|_0^2$$

$$= 2\left(\ln(\sqrt{5}+2) - \ln(1)\right)$$

$$= 2\ln(\sqrt{5}+2)$$

Ert $2\sinh^{-1}(2)$

2.



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

En parametrisering er:

$$x(\theta) = \begin{bmatrix} a\cos(\theta) \\ b\sin(\theta) \end{bmatrix}, \quad 0 \leq \theta < 2\pi$$

$$\therefore \quad \mathbb{X}'(\theta) = \begin{bmatrix} -a\sin(\theta) \\ b\cos(\theta) \end{bmatrix}$$

$$\therefore \quad \|\mathbb{X}'(\theta)\| = \sqrt{a^2\sin^2\theta + b^2\cos^2\theta}$$

Dermed er omkretsen til en ellipse gitt ved:

$$\int_0^{2\pi} \sqrt{a^2\sin^2\theta + b^2\cos^2\theta}\, d\theta$$

Omkretsen ble omtrent ≈ 22.1

3. $f(x_1, x_2) = 2 + x_1 x_2 / 10$ (Grunnflate)

$$\mathbb{x}(\theta) = \begin{bmatrix} 4\cos(\theta) \\ 3\sin(\theta) \end{bmatrix}, \quad \theta \in [0, \pi/2]$$

Ettersom arealet kan tenkes som summen av flere rektangler med høyde $f(x_1, x_2)$, og bredde, forklart tidligere som $\|\mathbb{x}'(\theta)\| \, d\theta$

Dersom vi bytter x_1 og x_2 med parametriseringen og summerer opp alle rektanglene får vi:

$$\int_{\Gamma} f\,ds = \int_0^{\pi/2} f(\mathbf{x}(\theta))\,\|\mathbf{x}'(\theta)\|\,d\theta$$

$$f(\mathbf{x}(\theta)) = 2 + \frac{1}{10}\left(4\cos(\theta)\,3\sin(\theta)\right)$$

$$= 2 + \frac{3\cdot 4}{10}\cos\theta\sin\theta$$

Fra før finner vi at:

$$\|\mathbf{x}'(\theta)\| = \sqrt{16\sin^2\theta + 9\cos^2\theta}$$

$$\therefore \int_{\Gamma} f\,ds = \int_0^{\pi/2} \left(2 + \frac{3\cdot 4}{10}\cos\theta\sin\theta\right)\sqrt{16\sin^2\theta + 9\cos^2\theta}\,d\theta$$

$$\approx \underline{\underline{13.16}}$$

4. Vi må starte med å finne parametriseringen av banen til brugden;

$$\mathbb{x}(\theta) = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$$

$$\therefore \text{Plankton} = \int_0^{2\pi} \delta(\mathbb{x}(\theta)) \|\mathbb{x}'(\theta)\| d\theta$$

$$\|\mathbb{x}'(\theta)\| = 1$$

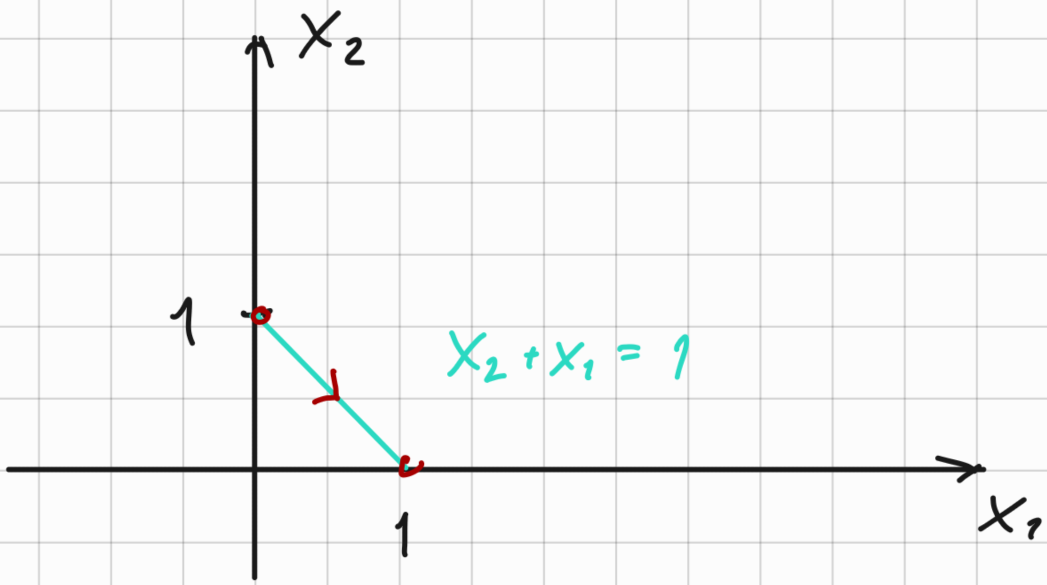
$$\therefore P = \int_0^{2\pi} (1 + \cos\theta \sin\theta) d\theta$$

$$= \int_0^{2\pi} \left(1 + \frac{1}{2}\sin(2\theta)\right) d\theta$$

$$= \left[\theta - \frac{1}{4}\cos(2\theta)\right]_0^{2\pi} = 2\pi - \frac{1}{4} + \frac{1}{4}$$

$$= \underline{\underline{2\pi}}$$

5.



Parametrisering:

$$\mathbf{x}(t) = \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} t = \begin{bmatrix} t \\ 1-t \end{bmatrix}, \quad t \in [0, 1]$$

$$\|\mathbf{x}'(t)\| = \left\| \begin{matrix} 1 \\ -1 \end{matrix} \right\| = \sqrt{2}$$

$$\therefore P = \int_0^1 \left(1 + t(1-t)\right)\sqrt{2}\, dt$$

$$= \sqrt{2} \int_0^1 1 + t - t^2 \, dt$$

$$= \sqrt{2}\left[t + \frac{1}{2}t^2 - \frac{1}{3}t^2\right]_0^1$$

$$= \sqrt{2}\left[1 + \frac{1}{2} - \frac{1}{3}\right] = \underline{\underline{\sqrt{2}\,\frac{7}{6}}}$$

6. En like gyldig parametrisering (selv om litt cursed):

$$\mathbb{X}_2(t) = \mathbb{X}(t^2) = \begin{bmatrix} t^2 \\ 1-t^2 \end{bmatrix}, \quad t \in [0,1]$$

$$\mathbb{X}_2'(t) = \begin{bmatrix} 2t \\ -2t \end{bmatrix} \implies \|\mathbb{X}_2'(t)\| = \sqrt{2}\,2t$$

$$\therefore \rho = \int_0^1 \left[1 + t^2(1-t^2)\right]\sqrt{2}\,2t\,dt$$

$$= \sqrt{2}\,2\int_0^1 \left[t + t^3 - t^5\right]dt$$

$$= \sqrt{2}\, 2 \left[\frac{1}{2} t^2 + \frac{1}{4} t^4 - \frac{1}{6} t^6 \right]_{t=0}^{t=1}$$

$$= \sqrt{2}\, 2 \left[\frac{6+3-2}{12} \right] = \underline{\underline{\sqrt{2}\, \frac{7}{6}}}$$

7. $F(x_1, x_2) = \begin{bmatrix} x_1 \\ (x_1 + x_2)^2 \\ x_2 \end{bmatrix} \quad F: \mathbb{R}^2 \to \mathbb{R}^3$

$$G(z_1, z_2, z_3) = \begin{bmatrix} -z_1 - z_2 \\ z_1 + z_2 + z_3 \end{bmatrix} \quad G: \mathbb{R}^3 \to \mathbb{R}^2$$

$$J_F(x_1, x_2) = \begin{bmatrix} \frac{\partial F_1}{\partial x_1} & \frac{\partial F_1}{\partial x_2} \\ \frac{\partial F_2}{\partial x_1} & \frac{\partial F_2}{\partial x_2} \\ \frac{\partial F_3}{\partial x_1} & \frac{\partial F_3}{\partial x_2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2(x_1 + x_2) & 2(x_1 + x_2) \\ 0 & 1 \end{bmatrix}$$

$$J_G(z_1, z_2, z_3) = \begin{bmatrix} -1 & -1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

8. $H(z) = F(G(z))$

$$H'(z) = F'(G(z))\, G'(z)$$

$$F' = J_F(x) = \begin{bmatrix} 1 & 0 \\ 2(x_1+x_2) & 2(x_1+x_2) \\ 0 & 1 \end{bmatrix}$$

$$G' = J_G(\quad) = \begin{bmatrix} -1 & -1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\therefore\ H' = F'(G(z))\, G'(z)$$

$$= \begin{bmatrix} 1 & 0 \\ 2(-z_1 - z_2 + z_1 + z_2 + z_3) & 2(z_3) \\ 0 & 1 \end{bmatrix} G'(z)$$

$$= \begin{bmatrix} 1 & 0 \\ 2z_3 & 2z_3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & -1 & 0 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -1 & -1 & 0 \\ 0 & 0 & 2z_3 \\ 1 & 1 & 1 \end{bmatrix}$$

9. $x:\mathbb{R}\to\mathbb{R}^2$, $f:\mathbb{R}^2\to\mathbb{R}$

Høydemeter gitt tid vil være gitt ved:

$f(x(t))$

Høydemeter per sekund:

(1)

$$\frac{d}{dt}\left[f(x(t))\right] = f'(x(t))\cdot x'(t) \quad \blacksquare$$

2:

$f'(x(t))$: høydemeter per meter.

$= \nabla f(x(t))$. For å få høydemeter per meter i retningen av banen:

$$\nabla f(x(t))\cdot\frac{x'(t)}{\|x(t)\|} \quad \blacksquare$$

3: Ettersom $f'(x(t))$ er det samme som gradienten til f i punktet $x(t)$, vil den beskrive retningen som er brattest.

4: Ettersom $f'(x) \cdot \frac{x'(t)}{||x(t)||}$ tilsier stigning i en retning, vil en vektor ortogonal på $f'(x)$ tilsvare et skalarprodukt 0, altså 0 stigning

10. $h(x_1, x_2) = 1 - x_1^2 - x_1 x_2 - x_2^2$

$x_0 = (1/2, \sqrt{3})$, $\dot{x}_0 = (1/2, 1/3)^T$

$\nabla h(x_1, x_2) = [-2x_1 - x_2, -2x_2 - x_1]$

$\therefore \nabla h(x_0) = [-1 - \sqrt{3}, -2\sqrt{3} - 1/2]$

$$\frac{\dot{x}_0}{\|\dot{x}_0\|} = \frac{\begin{bmatrix} 1/2 \\ 1/3 \end{bmatrix}}{\sqrt{1/4 + \frac{1}{9}}} = \begin{bmatrix} 1/2 \\ 1/3 \end{bmatrix} \cdot \frac{6}{\sqrt{13}}$$

1. $\nabla h(x_0) \cdot \dot{x}_0$

$$= \frac{1}{6}\left(-7\sqrt{3} - 4\right) \approx -2.64$$

2. $\nabla h(x_0) \cdot \frac{\dot{x}_0}{\|\dot{x}_0\|}$

$$= \frac{1}{13}\left(-4\sqrt{13} - 7\sqrt{39}\right) \approx -4.47$$

3. $\nabla h(x_0) = \left[-1 - \sqrt{3},\ -2\sqrt{3} - 1/2\right]$

4. $\nabla h(x_0) \cdot v = 0$

Dette kan løses på flere måter.
Min favoritt er rotasjonsmatrisen.

$$R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Vi ønske å stå vinkelrett på gradienten. Derfor $+90° = \frac{\pi}{2}$

$$\therefore R = \begin{bmatrix} \cos(\frac{\pi}{2}) & -\sin(\frac{\pi}{2}) \\ \sin(\frac{\pi}{2}) & \cos(\frac{\pi}{2}) \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$\therefore v = R \nabla h(x_0)^T$$

$$= \begin{bmatrix} -2\sqrt{3} - \frac{1}{2} \\ 1 + \sqrt{3} \end{bmatrix}$$

$$v_{norm} \approx \begin{bmatrix} -0.82 \\ 0.57 \end{bmatrix} \vee \begin{bmatrix} 0.82 \\ -0.57 \end{bmatrix}$$

11. $x(t) = \begin{bmatrix} \cos(2t) \\ \sin(2t) \end{bmatrix}, \quad t \in [0, \pi)$

$f(x_1, x_2) = 1 + x_1 x_2$

$$\therefore \int_0^{\pi} f(x(t)) \|x'(t)\| \, dt$$

$$= \int_0^{\pi} \left(1 + \cos(2t)\sin(2t)\right) \sqrt{4\left(\sin^2(2t) + \cos^2(2t)\right)} \, dt$$

$$= 2 \int_0^{\pi} \left(1 + \frac{1}{2}\sin(4t)\right) dt$$

$$= 2\pi + 16 \cdot \frac{1}{2} \cos(4t) \Big|_0^{\pi}$$

$$= \underline{\underline{2\pi}}$$

Samme som ~~før~~ før.

12. $F: \mathbb{R}^3 \to \mathbb{R}^3$

$$F(x) = \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix}$$

Ettersom

F er et konservativt vektorfelt.

Dette vet vi da potensialet

kan beskrives med $E_p = -mgz = -mgh$.

Ettersom $\nabla E_p = \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix}$

Dermed er arbeidet gjort:

$$-mg(-h) + mg0 = mgh$$

15. $V(\underline{x}) = -mgz$

16. $V(\underline{x}) = \frac{1}{\|\underline{x}\|} = \frac{1}{\sqrt{x_1^2 + x_2^2 + x_3^2}}$

$\therefore \nabla V(\underline{x}) = -(x_1^2 + x_2^2 + x_3^2)^{-\frac{3}{2}} [x_1, x_2, x_3]$

Den peker innover mot origo, med større kraft desto nærmere.

Nivåflate:

$V(\underline{x}) = r$

$\therefore 1 = r\sqrt{x_1^2 + x_2^2 + x_3^2}$

$\therefore \frac{1}{r^2} = x_1^2 + x_2^2 + x_3^2$

$\Rightarrow$ Kule med radius $\frac{1}{r}$

17. $\nabla V(x) / (-Gm_1m_2) = F(x)$

18. Formel for kraft:

$F = G \frac{m_m m_j}{r^2}$, $G \approx 6.674 \cdot 10^{-11} m^3/kgs^2$

$m_m \approx 7.35 \cdot 10^{22}$ kg

$m_j \approx 5.97 \cdot 10^{24}$ kg

$r_1 = 362600 \cdot 10^3 m$

$\Downarrow$

$F_1 \approx 2.23 \cdot 10^{20} N$

$r_2 = 405\,400 \cdot 10^3 m$:

$F_2 \approx 1.78 \cdot 10^{20} N$

Ettersom $\frac{d}{dr}\left(-\frac{G m_m m_j}{r}\right) = F$, er

$-\frac{G m_m m_j}{r}$ potensialet til gravitasjons-

Feil tekst.

Derfor i absoluttverdi, vil den potensielle energien E_p være;

E_{p_1} : $8.08 \cdot 10^{28}$ J

E_{p_2} : $7.22 \cdot 10^{28}$ J

$\Delta E_p = 8.56 \cdot 10^{27}$ J ish

14. $$E(\vec{x}) = \frac{q}{4\pi\varepsilon_0} \frac{\vec{x}}{\|\vec{x}\|^3}$$

$$= \frac{q}{4\pi\varepsilon_0} \left(x_1^2 + x_2^2 + x_3^2\right)^{\frac{3}{2}} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Hm. Denne har vi vel sett før?

Denne likner jo på Newtons gravitasjons-lov (med litt ulike konstanter).

$\therefore E = -\nabla V$:

$$V = \frac{q}{4\pi\varepsilon_0} \cdot \frac{1}{r} = \frac{q}{4\pi\varepsilon_0} \frac{1}{\|x\|}$$

$$= \frac{q}{4\pi\varepsilon_0} \frac{1}{\sqrt{x_1^2 + x_2^2 + x_3^2}}$$

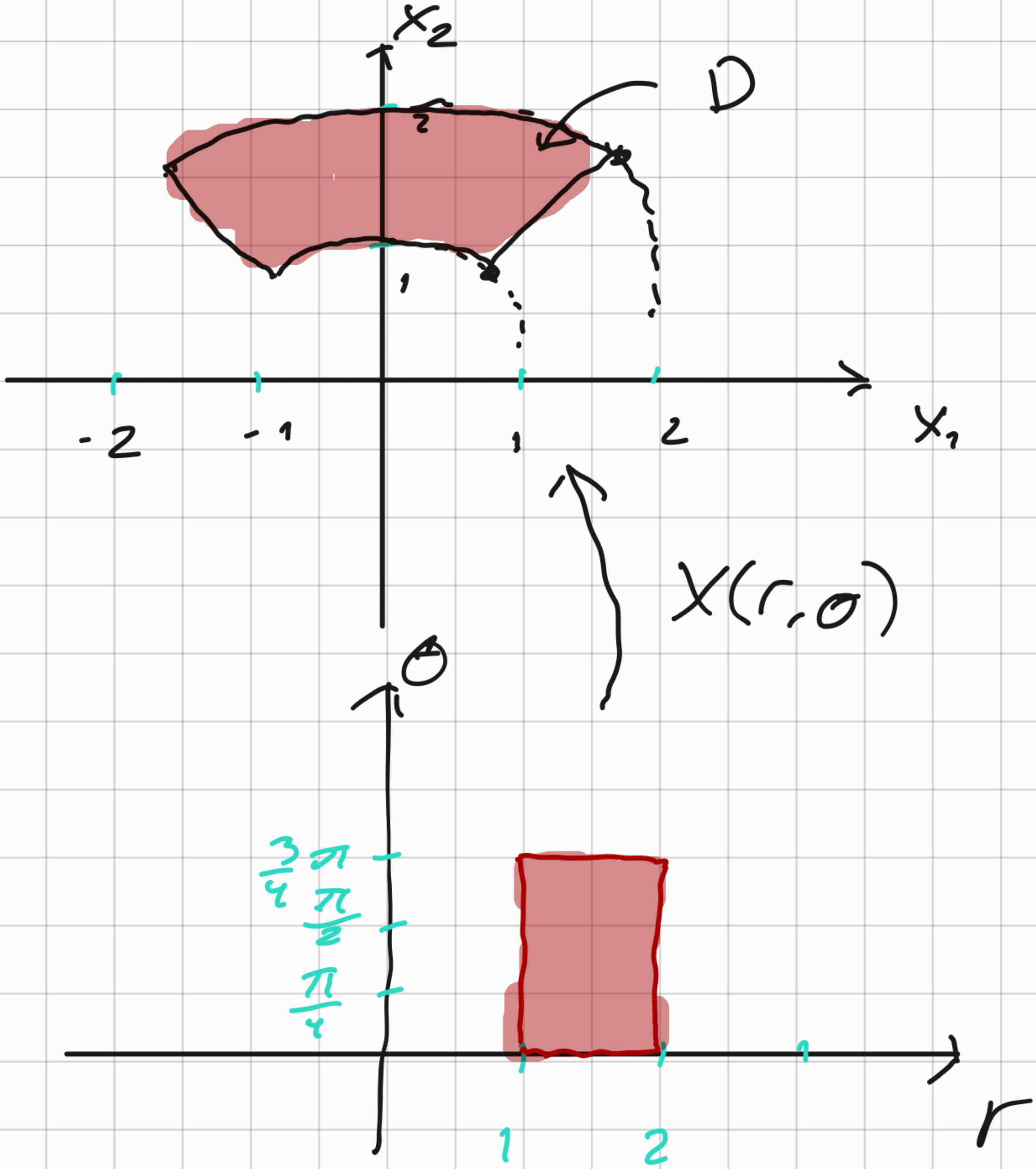
Denne har enhet $\frac{C}{C^2 N^{-1} m^{-2}} \frac{1}{m}$

$$= \frac{Nm}{C} = \frac{J}{C} = V$$

$\Rightarrow$ Det er det elektriske potensialet.

20. LF for denne ligger i samme dokument, og e langt bedre forklart enn jeg får til. Den ligger under 13.1

21.



23.

$$X(r,\theta,\phi) = r \cdot \begin{bmatrix} \cos\theta \sin\phi \\ \sin\theta \sin\phi \\ \cos\phi \end{bmatrix}$$

$$G(X) = \frac{q}{4\pi\varepsilon_0} \frac{X}{\|X\|^3}$$

$$E = H(r,\theta,\phi) = \frac{q}{4\pi\varepsilon_0 r^2} \cdot \begin{bmatrix} \cos\theta \sin\phi \\ \sin\theta \sin\phi \\ \cos\phi \end{bmatrix}$$

$$\therefore E' = \nabla H(r,\theta,\phi) = \nabla G(\mathbb{X}(r,\theta,\phi)) \cdot \nabla \mathbb{X}$$

↓ Letters

$$\frac{q}{4\pi\varepsilon_0}\begin{bmatrix} -\frac{2}{r^3}\cos\theta\sin\phi, & -\frac{1}{r^2}\sin\theta\sin\phi, & \frac{1}{r^2}\cos\theta\cos\phi \\ -\frac{2}{r^3}\sin\theta\sin\phi, & \frac{1}{r^2}\cos\theta\sin\phi, & \frac{1}{r^2}\sin\theta\cos\phi \\ -\frac{2}{r^3}\cos\phi, & 0, & -\frac{1}{r^2}\sin\phi \end{bmatrix}$$

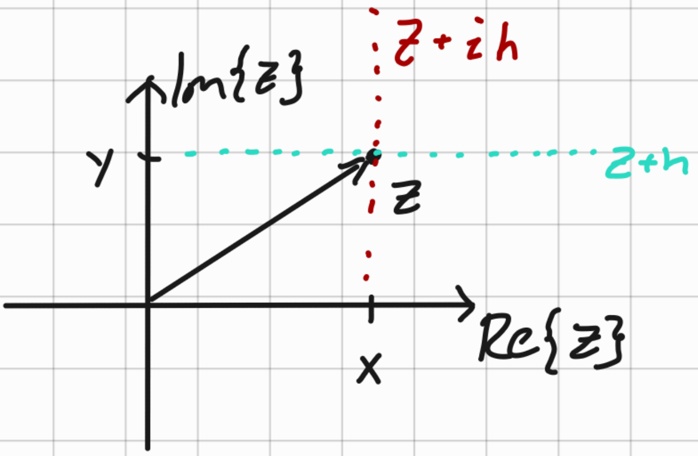
$$= \frac{q}{4\pi\varepsilon_0 r^3}\begin{bmatrix} -2\cos\theta\sin\phi, & -r\sin\theta\sin\phi, & r\cos\theta\cos\phi \\ -2\sin\theta\sin\phi, & r\cos\theta\sin\phi, & r\sin\theta\cos\phi \\ -2\cos\phi, & 0, & -r\sin\phi \end{bmatrix}$$

24. $f'(z) = \dfrac{f(z+h) - f(z)}{h}$

$f(x+iy) = u(x,y) + i\,v(x,y)$, $\quad u, v : \mathbb{R}^2 \to \mathbb{R}$

$f : \mathbb{C} \to \mathbb{C}$

$h, x, y \in \mathbb{R}$



$z+h$: $f'(x+iy) = \lim_{h\to 0} \frac{f(z+h)-f(z)}{h}$

$$= \lim_{h\to 0} \frac{u(x+h,y)+iv(x+h,y)-u(x,y)-iv(x,y)}{h}$$

$$= \lim_{h\to 0} \frac{u(x+h,y)-u(x,y)}{h} + i\cdot \lim_{h\to 0} \frac{v(x+h,y)-v(x,y)}{h}$$

$$= \boxed{\frac{\partial u}{\partial x} + i\cdot \frac{\partial v}{\partial x}}$$

$z+ih$: $f'(z) = \lim_{h\to 0} \frac{f(z+ih)-f(z)}{ih}$

$$= \lim_{h\to 0} \frac{u(x,y+h)+iv(x,y+h)-u(x,y)-iv(x,y)}{ih}$$

$$= \lim_{h\to 0} \frac{u(x,y+h)-u(x,y)}{ih} + \cancel{i}\cdot \lim_{h\to 0} \frac{v(x,y+h)-v(x,y)}{\cancel{i}h}$$

$$= \boxed{-i\frac{\partial u}{\partial y} + \frac{\partial v}{\partial y}}$$

Siden deriverte må være lik i alle retninger dersom den deriverte skal eksistere;

$$\frac{\partial u}{\partial x} + i\cdot\frac{\partial v}{\partial x} = \frac{\partial v}{\partial y} - i\frac{\partial u}{\partial y}$$

$$\Rightarrow \boxed{\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}}$$

25. Definisjonen på at en funksjon f er harmonisk, er at den oppfyller Laplaces likning:

$$\Delta f = 0 \iff \nabla^2 f$$

$$\Delta = \nabla \cdot \nabla \quad \text{(Laplace-operator)}$$

$$\nabla : \frac{\partial}{\partial x_1} + \frac{\partial}{\partial x_2} + \cdots + \frac{\partial}{\partial x_n} : \text{Divergens operator}$$

$$\therefore \; u: \quad \Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

CR-likningene:

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

$$\therefore \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 v}{\partial y \partial x}, \quad \frac{\partial^2 u}{\partial y^2} = -\frac{\partial v}{\partial x \partial y}$$

Under antagelsen om at v er kontinuerlig, kan vi bytte rekkefølgen på derivasjonsdelen.

$$\therefore \frac{\partial^2 v}{\partial x \partial y} = \frac{\partial^2 v}{\partial y \partial x} :$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 v}{\partial x \partial y} - \frac{\partial^2 v}{\partial x \partial y} = 0$$

$\Rightarrow u$ harmonisk.

v: $$\Delta v = \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0$$

$$\frac{\partial^2 v}{\partial x^2} = -\frac{\partial^2 u}{\partial y \partial x}, \quad \frac{\partial^2 v}{\partial y^2} = \frac{\partial^2 u}{\partial x \partial y}$$

Under antagelsen om at u kontinuerlig:

$$\frac{\partial^2 u}{\partial y \partial x} = \frac{\partial^2 u}{\partial x \partial y} :$$

$$\Delta V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = -\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial x \partial y} = 0$$

$\Rightarrow$ V harmonisk.

26. En sirkel sentrert i origo med $r=2$ kan parametriseres med:

$$z(t) = 2e^{it}, \quad t \in [0, 2\pi)$$

Derfor vil en sirkel sentrert i $1+i$ bli:

$$z(t) = 1 + i + 2e^{it}$$

27. Parametrisering for kvart enhetssirkel:

$$z(\theta) = e^{i\theta}, \quad \theta \in [0, \tfrac{\pi}{2}]$$

$$\therefore \int_\Gamma z^2 dz = \int_0^{\frac{\pi}{2}} (e^{i\theta})^2 \, i \, e^{i\theta} \, d\theta$$

$$= \int_0^{\frac{\pi}{2}} e^{3i\theta} \, i \, d\theta$$

$$= i\,\frac{1}{3i}e^{3i\theta}\Big|_{\theta=0}^{\theta=\frac{\pi}{2}} = \underline{\underline{\frac{1}{3}(-i-1)}}$$

28. Parametisering av rett linje:

$$z(t) = 1 + (-1+i)t \quad , \quad t\in[0,1]$$

$$\Rightarrow z'(t) = -1+i$$

$$\therefore \int_\Gamma z^2\,dz = \int_0^1 (1+(-1+i)t)^2(-1+i)\,dt$$

$$= \int_0^1 (1+2(-1+i)t+(-1+i)^2t^2)(-1+i)\,dt$$

$$= \int_0^1 ((-1+i)+2(-1+i)^2t+(-1+i)^3t^2)\,dt$$

$$= \Big[(-1+i)t+(-1+i)^2t^2+\frac{1}{3}(-1+i)^3t^3\Big]_{t=0}^{t=1}$$

$$= (-1+i)+(-1+i)^2+\frac{1}{3}(-1+i)^3$$

$$= \frac{1}{3}(-1-i)$$

29. $\int_\Gamma z^n \, dz \quad , \quad z(\theta) = e^{i\theta} , \quad \theta \in [0, 2\pi)$

$$= \int_0^{2\pi} e^{i\theta n} \, i e^{i\theta} \, d\theta \quad , \quad n \in \mathbb{Z}$$

$$= i \int_0^{2\pi} e^{i\theta(n+1)} \, d\theta = \frac{i}{i(n+1)} e^{i\theta(n+1)} \Big|_{\theta=0}^{\theta=2\pi}$$

$$= \frac{1}{n+1}(1-1) = 0 \qquad n \neq -1$$

$n = -1$:

$$i \int_0^{2\pi} e^{i\theta(-1+1)} \, d\theta = i \int_0^{2\pi} 1 \, d\theta = \underline{\underline{2\pi i}}$$

$$\therefore \int_\Gamma z^n \, dz = \begin{cases} 0 & n \neq -1 \\ 2\pi i & n = -1 \end{cases}$$

30. Her er det en del detaljer dere mangler for å få det fulle bildet. Residyregning er kanskje den største. Dette kommer i Matte 4.

Likevel, Cauchys derivasjonsformel er gitt ved:

$$f^{(n)}(a) = \frac{1}{2\pi i} \oint_C \frac{f(z)}{(z-z_0)^{n+1}}$$

Ringen i integralet tilsier at vi integrerer langs en lukket kurve. Eksempelvis som enhets-sirkelen, som biter seg selv i halen.