ABSTRACT

This paper presents two new methods, one for weldments, and one for homogenous materials, for determining a material’s whole range true stress – logarithmic strain curve, including both the pre- and post-plastic localization regimes. A material’s whole range true stress-strain curve is of crucial importance for plastic forming analysis, and also for the analyses of damage and ductile fracture, where large plastic deformation is involved. For homogenous materials, only smooth round tensile specimens are used for determining the true stress strain curve. In many cases, it is demanding to use rectangular tensile specimens. Recently, an extensive study has been carried out by the authors on the diffuse necking behavior of specimens with rectangular cross section. It is found that the material’s true stress-strain relation can be determined from the load versus thickness reduction curve of a rectangular specimen. An approximate equation for calculating true area reduction from thickness reduction for different materials and aspect ratio, is given. For weldments the conventional cross weld tensile specimens give practically little useful information. A novel method is developed for determining the true stress-strain curves of different material zones of a weldment, by using notched cross weld tensile specimens. The method can be applied to determine the true stress-strain curves for base metal, weld metal and HAZ. The effects of various parameters on the accuracy of both methods has been investigated.

INTRODUCTION

The whole range of the true stress-strain curve of a material including material response in both the pre- and post necking stages, is very important for metal forming analysis and for the analysis of ductile fracture. For a homogenous material, its true stress strain curve before diffuse necking can be easily determined by using either round or rectangular tensile bars. There is, however, no method available for determining the whole range stress-strain curve with rectangular tensile bars. In many cases, it is preferred to use rectangular tensile specimens, for example, for thin materials and materials with plastic anisotropy. The main problem of using rectangular tensile specimens is the difficulty to calculate and measure the current area reduction. Before diffuse necking, the deformation of the cross section is proportional. After diffuse necking, however, the cross section will change shape and the relation between area reduction and thickness reduction will depend on the initial aspect ratio and material strain hardening behaviour, Fig. 1. It has been shown by an extensive numerical study that the relation between the area reduction and the thickness reduction of a rectangular tensile specimens can be normalized by the initial aspect ratio and a material hardening parameter – the strain at the onset of diffuse necking. As long as the true area is used for calculating the logarithmic strain, the material’s true stress-strain curve is...
independent of the cross section shape of a tensile specimen. This implies that Bridgman’s correction can still be used for necking correction of the true stress strain curve from rectangular specimens.

For weldments where different material zones are involved, cross weld tensile testing is a common practice in industries to qualify welds. Such testing, however, yields very little practical information for further analysis. The load versus elongation curve from cross weld tensile testing strongly depends on the initial measuring gauge length, and also on where the fracture occurs. In this paper, it has been shown that by making a notch in a cylindrical cross weld tensile specimen, direct testing of notch cross weld tensile specimens can result in very useful information for analysis. The load versus diameter reduction curve of a notched cross weld tensile specimen can be converted to the true stress-strain curve of a material zone where the notch is located. The notch can be located in the base metal, weld metal or in the heat affected zone (HAZ). Parameter study indicates that when the material zone length is larger than the diameter of the specimen, the material zone length has practically no effect on the accuracy of the method.

**RECTANGULAR SPECIMENS FOR HOMOGENEOUS MATERIALS**

**Theory**

As mentioned in the Introduction, the main concern of this method is the relation between the total area reduction and the thickness reduction. In the following an approximate relation has been derived. The relation consists of three parts – a geometry function, a material function and a basic necking curve. Fig. 1b shows the initial rectangular cross section and current cross section. The two black dots represent the measuring points for thickness reduction. Before diffuse necking, the deformation of the cross section is proportional, and the total area reduction can be written:

\[
\frac{\Delta A}{A_0} = 2 \left( \frac{\Delta t}{t_0} \right) \left( \frac{\Delta t}{t_0} \right)^2,
\]

where \(A_0\) and \(t_0\) are the initial cross section area and initial thickness, \(\Delta A\) and \(\Delta t\) are the total area reduction and thickness reduction, see Fig. 1b. After diffuse necking, the deformation becomes non-proportional and the total area reduction is smaller than what is calculated by Eqn. (1):

\[
\frac{\Delta A}{A_0} = 2 \left( \frac{\Delta t}{t_0} \right) \left( \frac{\Delta t}{t_0} \right)^2 - \frac{\Delta A_s}{A_0},
\]

where \(\Delta A_s\) (negative) is the area reduction caused by shape change and a correction to the proportional deformation. It must be noted that \(\Delta A_s\) is zero before the diffuse necking. An extensive 3D finite element study of rectangular tensile specimens has been carried out to investigate the relation between \(\Delta A_s\) and the initial aspect ratio, \(S\) and material strain hardening behavior [1]. Different Young’s modulus/yield stress ratios were also studied, however, it was found that this ratio is not a parameter in the area reduction and thickness relation. The initial aspect ratio, \(S\), varied from 1 to 8. Materials with a power-hardening law [1] were considered first. Four materials with strain hardening exponent \(n=0.05, 0.1, 0.15, 0.2\) [1] together with two real materials where the stress strain curves couldn’t be described by a power law were considered.
For a given material, numerical results show that $\Delta A_s$ is an increasing function of the initial aspect ratio $S$, which implies that specimens with large aspect ratio tend to have large shape change, while specimens with square cross section remain nearly square after diffuse necking. Furthermore, at a given thickness reduction, unloading occurs much faster in the specimens with low aspect ratio than that with high aspect ratio. Fig. 2a shows $\Delta A_s$ versus thickness reduction curves for different initial aspect ratios for the material with hardening exponent $n=0.15$, Young’s modulus/yield stress ratio 500 and Poisson ratio 0.3, and Fig. 2b the results by normalizing the curves by the corresponding value at 50% thickness reduction. It is very interesting to observe that area reduction due to shape change versus thickness reduction curves for different aspect ratios can be normalized by a function of the aspect ratio. This normalization procedure has been applied to materials with different strain hardening behavior. It is shown that the normalization function of the aspect ratio is independent of the material hardening. This geometry (aspect ratio) function has been fitted as (Fig. 3):

$$f_g(S) = 0.1686 + 0.6\ln(S).$$

For a given aspect ratio, the effect of strain hardening exponent, $n$, on $\Delta A_s$ has been studied. At a given thickness reduction, it was found that low strain hardening will display large shape change, and vice versa. Efforts have been spent to normalize the $\Delta A_s$ by the strain hardening. Because the strain at the onset of diffuse necking ($\Delta A_s$ starts to be active) is approximately equal to the strain hardening exponent, materials with different hardening have different origin for $\Delta A_s$. After transforming all the curves to the same origin, it was found that all the curves can be normalized by a linear material hardening function, $f_m$:

$$f_m(n) = 0.2845 - 0.956n,$$

where $n = \left( \frac{\Delta t}{t_0} \right)_{\text{max}}$.

This material function $f_m$ is shown in Fig. 4. After the geometry and material hardening effect has been taken out, a material and geometry independent basic necking curve for the relation between $\Delta A_s$ and thickness reduction is fitted as follows:

$$f_s(x) = c_0 + c_1x + c_2x^2 + c_3x^3 + c_4x^4,$$
where $x \ (x \geq 0)$ implies the net thickness after diffuse necking:

$$x = \frac{\Delta t}{t_0} - \left( \frac{\Delta t}{t_0} \right)_{\text{max}}$$

(7)

The final equation to calculate the total area reduction based on thickness reduction is given by:

$$\frac{\Delta A}{A_0} = 2 \left( \frac{\Delta t}{t_0} \right) - \left( \frac{\Delta t}{t_0} \right)^2 - f_g(S) f_m \left( \frac{\Delta t}{t_0} \right)_{\text{max}} f_i \left( \frac{\Delta t}{t_0} \right)_{\text{max}}$$

(8)

Figure 2: a) The effect of aspect ratio on the shape change, b) normalized curves of a). The hardening exponent for this material is $n=0.15$.

Figure 3: The geometry function $f_g$.

Figure 4: The material function $f_m$. 

$c_0 = -0.03069$

$c_1 = 1.09016$

$c_2 = 11.1512$

$c_3 = -25.1$

$c_4 = 14.8718$

(6)
Verifications

The proposed approximate area reduction – thickness reduction relation, Eqn. (8), has been verified numerically. Fig. 5a shows the results for a high strength offshore steel and Fig. 5b for a 7XXX aluminium alloy. The true stress-strain curves of both materials can not be fitted by a power hardening law. In Fig. 5, the true stress-strain curve calculated directly from the finite element analysis has been compared with results with Eqn. (8) based on the load-thickness reduction curves which are also from finite element analysis. Fig. 5 shows that Eqn. (8) is very accurate.

![Figure 5 a) Numerical verification for a high strength steel and b) a 7XXX aluminium alloy.](image)

Eqn. (8) has recently been extended to materials with plastic anisotropy [3], and experimental verification has been carried out [4]. The experimental true stress-strain curves from rectangular tensile specimens based on Eqn. (8) have excellent agreement with the curves from round tensile specimens for the same material.

Fig. 5 also shows that the material’s true stress-strain curve is unique and independent of the cross section. This indicates Bridgman correction can be applied to the true stress-strain curve obtained from rectangular tensile specimen for correction of stress triaxiality, $\sigma_b$:

$$\frac{\sigma_b}{\sigma} = 1/[1+\alpha(\varepsilon))ln(1+\alpha(\varepsilon))]$$

(9)

The neck geometry parameter, $\alpha(\varepsilon)$, does not need to be measured. Results in [1] have shown that the empirical estimation by Le Roy et al. [5], $\alpha(\varepsilon)=1.1(\varepsilon - \varepsilon_{\text{max}})$, works very well for most of the materials.

NOTCHED CROSS WELD TENSILE SPECIMENS FOR WELDMENTS

For the safety assessment of weldments, it is of crucial importance to obtain the true stress-strain curves of each material zone. All weld testing and simulated HAZ testing are common practice. Both are, however, indirect and expensive. For all weld testing, the testing direction is often different to the actual loading direction. Another conventional testing used in industries for qualifying welds is called cross weld tensile testing. Load versus elongation curves can be measured. These curves, however, provide little useful information for further analysis, because the result is strongly dependent on the measuring gauge distance and fracture location. Recently, a method called notched tensile testing has been established [6]. The central idea of the method is to force plastic deformation at a notch in the material zone of interest, and to obtain the true stress-strain curve of that material zone from the recorded load versus diameter reduction curve. Notched cylindrical tensile specimens are proposed, see Fig. 6.
There is a so-called load-separation principle in the literature of fracture mechanics [7], which states that the load-deformation behaviour of a fracture mechanics specimen can be separated into two parts, one is a geometry function and another is a material response curve. The idea of this paper is based on the load-separation principle. Finite element analyses of smooth and notched round tensile specimens have been carried out for materials with different strain hardening exponents. The tensile stresses (load/current cross section area) for different geometry has been normalized to observe whether a unique material curve exists. Fig. 7a shows the tensile stress versus true strain curves for the material with hardening exponent $n=0.1$. As it is well known, a notch will increase the tensile stress, the smaller the notch radius the higher the tensile stress. Fig. 7b shows the normalized curves of Fig. 7a. The normalization was carried out by dividing the whole stress strain curve by its value at maximum load. For the material shown in Fig. 7, the strain at maximum load is 0.1. It is interesting to observe from Fig. 7b that, to a certain degree, the stress-strain curves for different geometries collapse into one. Similar behaviour has been observed for materials with different hardening. This finding indicates that the true stress-strain curve of a smooth specimen can be obtained from the tensile stress strain curve of a notched specimen by dividing a geometry constant, $G$:

$$\sigma^\text{Smooth}(\varepsilon) = \sigma^\text{Notched}(\varepsilon)/G$$

The geometry function $G$, is certainly dependent on the material hardening exponent. However, the dependence is relatively weak. By taking the material with a hardening exponent $n=0.1$ as a reference material, Fig. 7 shows the effect of hardening on the relative difference of $G$ for two notch geometries. It seems that by taking a relatively large notch ($R/D>0.33$) the relative difference can be below 5% for hardening in the range from 0.05 to 0.20. The $G$ obtained in this study for the notch geometry $R/D>0.33$ is 1.45 and for the $R/D>0.13$ notch is 1.74. A table showing $G$ as a function of notch radius will be issued later on.

![Figure 6: Notched cross weld tensile specimen for determining true stress-strain curves for the weld metal, HAZ as well as base metal of a weldment.](image)

![Figure 7: a) Tensile stress from smooth and two notch specimens, b) normalized curves of a). The material strain hardening exponent is $n=0.1$.](image)
The effect of material zone length on the accuracy of the method has been investigated in a bi-material system. The notch was positioned always in the centre of the material zone of interest. Fig. 9 shows the results for a material system with $n=0.1$ and mismatch ratio $m=0.5$ (the notch zone material is undermatching). Five different zone lengths were analyzed. It shows that when the zone length is very small, the resulting tensile stress is amplified due to the enforced constraint. However, when the zone length is approaching the minimum diameter of the tensile specimen, the tensile stress of a bi-material system is very close to that of one material system made of the notch zone material. Both undermatch and overmatch have been studied and the conclusions are the same [6]. The same behaviour can be observed in the specimens with a smaller notch radius, $R/D=0.13$.

**Figure 8:** Relative difference, $(G_n - G_{n=0.1})/G_{n=0.1}$ as a function of hardening exponent $n$.

**Figure 9:** Effect of material zone length ($H/D$) on the tensile stress - strain curves. The notch radius is $1/3$ of the diameter. The material system is characterized as $n=0.1$ and $m=0.5$. The layers in the legend means the element layers.
SUMMARY

With the advances of computing technology, there is an increasing demand on a material’s whole range true stress – logarithmic strain curves. This paper presents two novel methods for material testing. The first method is developed for homogenous materials with thin plate thickness. In the proposed method, any rectangular cross section with aspect ratio less than 8 can be used. In practice, specimens with aspect ratio 4 are recommended. During the test a load versus thickness reduction curve should be recorded, from which materials true stress-strain curve can be readily determined by using Eqn. (8). It must be noted that before utilizing Eqn. (8), the thickness reduction at maximum load should be found first. The correction to the proportional deformation starts only after the onset of diffuse necking. There is one condition for using this method. The material should be isotropic. For materials with plastic anisotropy, another equation should be used [3,4].

The second method proposed is called notched cross weld tensile testing. This method utilizes cross weld round tensile specimens with a notch in the respective material zone of interest. For both steel and aluminium weldments, this method can be applied to determine the true stress-strain curve of weld metal and base metal. For aluminium weldments, the HAZ zone is quite large and the method can give a reasonable representation of the HAZ properties. For steel HAZ, care must be taken, because the HAZ zone length is quite small. There is also a strong gradient of the mechanical properties across the HAZ. It must be noted that, in the proposed method, diameter reduction rather than elongation should be measured. The true stress-strain curve of that material zone can be obtained by dividing the true stress by a constant geometry factor. The material zone length will influence the accuracy of the method, however, when the zone length is larger than the initial diameter, this effect can be neglected. The notched cross weld tensile testing method can also be applied to homogenous materials. It must be reminded that this method is approximate in nature.

Acknowledgements - The financial support from the Norwegian Research council via a SIP project at SINTEF Materials Technology, and via the PROSMAT project is greatly appreciated.

REFERENCES