Numerical study on the effect of prestrain history on ductile fracture resistance by using the complete Gurson model

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ABSTRACT

During installation and operation, pipeline steels may suffer from plastic pre-deformation (prestrain) due to accidental loading, cold bending or ground movement. The plastic prehistory not only modifies steels' yield and flow properties but also influences their fracture performance. This paper focuses on the effect of plastic prestrain history on ductile fracture resistance. Single edge notched tension (SENT) specimens have been selected for the numerical study and the crack is assumed to exist before a prestrain history was applied. The complete Gurson model has been applied to simulate the ductile fracture behaviour. The results show that prestrain history can reduce the fracture resistance significantly and neither the history-independent resistance model nor material-memory resistance model existing in the literature can be used to describe the prestrain history effect. Based on the numerical results an approximate history-dependent resistance model is proposed. The results also suggest that it is important to take the prestrain effect into consideration in future structural integrity assessment procedure for pipelines.

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1. Introduction

There are many engineering scenarios where steel components are subject to plastic deformation (prestrain) due to accidental loading, cold bending and ground movement and subsequent loading and deformation capacity of the components should be assessed. One of the scenarios is the pipe reeling process in the offshore industry. The reeling process is an efficient and economical way to install pipelines on the seabed, see Fig. 1. During the installation, welded pipes are reeled onto a drum, reeled off, aligned and straightened before it is deployed in the sea. During the process axial plastic deformation as large as 2–3% can occur in the pipe section. In order to properly tackle the problem, the distribution of the prestrain history over the pipe cross section should be precisely predicted first. The next step is to study the effect of the prestrain history on subsequent deformation and fracture behaviour. It is known that plastic deformation will not only modify the yield and strain hardening behaviour, but will also influence in a detrimental manner the local failure mechanisms, the initiation toughness and fatigue properties [1–9]. These findings indicate that the effect of plastic prestrain needs to be quantified. Today's recommended practise ignores the prestrain effect [10].

Most of the studies carried out in the literature focus on the effect of a homogenous prestrain history on material's fracture behaviour. The crack is then introduced after a homogenous prestrain history. In real pipe laying operation, cracks due to welding defects may exist before a reeling process is started and it is important to study how a reeling induced prestrain history affects the fracture resistance of an existing crack.

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Recently, a theoretical and experimental program has been carried out by Ernst et al. [6] to investigate the effect of pipe reeling induced prestrain cycles on the ductile fracture resistance curve (R curve). Ernst et al. introduced two models to describe the resistance curves with prestrain histories, a history-independent R curve and a material-memory R curve, see Fig. 2. The history-independent resistance model is used in today’s recommended practise [10]. In the history-independent resistance model the shape of the resistance curve is the same for all loading cycles, independent of the prestrain history. If crack extension takes place in the current cycle the resistance curve is shifted towards the current crack length before the next cycle starts. In the material-memory resistance model the resistance curve does not shift to a new origin from one cycle to another one. The crack driving force achieved in the previous cycle needs to be reached in the next cycle in order for further crack extension to take place. Although it was concluded that material-memory R curve seems to better fit their experimental results, the effect of prestrain history on the ductile fracture resistance is not well understood.

In this work, a numerical study has been carried out to investigate the effect of prestrain history on the crack resistance curve. Modelling ductile crack growth needs an appropriate constitutive model which can describe the ductile fracture mechanism. In this regard, the complete Gurson model developed by the authors [11] has been utilized. 2D single edge notched tension (SENT) specimens are specially selected for this study. Recently it has been demonstrated by Nyhus et al. [12] that the SENT specimen is a good representation of the pipe sections in terms of fracture toughness measurement. Both symmetrical and non-symmetrical prestrain cycles are investigated. Symmetrical prestrain cycles with amplitudes up to 0.55% nominal strain have been considered. The non-symmetrical prestrain cycles considered consist of an elongation up to 0.6% nominal strain followed by a compression down to about the initial specimen length. Because of 2D plane strain models are used, the prestrain cycles considered are somewhat smaller and simpler then an actual reeling prestrain cycle. After the prestrain cycle the specimen was re-loaded by tension.

In the following, the specimen dimensions, constitutive equations used for modelling ductile crack growth and the finite element model will be briefly described first. In Section 3 the effect of basic parameters such as initial void volume fraction as well as material hardening exponent on the resistance curve of materials without a plastic prestrain history is revisited. The
results of plastic prestrain history on ductile fracture resistance as well as a new resistance model to consider the prestrain history effect are presented in Section 4. The paper is closed with summary and conclusions.

2. The complete Gurson model and numerical procedure

In this section, the complete Gurson model used for simulating ductile crack growth and its associated material parameters, the numerical algorithms for implementing the complete Gurson model in ABAQUS, the details of the finite element model as well as the procedures for calculating the crack driving forces and measuring the crack growth are presented.

2.1. The complete Gurson model

Engineering steel alloys inevitably contain secondary phases, inclusions and particles which will nucleate microvoids during plastic deformation. Ductile fracture is often the result of microvoid nucleation, growth and coalescence. Significant progress has been made in developing constitutive models for elastic–plastic materials incorporating void mechanisms and the best known constitutive model appears to be the one originally developed by Gurson [13] and later modified by Tvergaard and Needleman [14–16]. The Gurson model is in fact a void growth model. With the introduction of the so-called critical void volume fraction, the Gurson–Tvergaard–Needleman model can consider the effect of void coalescence. However, the model is lamed by the lack of a physical mechanism-based coalescence criterion. Application of the Gurson model as a predictive tool using realistic microvoid parameters demands a void coalescence criterion. Several analytical and theoretical void coalescence models are available in the literature [17–20]. Zhang and Niemi [21–22] have demonstrated that the plastic-limit load based coalescence model by Thomason [17] is easy to implement and very accurate compared with the finite element results by Koplik and Needleman [23]. By combing the Gurson–Tvergaard–Needleman model for void growth and Thomason’s plastic limit load model for coalescence a so-called complete Gurson model has been proposed by Zhang et al. [11,24]. One of the important advantages of the complete Gurson model is that we do not need to select or determine the critical void volume fraction before hand. The critical void volume fraction is not a material parameter any more. It is a result of plastic deformation and is determined automatically by the plastic-limit load model. With the complete Gurson model ductile fracture is exclusively linked to the void nucleation parameter(s) and the mesh size. Here mesh size is explicitly related to materials characteristic length parameter. Latest study has shown that artificially selected critical void volume fraction can give contradictory results [25].

The yield function of the Gurson–Tvergaard–Needleman model [13–16] has the following form:

\[ \phi(q, \sigma_f, f, \sigma_m) = \frac{q^2}{\sigma_f^2} + 2qf \cosh \left( \frac{3q \sigma_m}{2 \sigma_f} \right) - 1 - (q_f)^2 = 0 \]  

where \( f \) is the void volume fraction, \( \sigma_m \) is the mean stress, \( q \) is the von Mises stress, \( \sigma_f \) the flow stress. \( q_1 \) and \( q_2 \) are parameters introduced by Tvergaard [14–15] to modify the original Gurson model [13]. Fixed value of \( q_1 = 1.5 \) and \( q_2 = 1.0 \) are used in this study. Due to the incompressible nature of the matrix material the growth of existing voids can be expressed as

\[ df_{\text{growth}} = (1 - f)d\varphi \cdot \mathbf{I} \]  

where \( \varphi \) is the plastic strain tensor and \( \mathbf{I} \) is the second-order unit tensor.

There are several void nucleation models we can use [11]. In this study a cluster void nucleation model [11,24] has been selected. It assumes that all the microvoids will be nucleated in the beginning of plastic deformation and the increase in void volume fraction is solely due to the growth of existing voids. The critical void volume fraction \( (f_c) \) is automatically determined by Thomason’s plastic limit load criterion [17]. For 2D plane strain problem, Thomason’s plastic limit load criterion states that coalescence occurs when the following condition is satisfied:

\[ \frac{\sigma_1}{\sigma_f} = \frac{0.3}{r(R - r)} + 0.6 \]  

In Eq. (3) \( \sigma_1 \) is the maximum principal stress, \( r \) and \( R \) are current average void radius and intervoid distance which can be calculated from the current principle strains. Readers are referred to [11,24] for a full account of the complete Gurson model. Thomason’s plastic limit model was based on slip-line theory with rigid-plastic materials. Recently Pardoen and Hutchinson [18] and Zhang et al. [11] have shown that Thomason’s criterion can be further improved to consider the plastic strain hardening. In the present study the original Thomason criterion is used and the effect of strain hardening on the prediction of crack resistance has been studied in detail and is reported in a separate study [25].

The post–coalescence deformation behaviour of the Gurson model is numerically simulated by an artificial acceleration of void growth, as suggested by Tvergaard and Needleman [14]:

\[ f' = \begin{cases} \frac{f}{f_c + \frac{R - r}{R - r} (f - f_c)} & \text{for } f \leq f_c \\ f_c & \text{for } f > f_c \end{cases} \]  

where \( f_c = 1/q_1 \) and \( f_c \) is the void volume fraction at the end of void coalescence, \( f_c = 0.15 + 2f_0 \), where \( f_0 \) is the initial void volume fraction [11]. When the coalescence started and \( f > f_c \), \( f' \) replaces \( f \) in Eq. (1). It should be noted that the critical void
volume fraction determined by the complete Gurson model in the present study is around 2–3%, which is much smaller than
the one used by Xia and Shih [26] that is 20%.

The stress–strain behaviour of the matrix material which is part of the specification of the complete Gurson model is
characterized by the following power law hardening model:

\[
\sigma_f = \sigma_0 \left( 1 + \frac{\varepsilon_p}{\varepsilon_0} \right)^n
\]

where \(\sigma_0\) is the yield stress, \(\varepsilon_p\) the equivalent plastic strain, \(\varepsilon_0\) the yield strain and \(n\) the hardening exponent. Eq. (5) states
that \(n \to 0\) corresponds to an elastic-perfectly plastic solid. In this study, \(\sigma_0 = 400\) MPa, Young’s modulus \(E = 200\) GPa and
Poisson ratio \(\nu = 0.3\) are fixed and independent of the hardening and plastic strain history.

### 2.2. Implementation of the complete Gurson model

The complete Gurson model is implemented into the ABAQUS using a set of generalized-midpoint algorithms developed
by Zhang via the material user subroutine UMAT\(^2\) [27–29]. Aravas [30] first presented an Euler backward algorithm for the
Gurson model and derived an equation for calculating the consistent-tangent-moduli. However, this equation involves matrix
inversion which is not efficient and not possible for non-hardening materials. Zhang [27] proposed a family of the generalized-
midpoint algorithms for the Gurson model and an explicit equation with seven constants for calculating the consistent tangent
moduli is presented [28–29]. This equation does not involve any matrix inversion and has thus avoided the singularity problem.
It was concluded that the Euler backward algorithm which is used in this study, may not be the most accurate but may be the
best choice for many practical applications in terms of the balance between accuracy and efficiency [28–29].

### 2.3. Specimen geometry and FE mesh

The single edge notched tension (SENT) specimens have been chosen for the present study. Same specimens were used in
the previous study on the effect of prestrain on the crack tip constraint [9]. The geometry and the finite element mesh of the
SENT specimen are schematically shown in Fig. 3. The specimen width (w) is fixed 28 mm and half length (L) is 115 mm.
Because of the symmetry, only half of the SENT specimen is modelled. Close to the crack tip a rectangular region (7.1 mm
ahead of the initial crack tip and 0.8 mm above the symmetrical line) with uniform mesh size 0.1 \(\times\) 0.1 mm is created. Simple
4-noded 2D plane strain elements have been used. Nonlinear geometry effects (NLGEOM in ABAQUS) are accounted for in the
analyses. An initial crack opening 0.02 mm is applied for all the models. Three initial crack sizes (a) have been considered;
a/w = 0.14, 0.26 and 0.60. The specimen dimensions and applied loading have been chosen such a way that buckling during
compression is avoided. A remote homogenous displacement controlled (clamped) boundary condition was used. During the
prestrain cycle the crack faces can be partly closed. A rigid analytical plane is defined in the model to simulate the contact of
crack faces. The rigid plane is shown in Fig. 3b as a horizontal line.

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\(^2\) A free copy of the ABAQUS UMAT Fortran code can be obtained from the corresponding author.
Because the J-integral is not well suited for cyclic plasticity problem, the CTOD has been used as the crack driving force parameter. It is known that for monotonic loading and nonlinear elastic material, an intrinsic relation between the CTOD and J-integral exists [31]. However, this relation is not clear in the present prestrain history problem. The CTOD was selected as a result of convenience, and it is beyond the scope of this paper to discuss which parameter is the most appropriate one for the prestrain problem.

2.4. Definition of crack initiation and propagation

In the simulation of crack growth using the complete Gurson model with ABAQUS, the element is not allowed to fail completely and a residual stiffness is necessary to avoid numerical oscillation and non-convergence problem. In the present analysis after critical void volume fraction is reached void growth continues until \( f' \) reaches 0.91\( f_0' \) [11]. Then no further void growth is allowed and the element can still have a small residual load carrying capacity.

The common way to estimate crack growth in a numerical simulation is to multiply the initial element length by the number of damaged elements, without accounting for the deformation of the damaged elements [11,26], see Fig. 4a. Considering the fact that elements deform during crack growth, a new method to calculate the crack growth is used in this study. The crack growth is calculated as the absolute horizontal distance from the node above the initial crack tip to the node forming the current crack tip, see Fig. 4b. This technique generally predicts shorter crack growth (especially when the crack

![Image](image-url)
growth is large) at a given crack driving force due to element contraction, Fig. 4c. The crack growth calculated by this technique should be more close to the crack growth measured in an experiment where the finite deformation is accounted for.

3. Resistance curves without prestrain history

In this section monotonic loading of the SENT specimens is analyzed and the results will be taken as the references for studying the prestrain history effect. We briefly comment on the effect of the following three material and geometry parameters that have a direct bearing on the computed fracture resistance. The three parameters considered are initial void volume fraction $f_0$, crack size/specimen width ratio $a/w$, and strain hardening exponent $n$.

3.1. Effect of initial void volume fraction

The initial void volume fraction represents the degree of damage in the material. The larger the initial void volume fraction is the larger damage has the material. Fig. 5 shows the predicted resistance curves by the complete Gurson model for three initial void volume fractions, $f_0 = 0.001$, 0.005 and 0.010, respectively. In the present case where a cluster void nucleation model is used, the initial void volume fraction is the only damage parameter and no additional parameters are necessary. The strain hardening exponent of the matrix material for the results shown in Fig. 5 is 0.05 and the crack size/specimen width ratio, $a/w$ is 0.14. It can be clearly seen that the initial void volume fraction influences both the crack initiation toughness and the crack growth resistance curve. Increasing the initial void volume fraction increases the damage and reduces the crack growth resistance.

![Fig. 5. Resistance curves for $f_0 = 0.001$, 0.005 and 0.010, $a/w = 0.14$ and $n = 0.05$.](image)

![Fig. 6. Resistance curves for $a/w = 0.14$, 0.26 and 0.60, $f_0 = 0.005$ and $n = 0.05$.](image)
3.2. Effect of crack size/specimen width aspect ratio $a/w$

Fig. 6 shows the fracture resistance for different initial crack size to width ratios (the specimen width is fixed). The initial crack size to width ratios analyzed are $a/w = 0.14, 0.26$ and $0.60$, respectively. For three-point-bending specimens, it is well

Fig. 7. Effect of strain hardening on the resistance curve. Four hardening exponents are considered, $n = 0.01, 0.05, 0.10$ and $0.20$. $f_0 = 0.005$ and $a/w = 0.14$.

Fig. 8. (a) CTOD versus nominal strain for different symmetrical prestrain cycles (b) effect of symmetrical prestrain on the fracture resistance. $n = 0.05$, $a/w = 0.14$ and $f_0 = 0.005$. 
known that shallow crack, for example the specimen with $a/w = 0.14$, corresponds to a low constraint case with negative $T$ stress, while deep crack specimen with $a/w = 0.60$ has high crack tip constraint with positive $T$ stress. Both experimental results and numerical simulations have shown that shallow and deep cracked three-point-bending specimens give rise to different fracture resistance behaviour – a significantly higher resistance curve was obtained from the shallow crack geometry [26]. However, for the present clamped SENT specimens, Fig. 6, only a relatively small increase of the resistance for the shallow crack specimen has been observed. This is somewhat surprising and we may expect a larger difference between the case with $a/w = 0.14$ and $a/w = 0.60$. However, the present results support the experimental findings by Nyhus et al. [32] which indicate that the effect of initial crack size to width ratio on the fracture resistance is insignificant.

3.3. Effect of strain hardening

The effect of strain hardening on the ductile fracture resistance is controversial. Xia and Shih [26] reported that raising materials’ hardening capacity, and keeping other parameters fixed, increases the ductile resistance. Using the complete Gurson model, the effect of the material strain hardening exponent ($n$) on the crack resistance curves are plotted in Fig. 7 for materials with $n = 0.01, 0.05, 0.10$ and $0.20$, respectively. Similar to the results of [26], the crack initiation toughness measured by CTOD is rather independent of the strain hardening exponent. However, it is interesting to observe that decreasing the hardening exponent will significantly raise the resistance curve. The results of the strain hardening effect on the resistance curves reported by [26] and the ones shown in the present study are just opposite. The present results are similar to the recent results of Østby et al. [33–34]. It should be noted that it is very difficult to experimentally verify the strain hardening effect on the fracture resistance, because it is nearly impossible to produce materials with different hardening capacities while keeping other parameters (yield stress, initial volume fraction) unchanged. A detailed study on the effect of strain hardening effect on the crack resistance behaviour has been carried out by the authors [25] and it has been found that the effect of strain hardening is very sensitive to the constitutive model used.

![CTOD versus nominal strain for different symmetrical prestrain cycles and (b) effect of symmetrical prestrain on the fracture resistance. $n = 0.10$, $a/w = 0.14$ and $f_0 = 0.005$.](image-url)
4. Resistance curves with a prestrain history

In this section we present the results of the ductile fracture resistance curves with a prestrain history. It should be noted that void growth is only allowed during tensile deformation and is suppressed during compression. Both symmetrical and non-symmetrical prestrain cycles have been considered. Nominal strain is used as the global loading parameters. Nominal strain is measured by the ratio of the applied displacement divided by the initial length. The SENT specimens with different crack size to width ratios and different strain hardening have also been studied. The initial void volume fraction has been kept constant 0.5%.

4.1. Effect of prestrain on ductile crack growth

For each SENT specimen three prestrain cycles have been analyzed in addition to the case with monotonic tension. It must be noted that the crack models have an initial opening 0.02 mm and therefore a maximum closure 0.02 mm is allowed. In all the cases considered the specimens were loaded by tension first. Reverse loading was applied at different nominal strain levels. The CTOD versus nominal strain curves for the four cases with symmetrical prestrains are shown in Fig. 8a for \( a/w = 0.14 \) and \( n = 0.05 \). Prestrains 0.2\%, 0.3\% and 0.4\% were applied. It can be seen that during the loading a typical elastic–plastic CTOD versus nominal strain behaviour can be observed. The CTOD increases slowly in the beginning and rapidly when substantial yielding has reached. It is interesting to note that during the reverse loading, the CTOD decreases also slowly in the beginning (elastic unloading) and then rapidly when compressive loading has re-induced yielding.

Prestrain will not only change the shape of CTOD versus nominal strain curve, but will also affect the fracture resistance behaviour. The effect of the three symmetrical prestrains on the ductile fracture resistance of the SENT specimen is shown in Fig. 8b. The red line indicates crack growth during prestrain. The result of monotonic loading without a prestrain history is

![CTOD versus nominal strain for different symmetrical prestrain cycles](image)

![Effect of symmetrical prestrain on the fracture resistance](image)

**Fig. 10.** (a) CTOD versus nominal strain for different symmetrical prestrain cycles and (b) effect of symmetrical prestrain on the fracture resistance. \( n = 0.05 \), \( a/w = 0.26 \) and \( f_0 = 0.005 \).
plotted in the same figure as a reference case. It can be clearly observed that a symmetrical prestrain up to 0.2% can already result in an observable reduction in the fracture resistance both in terms of initiation toughness and crack growth resistance. The crack initiation toughness for the case with 0.3% prestrain reduced to about zero. For the 0.4% prestrain case a small crack growth occurred in the prestrain cycle and crack growth in the re-loading stage starts almost immediately when the CTOD becomes positive. The general observation is that crack initiation toughness in the re-loading stage reduces with increasing prestrain amplitudes and approaches to zero when the applied prestrain is large enough. It should be noted that the exact prestrain level at which the re-loading crack initiation toughness becomes zero is dependent on the material and geometry combination. It is not the intention of this study to establish a quantitative link between the prestrain and these parameters.

Similar analyses have been extended to the same SENT specimen but with a high hardening material, $n = 0.10$. Three prestrains, 0.2%, 0.4% and 0.55% have been analyzed. The results are displayed in Fig. 9. The general trend is the same – increasing symmetrical prestrain levels, while keeping other parameters fixed, will reduce both the initiation toughness as well as the resistance. For the high strain hardening material ($n = 0.10$) no crack growth occurs in the prestrain cycle for the case with 0.4% prestrain, as did for the low hardening material with $n = 0.05$. A larger prestrain (0.55%) is needed to induce crack growth in the prestrain cycle.

The effect of prestrain has also been explored for the SENT specimen with a deeper crack length to width ratio, $a/w = 0.26$ and $n = 0.05$. Three symmetrical prestrain cycles, 0.2%, 0.25% and 0.3% are analyzed. The results are displayed in Fig. 10. Crack growth is observed during the 0.3% prestrain cycle. As it can be expected, the prestrain level at which crack growth first occurs in the prestrain cycle for the deep crack is smaller than for the shallow crack with $a/w = 0.14$.

So far, only symmetrical prestrain cycles have been considered. The effect of non-symmetrical prestrain cycles has also been studied. Here the specimen is loaded in tension to the specified prestrain level and then unloaded and followed by a compression to about the initial specimen length. Three prestrains, 0.4%, 0.5% and 0.6% are applied. The CTOD versus nominal strain is shown in Fig. 11 (a). The effect of non-symmetrical prestrain on fracture resistance is shown in Fig. 11 (b).

![Fig. 11](image-url) (a) CTOD versus nominal strain for the non-symmetrical prestrain levels and (b) Effect of non-symmetrical prestrain on fracture resistance. $n = 0.05$, $a/w = 0.14$ and $f_0 = 0.005$. 
strain curves and the corresponding fracture resistance curves are shown in Fig. 11. Crack growth occurred in all the applied prestrain cycles. After the prestrain cycle crack growth starts at a lower CTOD than the CTOD at the end of pre-tension loading, but greater than the initiation CTOD in the monotonic loading. It must be noted that the 0.4% symmetrical prestrain cycle and the 0.4% non-symmetrical prestrain cycle have the same amount of elongation during prestrain, but different degrees of compression. Their fracture resistances in the re-loading stage differ significantly with much lower initiation toughness and resistance for the case with symmetrical prestrain cycle. Figs. 8 and 11 clearly indicate that the compression in the prestrain cycle plays a significant role in the resistance behaviour after the prestrain cycle.

Fig. 12. Damage (void volume fraction) and crack growth during the prestrain cycle and further tearing. The prestrain applied was a 0.4% symmetrical prestrain cycle. \( \eta/w = 0.14 \), \( n = 0.05 \) and \( f_0 = 0.005 \).

Fig. 13. Residual stress distribution of the SENT specimen after a 0.4% symmetric plastic strain cycle, \( f_0 = 0.005 \) and \( n = 0.05 \).
The general conclusions which can be drawn from Figs. 8–11 are that a plastic strain history will reduce both the initiation and resistance curves. The degree of reduction is dependent on materials hardening exponent, prestrain level as well as the crack tip geometry constraint level. The prestrain effect becomes stronger for the cases with low hardening and high

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**Fig. 14.** History-dependent $R$ curve.

**Fig. 15.** Comparison between approximated and numerical resistance curve, after a symmetrical prestrain cycle. (a) 0.4% prestrain, $n = 0.05$ and $a/w = 0.14$ and (b) 0.55% prestrain, $n = 0.10$ and $a/w = 0.14$. 
geometry constraint. Prestrain nature (type) seems to influence the reduction of resistance curve strongly. Symmetrical prestrain cycle is more detrimental compared with the non-symmetrical prestrain analyzed in the paper. Figs. 8–11 also show that the resistance curves after the prestrain cycle are to a large degree similar to each other, and the shapes of these resistance curves resemble to the resistance curve without a prestrain history. This observation will be the basis for the history-dependent resistance model proposed and will be further elaborated in the following.

4.2. Prestrain induced crack tip damage

The observed reduction in fracture resistance can possibly be explained by Fig. 12. Fig. 12 shows the development of crack tip damage (void volume fraction) for the case with 0.4% symmetrical prestrain cycle. Fig. 12a shows the condition at the end of the pre-tension loading. A small amount of crack growth is observed in the first tensile loading (red indicates totally damaged element). The crack is closed and the near tip crack field is distorted during reverse compressive loading (Fig. 12b). When the second tensile loading starts the damage will starts to accumulate immediately (Fig. 12c). New crack growth initiates at a CTOD approximately equal to zero in the second tensile loading loading due to the accumulated damage during the first tensile loading. In Fig. 12d it can be observed that at approximately the same CTOD as in Fig. 12a, the ductile damaged zone and ductile crack growth in the second tensile cycle is much greater than that in the first tensile cycle.

As noted previously, it is interesting to observe that even though crack initiation in the second tension starts at different CTOD levels and crack growth amounts, the shapes, and in particular the curvature of the resistance curves are similar to the resistance curve for the monotonic loading. It seems that the resistance curve in the monotonic loading case will be vertically shifted down when a plastic prestrain cycle is applied. It is apparent that the initial stage of crack growth after the prestrain cycle is disturbed by the prestrain induced damage. The resistance curve starts with a small flat segment before it increases rapidly to the similar shape as the resistance curve for monotonic loading. This behaviour is most evident for prestrain cycles with a great amount of plastic yielding during reverse loading. The behaviour can be explained by the distortion of the near

![Fig. 16](image-url) **Fig. 16.** Comparison between approximated and numerical resistance curve after (a) a 0.3% symmetrical prestrain cycle, n = 0.05 and a/w = 0.26 and (b) a 0.6% non-symmetrical prestrain cycle, n = 0.05 and a/w = 0.14.
crack tip field and the strain hardening during the reverse compressive loading. In the second tension loading (re-loading) accumulation of ductile damage occurs close to the crack tip even though the global load is negative. This behaviour can originate from the distortion of the near crack tip field and explain the flat segment of the resistance curve in the initial stage of re-loading. During compression significant strain hardening occurs. This results in a lower effective material hardening exponent \( n \). According to the effect of strain hardening reported in Section 2, decreased hardening increases the tangent to the resistance curve. The competition between the effect of strain hardening and distortion of the near crack tip field can possibly explain the complex shape of the resistance curve after the prestrain cycle. It is clear that only the area close to the crack tip is significantly affected by the stress concentration, see Fig. 13.

4.3. History-dependent \( R \) curve

The present results tend to conclude that the resistance curve depends on the prestrain history. Neither today’s recommended practice (history-independent \( R \) curve) nor the material-memory resistance curve presented by Ernst et al. [6] fit the present numerical results. Based on the observation in the previous section, a new resistance model is proposed to describe the present results. The new model is called history-dependent \( R \) curve which is schematically explained in Fig. 14. The resistance curve for the monotonic loading is used as the basis for the history-dependent \( R \) curve model. The point on the monotonic resistance curve where the first tensile loading ended becomes the new origin of the resistance in the second tension. After the first prestrain cycle (both the tension and the compression stage) the CTOD and crack growth are reset and the origin will be moved to a new position. The position of the new origin depends on the prestrain levels and type.

If the CTOD versus crack growth curve for a prestrain cycle followed by re-tension is plotted in the same coordinate system, the resistance curve for the second tensile cycle is located below the resistance curve of monotonic loading. According to the history-dependent \( R \) curve model the vertical shift of the resistance curve equals the change in CTOD during the compression stage of the prestrain cycle. In Figs. 15 and 16 the resistance curve constructed according to the new history-dependent \( R \) curve model is compared with the original numerical fracture resistance. It can be seen that the estimated resistance curve by the history-dependent \( R \) curve model is in reasonable agreement with the numerical resistance curve both for the symmetrical prestrain cycles and the non-symmetrical prestrain cycle.

5. Concluding remarks

The complete Gurson model has been used to numerically predict ductile crack growth of SENT specimens. A new technique has been applied to calculate the crack growth by taking the plastic deformation of the elements into account. To the authors’ knowledge such technique has not been used or reported by others. This new technique should not alter the trends of the predicted ductile fracture behaviour. However, the predicted resistance curve is supposed to be more close to the experimental measured ones.

The effect of the hardening exponent \( n \) on the fracture resistance is controversial. Opposite trends have been observed in this study compared with the early studies reported in the literature. The readers are referred to [25] for a detailed account on this topic.

The results show that plastic prestrain cycles (first tension followed by compression) influence the fracture resistance with further loading. Reduced fracture resistance in the second tensile cycle comes from accumulated ductile damage in the first tensile cycle. How much the resistance curve is reduced is dependent on the amount of plastic compression between the two subsequent tensile cycles. In the study a new and simple model to predict the resistance curve through deformation cycles is proposed. The history-dependent \( R \) curve model takes no account for change in material properties and redistribution of stress due to compressive loading. In the present study only small prestrain amplitudes have been investigated. In case of greater prestrain amplitudes significantly change of material properties and near crack tip field distortion may occur during compression. This may cause that the history-dependent \( R \) curve model function improperly.

This study has only considered simple prestrain cycles. Pipe reeling or other industrial and service processes may involve multi prestrain cycles and the first cycle may not necessarily be in tension. Finally it should be noted that only the influence of prestrain on ductile fracture has been considered. The prestrain can also change the failure mechanisms and failure modes. A systematic experimental work is planned to verify the numerical results.

References
