Feasibility Study of a Circularly Towed Cable-Body System for UAV Applications

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Abstract—This paper presents a feasibility study for endbody positioning maneuvers using a towed cable-body system where a fixed wing Unmanned Aerial Vehicle (UAV) is stabilized in a steady-state circular motion. As high precision maneuvers such as object pickup/dropoff are typically performed by rotorcraft UAVs, a successful fixed-wing concept would greatly increase the possible range for this type of operation and enable missions into more remote locations. Circularly towed cable-body systems have been shown capable, both analytically and experimentally, of maintaining equilibrium configurations with the towed endbody stabilized in a small orbit respective to a point on the ground. However, no known efforts consider small to medium scale UAV operations for object pickup/dropoff. It is a primary goal of this paper to identify key parameters to consider when selecting a UAV and towable system to use for high precision maneuvers, specifically to consider the achievable steady state configurations for circularly towed UAV systems. The equations of motion for the towed cable are developed based on a Lumped Parameter Model (LPM). An optimization problem was formulated in order to determine configuration parameters for the towed system that minimizes the steady-state orbit of the towed endbody while staying within the performance constraints that apply to the towing UAV. It is shown feasible to achieve sufficiently small orbits of the endbody to warrant considering the concept further for precision maneuvers.

Keywords—Unmanned Aerial Vehicle/Systems; Fixed-Wing Precision Maneuvers; Circular Towing; Towed Cable-Body Systems, Steady-State Performance, Numerical Optimal Control.

I. INTRODUCTION

The recent years have seen significantly increased interest in the Arctic regions and open oceans from both commercial interests as well as government agencies. Due to the hostile environment and lack of infrastructure, alternate solutions for monitoring and support operations are needed. UAVs represent a promising technology platform to support these activities. As accessibility and cost of operations is a major limitation, the capable range of the UAV is expected to be critical. While high precision maneuvers such as object drop-off and retrieval are typically performed with rotorcraft UAVs, these systems have very limited range. This work is performed with a view to extend the capabilities of the high-endurance fixed-wing UAVs to include these types of maneuvers.

The main challenge to performing high precision maneuvers with fixed wing UAVs is the requirement to maintain a significant minimum forward speed at all times. There are two obvious candidate approaches to achieve autonomous precision pickup type operations with this constraint in place. The first approach involves attaching a mechanism to the UAV that can grab the object during a close fly-over. The critical aspect here is the considerable risk of unintended UAV impact with the ground or other objects. The second approach involves equipping the UAV with a towable cable that has some type of (potentially instrumented and/or controllable) device attached to the end. For this alternative, the UAV is less likely to be damaged, but the towed cable-body system increases the complexity of the problem considerably. The combination of complex dynamics, the likely presence of winds and sudden gusts and the fairly high airspeeds involved give prospects of a marginal success rate. An approach that would allow more time to perform the actual object pickup operation involves a "hover-like" capability for the fixed-wing UAV, where the UAV is stabilized in a steady-state circular motion while towing a long cable with a mass/body attached at the free endpoint. With the right combination of tow-cable parameters, UAV speed and towing radius, the body at the endpoint will stabilize in a low speed, small radius orbit relative to a point on the ground. The equilibrium cable shapes that result when viewed from the rotating reference frame attached to the towing UAV can be obtained from numerical methods. The "circle-tow" maneuver has also been put into practical use. In the 1950s missionary Nate Saints suspended a basket attached to a long rope from his small fixed wing airplane and performed pinpoint deliveries of supplies to remote villages in Ecuador. Also, the United States Navy TACAMO (Take Charge and Move Out) project employs a large aircraft (a heavily modified Boeing 707) to tow a 15,000 - 25,000 feet long wire in a circular orbit for use as a Very Low Frequency (VLF) antenna in order to communicate with the ballistic missile submarine fleet.

Solutions to the linearized eigenvalue problem for a cable rotating about its own axis in a vacuum were first derived by D. Bernoulli (1700-1782) and L. Euler (1707 - 1783), but as noted by T.K. Caughey [1] the linearized solution failed to explain the physics observed in simple practical experiments. In 1955, Kolodner [2] sparked renewed academic interest in rotating cable systems by publishing a mathematical study...
that demonstrated that the experimental results of these systems are better explained by nonlinear theory. Several efforts to study the circular towing problem were made in the 1960ies and 1970ies. Most notably by Skop and Choo who in their 1971 paper [3] identify the important parameters to consider for the circularly towed system, their effect on the tow-cable shape and also include numerical results for airborne circular towing. In 1977 Russell and Anderson publishes equilibrium and stability results of the circularly towed cable system based on a Finite Element Model (FEM) of the cable [4]. Clifton, Schmidt and Stuart developed a steady-state and dynamic model of the circularly towed system for use in the TACAMO project, and showed that the model correlated well with flight test data [5]. Murray proposes to treat the system as differentially flat and applies an inversion procedure to obtain the cable shape, thereby greatly reducing the computational effort required [6]. A number of papers that considers the use of circularly towed systems for pickup and delivery of payloads in remote areas have been published by researchers at the Royal Melbourne Institute of Technology (RMIT) from 2004 until 2012 [7], [8]. A dynamical model based on a discrete lumped mass approach is developed, and various ways to achieve motion suited for high precision object placement and retrieval are studied by the group. In particular, they consider ways to modify the towing aircraft motion to compensate for wind and disturbances. Another recent research objective tied to towed cable systems involves development of a method for recovering Micro Air Vehicles (MAVs) in flight using a towing vehicle and a towed drogue [9]-[11]. This work is performed by researchers with Brigham Young University (Utah, USA), and it represents the only known work where circularly towed cable systems are formed using UAVs. The larger towing vehicle will have to maintain a speed outside the capability of the MAVs, so the circle-tow maneuver is used to place a “docking drogue” in a smaller circular orbit at lower speeds.

The present paper builds on the work performed at RMIT related to the modelling, steady-state and dynamical analysis as well as the towing configuration optimization of large (manned) towed systems and adapts the techniques to smaller scale (UAV) systems. While Williams and Trivailo [7] performed optimization analysis for 3 existing aircraft to determine feasible circular towing configurations considering aircraft operating constraints, the approach selected here is to eliminate only the physically impossible portions of the candidate design space to derive the achievable towing configurations for a UAV system. This strategy supports using the results from the optimization analysis to derive desired properties for a UAV designed with circular towing in mind. Furthermore the results contained in this paper will provide the analysis and verification basis for a dynamical Optimal Control Problem that will be used to develop optimal trajectories and control strategies for circularly towed systems when subjected to winds and other disturbances.

II. MATHEMATICAL MODELLING OF TOWED CABLE SYSTEMS

A. Modelling Overview, Assumptions and Simplifications

In order to obtain useful and representative results from the optimization analysis it is critical that the analytical system model reflects the actual behavior of the physical system. Hence, a simple yet representative model must be developed for the towing aircraft, the towing cable and the towed body. Early research efforts related to the dynamics of towed systems revealed that for scenarios involving a long tow-cable and/or a comparatively light weight towed body, the overall system response depends so much on the cable dynamics that the cable typically needs to be treated as a complete hydro/aero-dynamic body with properties such as shape, size, mass distribution and flexibility [12].

The equations of motion for the towing cable are approximated by replacing the continuous cable with a set of N mass points that are connected with massless, elastic thin rods (to model stretching of the cable). The point mass associated with each cable element is numbered from 1 at the cable end-point through N at the last deployed cable element (at the tow-aircraft attachment point). A simplification is made to lump all the external forces acting on each element at the point mass (node), which effectively uncouples the acceleration terms between nodes and thereby eliminates the inertial coupling between elements that complicates a standard Finite Element Model (FEM). The simplification allows the motion of each node to be uniquely determined at each time step. The degrees of freedom at each node are coupled to the neighboring nodes through the tension and strain which acts along the connections. The towed body attached at the end of the cable, will simply be added to the bottom cable point mass. For the purposes of this study, it is assumed that the unstrained cable segment lengths (l) between the different point-masses are equal and also that the required length of the cable is equivalent to the total available length (Lc). This type of cable model is known as a discrete Lumped Parameter Model (LPM). Due to the balance between accuracy, computational cost and versatility, this approach has been adopted in most of the recent studies involving towed cable systems, including all the previously mentioned research performed by Paul Williams et al. at RMIT and the group studying UAV in-air retrieval at Brigham Young University. While the LPM method adopted for use in this paper is less accurate than a standard FEM approach, the fact that the acceleration terms are uncoupled between nodes greatly simplifies the analysis.

The geometries and associated coordinate systems that describe the towed system are shown in Fig. 1. The right-handed Cartesian coordinate system XYZ with origin O and unit vectors I, J and K is the inertial reference frame which is located on the ground level at the center of the steady-state orbit formed by the towed object. The cable dynamics is described in the xyz system with origin TP and unit vectors i, j and k, which is attached to, and hence rotates with, the UAV towpoint. The local y-axis points in the direction of motion of the towing UAV and the local z-axis and the inertial Z-axis are parallel axes. The aircraft rotates about the center C which is given by the coordinates xc, yc, and zc relative to the inertial origin O. In the absence of winds and disturbances (as will be assumed in this concept study), xc = yc = 0, and zc are fixed, hence xyz will rotate about the inertial Z-axis at a constant height (H) and constant angular velocity (ω = θ) as illustrated in Fig. 1. The angle θ represents the rotation of the local x-axis
with respect to the inertial X-axis. The lumped masses are related to the towpoint on the towing aircraft using Cartesian coordinates \((x_j, y_j, z_j)\).

![Diagram](image.png)

**Fig. 1. Coordinate systems and Geometries for the Towed Cable-Body System**

As the focus of this study is on the achievable steady-state performance of a UAV-sized towed system, the towing UAV is modelled as constraints applied to the top of the tow cable. The towable attachment point is limited to follow an orbital motion where the speed and orbit radius is restricted to lay within a realistic range. The towed body will be modelled as a small sphere.

For the analysis described in this paper, the equations of motions were derived from Newton’s second law. For the jth point-mass along the towed cable we have:

\[
F_j = m_j \ddot{x}_j, \quad j = 2, 3, ..., N
\]  

(1)

Similarly, for the bottom cable element that the towed body is attached to, we have:

\[
F_B + F_1 = (m_B + m_1) \ddot{a}_1
\]  

(2)

The relevant forces to include in the analysis are detailed in the next section. Due to the assumption of constant cable length between point masses (and of a constant cable cross-section), the cable point masses for this problem are identical and are equal to:

\[
m_j = \rho_c l \frac{\pi d^2}{4}, \quad j = 1, 2, 3, ..., N
\]  

(3)

In the above equation, \(\rho_c\) is the material density, and \(d\) is the diameter of the cable. The inertial acceleration of each point-mass can be obtained from the second derivative of its position relative to the inertial frame. The equations of motion will be expressed in terms of coordinates relative to the rotating frame to allow the equilibrium configuration of the cable to be determined more easily. The inertial position vector \(p_j\) of the jth point mass written in terms of the xyz rotating coordinates is given by:

\[
p_j = (R_{tp} + x_j)i + y_jj + (H + z_j)k
\]  

(4)

The inertial velocity \(v_j\) and acceleration \(a_j\) can thus be obtained, including the centripetal terms related to the rotation of the xyz coordinate system:

\[
v_j = \left(\frac{dp_j}{dt}\right) + \omega \times p_j = (\ddot{R}_{tp} + \dot{x}_j + y_j \dot{\theta})i + (\dot{\theta} + z_j)k
\]  

(5)

\[
a_j = (\ddot{R}_{tp} + \dot{x}_j - 2y_j \dot{\theta} + y_j \ddot{\theta} - R_{tp} \dot{\theta}^2 + x_j \dot{\theta}^2)i + (\dot{y}_j + 2\dot{R}_{tp} \dot{\theta} + R_{tp} \ddot{\theta} + 2x_j \ddot{\theta} + x_j \dot{\theta} - y_j \dot{\theta}^2)j + (H + z_j)k
\]  

(6)

The second derivatives (accelerations) of the position of each lumped mass relative to the towpoint can be solved for by inserting the above expression for \(a_j\) into Newton’s second law, noting that all the forces should also be expressed in coordinates relative to the coordinate frame at the towpoint.

\[
\ddot{x}_j = \frac{F_{xj}}{m} - \ddot{R}_{tp} + 2\dot{y}_j \dot{\theta} + y_j \ddot{\theta} + R_{tp} \ddot{\theta}^2 + x_j \dot{\theta}^2
\]  

(7)

\[
\ddot{y}_j = \frac{F_{yj}}{m} - 2\dot{R}_{tp} \dot{\theta} - R_{tp} \ddot{\theta} - 2\dot{x}_j \dot{\theta} - x_j \ddot{\theta} + y_j \dot{\theta}^2
\]  

(8)

\[
\ddot{z}_j = \frac{F_{zj}}{m} - H
\]  

(9)

It is also of interest to compute the vector connecting adjacent point masses to obtain a measure of the deformation and orientation of each cable element. The vector for the jth element is:

\[
E_j = (x_j - x_{j+1})i + (y_j - y_{j+1})j + (z_j - z_{j+1})k
\]  

(10)

The magnitude of this vector corresponds to the final stretched length of the particular cable element. The direction of the vector will be used to relate the forces derived in the next section to the rotating coordinate system xyz, and can be found by dividing the vector by the vector magnitude.

**B. Overview of Cable Forces**

The forces that are considered relevant to model to estimate the steady-state cable configurations, includes the internal tension force of the towable as well as the external forces due to aerodynamic effects and gravity. In line with common practice for heavier-than-air aerial systems, forces due to Added/Virtual mass and Buoyancy have been neglected. Also effects due to internal damping, shear forces, bending moments and torsional stiffness of the tow-cable have been ignored for the purposes of this analysis. Fig. 2 illustrates the forces acting on the jth point mass along the tow-cable.
C. Aerodynamic Forces

Aerodynamic forces are the forces exerted on an object by the fluid flowing past it. The component of the force that is perpendicular to the velocity vector of the fluid flow is typically referred to as lift, while the component parallel to the flow direction is referred to as drag. The common approach for modelling the aerodynamic forces acting on a cable, involves treating each cable element as a slender cylinder and to use well-known methods from the literature to model the forces generated by the fluid flowing over each element.

The crossflow method is considered to be the simplest method that breaks the aerodynamic force down into a normal and a tangential component. The method is described in some detail in the book on Fluid-Dynamic Drag by Hoerner [13], and it has been used in the tow-cable modelling for a large number of the studies that was reviewed for this report. The aerodynamic form or pressure force that arises because of the shape of the object is given by:

\[
C_D = C_f + C_{D_{\text{basic}}} \left( \sin^2 \alpha \right) \quad (11)
\]

\[
C_L = C_{D_{\text{basic}}} \left( \sin \alpha \cos \alpha \right) \quad (12)
\]

\(C_f\) is the skin friction drag, \(\alpha\) is angle of attack between the inclined cylinder and the wind and \(C_{D_{\text{basic}}}\) is the drag coefficient of the cylinder if \(\alpha\) is equal to zero. Since this analysis assumes calm air (no winds or turbulence), the angle of attack is simply the angle between the orientation of the cable element and its velocity vector. From the geometric definition of vector dot product the angle of attack for each cable element can be obtained as follows:

\[
\cos \alpha_j = -\frac{E_j \cdot v_j}{|E_j||v_j|} \quad (13)
\]

The lift and drag forces acting on each towcable element can be obtained from the standard lift and drag equations by setting the reference area to the diameter times the axial length of the cable:

\[
F_{D,j} = \frac{1}{2} \rho_a |v_j|^2 C_{D_j} l d e_{Dj} \quad (14)
\]

\[
F_{L,j} = \frac{1}{2} \rho_a |v_j|^2 C_{L_j} l d e_{Lj} \quad (15)
\]

The mass density \(\rho_a\) of the surrounding air will be assumed to be constant and \(e_{Dj}\) and \(e_{Lj}\) are the unit vectors for the drag and lift forces on the \(j^{th}\) element respectively. The drag force acts in the direction of the fluid flow across the body, so in this case it acts in the opposite direction of the velocity vector of the \(j^{th}\) cable element. The following unit vectors with respect to the inertial frame are obtained for the drag and lift force on each segment of the cable:

\[
e_{Dj} = -\frac{v_j}{|v_j|} \quad (16)
\]

\[
e_{Lj} = \frac{(v_j \times E_j) \times v_j}{(|v_j \times E_j|)^2} \quad (17)
\]

The drag generated by the towed body can be computed from:

\[
F_{D_B} = \frac{1}{2} \rho_a |v_1|^2 C_{D_B} \pi r_B^2 e_{D1} \quad (18)
\]

\(C_{D_B}\) and \(r_B\) is the drag coefficient and radius of the towed body respectively. The total aerodynamic force on each point mass is:

\[
F_{a_j} = F_{D_j} + F_{L_j}, \quad j = 2, 3, ..., N \quad (19)
\]

\[
F_{a_1} = F_{D_1} + F_{L_1} + F_{D_B} \quad (20)
\]

D. Gravity Force

Since a circle-tow scenario is expected to take place at relatively low altitudes it is reasonable to ignore the altitude dependency of the gravity force, hence the gravity force on the \(j^{th}\) element is computed as follows:

\[
F_{g_j} = -m_j g z \quad (21)
\]

Here \(g\) is selected to be equal to the gravitational acceleration at sea level.

E. Elastic Tension Forces

By assuming that the tow cable material follows Hooke's law, a relatively simple relationship for the elastic tension forces between each mass in the lumped mass model can be derived. Under certain operational circumstances the tow cable may be slack (i.e. not experience any tension). As the tow cable can contract freely, it is assumed to not be able to sustain compression forces; hence the strain in the \(j^{th}\) cable element is taken to be:

\[
\varepsilon_j = \frac{|E_j| - l}{l}, \quad |E_j| \geq l \quad (21)
\]

\[
\varepsilon_j = 0, \quad |E_j| < l \quad (22)
\]

The tension along the cable that each element is subjected to can be computed from Hooke's law:

\[
|T_j| = \sigma_u A = E \varepsilon_j A \quad (23)
\]

\(E\) is the modulus of elasticity of the tow cable and \(A\) is the cross-sectional area of the tow cable (assumed to be constant along the length of the cable for this study). Note that it also follows that the minimum allowable cable diameter is a function of the maximum cable tension force (\(T_{\text{max}}\) and the ultimate tensile strength of the cable material (\(\sigma_{\text{ut}}\)).
\[ d_{\text{min}} \geq \sqrt[4]{\frac{4|T|_{\text{max}}}{\pi \sigma_{\text{ut}}}} \] (24)

The total tension force applied to the \( j \)th mass with respect to the rotating \( xyz \) coordinate system is:

\[ T_{\text{Tot},j} = T_j \left[ \frac{E_j}{E} \right] - T_{j+1} \left[ \frac{E_{j+1}}{E} \right] \] (25)

**F. Equilibrium Conditions**

The basic steady-state solution for the towed cable system, in the absence of any external disturbances, requires that the towing aircraft operates at constant speed and with a constant angular velocity, while the cable and towed body trail behind in a geometric configuration that is determined by the external forces acting on them (primarily aerodynamic and gravitational forces). The basic scenario with zero angular velocity corresponds to rectilinear flight and involves a relatively simple steady state solution. All other equilibrium scenarios (that are practical for towed cable systems) involve a constant, nonzero angular velocity in the horizontal plane such that the towing aircraft is engaged in a constant radius turn maneuver and the following is satisfied:

\[ \dot{R}_{TP} = \dot{R}_{TP} = \theta = H = H = 0 \] (26)

In this case the point masses and endbody along the towable have zero accelerations and velocities relative to the rotating coordinate system. Thus for a particular cable system where the motion of the UAV towpoint is specified, the only unknowns for the steady state solution are the positions of the point masses.

For a given towed vehicle system, three independent variables can be used to achieve the desired geometry of the tow cable and towed body/vehicle; the cable length, the orbit radius of the towing vehicle, and the velocity of the towing vehicle. Additionally, system parameters such as the cable properties (material and thickness) and the performance characteristics of the towing UAV and the towed body can be modified to influence the steady-state geometry. Solutions satisfying the equilibrium conditions are discussed in various level of detail by Skop and Choo [3], Russell and Anderson [4], Cohen and Manor [14], Clifton, Schmidt and Stuart [15], Williams and Trivailo [7] and others.

**G. Summary of Equilibrium Equations of Motion**

The equations of motion of the towable can be obtained by substituting the forces derived in section II B into Newton's Second Law applied to each point mass. The equations of motion should be expressed in terms of coordinates relative to the coordinate system at the towpoint.

When the equilibrium solution is derived from the differentially flat approach, all the relative accelerations and velocities will simply be set to zero and the positions of all the point masses will be derived from that of the bottom mass.

**H. Towing UAV Performance Constraints**

As already mentioned the towed UAV is simply modelled as constraints on the orbital motion possible for the towpoint at the top of the tow-cable. This is done for simplicity, but also to avoid enforcing unnecessary performance limitations based on existing UAV designs since it is of interest to know what type of performance may be feasible to achieve given a UAV that may be designed particularly for this purpose. Still it is relevant to define a realistic combination of tow-speed and tow-radius that obeys the general physical laws that applies to fixed wing air vehicles. First, the true airspeed of the UAV is limited to a realistic range by adapting realistic minimum and maximum speeds.

\[ v_{\text{UAV, min}} \leq v_{\text{UAV}} \leq v_{\text{UAV, max}} \] (26)

Second, reasonable bounds should be enforced on the minimum tow-radius. Fig. 3 is reproduced from Warren F. Phillips book on mechanics of flight [16] and illustrates the dependency of the minimum turning radius on airspeed for a particular general aviation airplane.

![Fig. 3. Minimum Turning Radius Constraints from [17]](image)

Insight to identify the combination of airspeed and towing radius that may lead to structural damages, can be gained from Newton's Second Law applied to the UAV engaged in a steady and level coordinated turn. Note that for steady-state towing the force applied by the tow cable is expected to be relatively small and will be neglected for the purposes of this feasibility study. The horizontal and vertical directions then give:

\[ \frac{m_{\text{UAV}} v_{\text{UAV}}^2}{r_{\text{UAV}}} = L \sin \varphi_B \] (27)

\[ r_{\text{UAV}} = \frac{2 n_{\text{UAV}}}{\rho_a C_{\text{L, UAV}} \sin \varphi_B} \] (29)

In the equation above, \( n_{\text{UAV}} = \frac{m_{\text{UAV}}}{S} \) is the wing loading of the UAV. Thus in order to facilitate a small orbit radius, the wing loading must be small. It follows that the load limited minimum towing radius is:
\[ r_{\text{min LL}} = \frac{2n_{\text{UAV}}}{\rho_a C_{\text{L UAV}} \sin \varphi_{\text{B max}}} \]  

(30)

It is useful to note that the wing loading is a good measure of the general maneuvering performance of an aircraft. An aircraft with a low wing loading (large wings relative to its mass) will be able to generate more lift at any given speed and will be capable of performing tighter turns, while a high wing load factor is better suited for high speed flight since the smaller wings offer less drag.

Combining the expressions obtained from Newton's Second Law in the horizontal and vertical direction allows us to solve for the minimum possible coordinated turn radius in terms of the desired true airspeed and maximum bank angle:

\[ r_{\text{min } \nu} = \frac{v_{\text{UAV}}^2}{g \tan \varphi_{\text{B max}}} \]  

(31)

The orbit radius for a balanced banked turn is proportional to the square of the true airspeed and the orbit radius decreases with increasing UAV bank angle. The minimum allowable orbit radius for the towing aircraft is equal to the larger of the load limited and airspeed limited orbit radii.

III. OPTIMAL EQUILIBRIUM SOLUTION METHOD

A. System Properties, Definitions and Setup

For the purposes of numerical analysis, key system properties must be defined. These are summarized in Table 1. The towable is assumed to be a multifilament fishing line, and the properties of Spectra have been used in the analysis work.

B. Selection of Solution Method

The traditional approach to solving the circular towing equilibrium problem involves solving an initial value problem by applying a discrete shooting method to converge by successive iterations. The method involves specifying the motion (turning radius and speed) of the towing UAV and to guess the equilibrium value for the towed body radius. Next, the set of nonlinear equations of motion are integrated forward in time using the nonlinear system

<table>
<thead>
<tr>
<th>Property (Symbol)</th>
<th>Value</th>
<th>Data Source/Justification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density of Air</td>
<td>1.225 kg/m³</td>
<td>Use ISA sea-level value and ignore altitude dependence.</td>
</tr>
<tr>
<td>Basic Drag Coefficient for towable ((C_{\text{D tow}}))</td>
<td>1.1</td>
<td>Data from Figure 18 in [14]. Assumes subcritical Reynolds numbers.</td>
</tr>
<tr>
<td>Skin Friction Drag Coefficient for towable ((C_{\text{s}}))</td>
<td>0.02</td>
<td></td>
</tr>
<tr>
<td>Drag Coefficient for Spherical Towed Body ((C_{\text{p}}))</td>
<td>0.47</td>
<td>Data from Figure 10 in [14]</td>
</tr>
<tr>
<td>Ultimate tensile strength ((\sigma_{\text{ ult}}))</td>
<td>3000 MPa</td>
<td>Values assume the tow-cable is Honeywell Spectra 1000 Fiber and the properties are taken from: <a href="http://www.matweb.com">http://www.matweb.com</a></td>
</tr>
<tr>
<td>Modulus of Elasticity ((E))</td>
<td>172 GPa</td>
<td></td>
</tr>
<tr>
<td>Cable Material Density ((\rho_c))</td>
<td>970 kg/m³</td>
<td></td>
</tr>
<tr>
<td>Towed Body Radius ((r_{\text{B}}))</td>
<td>0.03 m</td>
<td>A reasonable guess for a UAV towed system assuming an object pickup application</td>
</tr>
<tr>
<td>Towed Body Mass ((m_B))</td>
<td>2 kg</td>
<td></td>
</tr>
<tr>
<td>Maximum Bank Angle ((\varphi_{\text{B max}}))</td>
<td>50 deg</td>
<td>A reasonable guess for a UAV</td>
</tr>
<tr>
<td>Minimum True Airspeed ((v_{\text{UAV min}}))</td>
<td>15 m/s</td>
<td></td>
</tr>
<tr>
<td>Maximum True Airspeed ((v_{\text{UAV max}}))</td>
<td>50 m/s</td>
<td></td>
</tr>
<tr>
<td>Wing Loading ((\alpha_{\text{UAV}}))</td>
<td>25 kg/m²</td>
<td>Selected a low value assuming that the UAV will be designed to accommodate tight turns.</td>
</tr>
<tr>
<td>UAV Lift Coefficient ((C_{\text{L UAV}}))</td>
<td>1.5</td>
<td>Ranges from 0.8 to 5, but will typically be between 1.2 and 1.6 [18].</td>
</tr>
<tr>
<td>Default Cable Diameter ((d))</td>
<td>0.002 m</td>
<td>Fixed value used to generate Fig. 5 and 6 in order to limit the number of variables.</td>
</tr>
<tr>
<td>Safety Factor (SF)</td>
<td>10</td>
<td>Used to take into account that the towable will need to be designed for higher loads than steady-state circle tow.</td>
</tr>
</tbody>
</table>

The "differentially flat system" approach is used in many recent research efforts on towed system dynamics, including by Williams and Trivailo [7] and Sun and Beard [9]. However, it should be noted that both these research groups avoid using this approach in later publications [8], [11] when solving for optimal trajectories in the presence of winds, presumably due to the difficulty taking the performance constraints of the system into account and due to obtaining non-smooth motion for the towing vehicle.

In order to solve for the optimal (periodic) trajectory to minimize towed body motion in the presence of winds or other disturbances, an Optimal Control Problem (OCP) can be formulated as done in [8] and [11]. The (guessed) initial positions of the point masses that defines the cable shape are integrated forward in time using the nonlinear system.
equations of motion and the cost function (objective) is formulated with the aim of minimizing the endpoint motion.

As the primary objective of the present analysis is to investigate the feasibility of UAV-sized circularly towed systems for endpoint precision missions, it is of essence to collect and analyze data across the potential design space for the circular towing parameters. Secondary, as the analysis contained in this report is in part preparation for more advanced analysis of towed systems subjected to realistic winds and disturbances, it is a goal that the methods developed here can easily be adopted for such work. Since the achievable towing performance for circularly towed systems can be determined in a very efficient manner from the "differentially flat approach", this method is used to explore the candidate design space for the towing parameters. The results will also benefit future efforts to solve the OCP for the dynamic system subjected to disturbances in the following ways:

1. The equilibrium positions of the towcable point masses resulting from the differentially flat analysis will be used as initial guess of cable shape for the OCP.
2. An optimization analysis performed on the differentially flat solution will define the cost function (objective) and constraints to be used in the OCP.
3. The large set of data possible to (easily) subtract from the differentially flat analysis can be used to verify the solutions obtained with the OCP.

In order to limit the scope of this study, the stability of the equilibrium solutions is not considered. Refer to [7], where Williams et al. demonstrates that all the optimal solutions they compute for a light general aviation aircraft are shown to be dynamically stable.

C. Differentially Flat Shooting Procedure for Equilibrium

Table II summarizes the procedure that is used to solve for the equilibrium cable shape using the differentially flat system assumption. All relative velocities and accelerations are assumed to be zero and the position of the point masses can be derived from the position of the end-mass. This discrete shooting procedure is also described in some detail in [8], but has been included here for completeness.

D. Optimization Problem

To find the UAV velocity and orbit radius within the feasible performance envelope that minimizes the steady-state orbit radius of the towed body, the shooting procedure from Table II was slightly modified and solved as an optimization problem. In addition to optimizing the UAV velocity and orbit radius, the UAV altitude is optimized such that the towed object is placed near the ground for a given length towable. As pointed out by others [7], it is also a good idea to optimize the towable diameter due to the fact that a longer cable can be shown to decrease the towed body orbit, but also increases the internal tension force of the cable due to the increased cable weight and drag. We have seen that the increase in maximum cable tension force leads to an increased minimum allowable cable diameter (equation 24), which leads to an additional increase in cable weight and drag force, so we have an iterative process. Since the optimization is performed on the differentially flat solution, the towed body orbit is included as an optimization variable since an initial guess of the position of the bottom point mass is required.

Hence the optimization control variables includes, the UAV speed ($V_{UAV}$), the UAV towing radius ($R_{TP}$), the towed body radius ($R_b$), the diameter of the towable (d) and the orbit height of the UAV (H). The cost function to be minimized is:

$$f = R_b^2 = (R_{TP} + x_0)^2 + y_0^2 + (H + z_0)^2$$ (32)

The cost function is subject to the system equations of motion hence the following constraints must be enforced:

$$d \geq SF \sqrt{\frac{4|T|_{\text{max}}}{\pi \sigma_{ut}}}$$ (33)

$$v_{UAV_{\text{min}}} \leq V_{UAV} \leq v_{UAV_{\text{max}}}$$ (34)

$$r_{UAV} \geq max\left(r_{\text{min}_{LL}}, r_{\text{min}_{H}}\right)$$ (35)

$$\phi_B \leq \phi_{B_{\text{max}}}$$ (36)

The constraint on the towable diameter has been multiplied with a safety factor (SF) to account for the fact that upsets such as winds and transitory flight phases are expected to generate significantly higher loads on the towable than what will result from the steady-state analysis performed here. While an exact minimum diameter is impossible to derive from the present analysis, it is viewed as beneficial to include the effect that increased length leads to increased loads in the optimization problem.

To efficiently arrive at the optimal solution to the optimization problem numerical optimization software has been employed. The open-source framework CasADi [17] was used in combination with the open-source NLP solver IPOPT.
IV. RESULTS

The solution of the differentially flat problem provided a quick and simple way to narrow down the design parameter search space. It also allowed a fairly accurate estimation of the initial positions for the point masses. In order to obtain reliable analysis results it was necessary to determine the number of point masses needed to ensure convergence of the solution. Fig. 4 shows the steady-state solution for an unconstrained configuration (H = 772 m, $V_{UAV} = 40$ m/s, $R_{TP} = 60$ m) for different numbers of towable elements (N). Based on this plot, 50 point masses were used for the remainder of the analysis in this section.

![Fig. 4. Convergence of the Towsystem Solution](image)

A simple verification of the model was accomplished by reproducing the towing performance for two light aircraft cases provided in Williams and Trivailo [7]. Table III defines the two scenarios and Table IV shows the results from the two studies. The reason why the two studies do not result in identical results is due to minor modelling differences, such as different aerodynamic coefficients for the towable.

<table>
<thead>
<tr>
<th>Case</th>
<th>$L$ (m)</th>
<th>$d$ (mm)</th>
<th>$m$ (kg)</th>
<th>$C_wA$ (m$^2$)</th>
<th>$R_{TP}$ (m)</th>
<th>$\omega$ (rad/s)</th>
<th>$\rho_c$ (kg/m$^3$)</th>
<th>$E$ (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>V1</td>
<td>3000</td>
<td>1.27</td>
<td>10</td>
<td>2.0</td>
<td>213.78</td>
<td>0.246</td>
<td>970</td>
<td>120</td>
</tr>
<tr>
<td>V2</td>
<td>1000</td>
<td>2.29</td>
<td>10</td>
<td>2.0</td>
<td>168.84</td>
<td>0.291</td>
<td>970</td>
<td>120</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case</th>
<th>Williams and Trivailo [7]</th>
<th>Present Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$R_B$</td>
<td>Cable Extension</td>
</tr>
<tr>
<td>V1</td>
<td>1.48 m</td>
<td>2.29 m</td>
</tr>
<tr>
<td>V2</td>
<td>6.11 m</td>
<td>0.24 m</td>
</tr>
</tbody>
</table>

The parameter that singularly determines the achievable steady-state performance of the towed system is the deployed length of the towable. Essentially, the longer the towable, the smaller the possible towed body orbit becomes. Of course this depends on the ability of the towing UAV to maintain the required orbiting conditions given the weight of the towable. As this feasibility study assumes that the UAV is not affected by the towable tension, a reasonable length towable must be selected for the study to remain relevant. In an attempt to capture a plausible towable range, analysis in this report has been performed for a 200 m and a 600 m long towable, noting that the towable length should be finally optimized when the towing UAV and powerplant is selected.

The performance of a towed UAV-sized system for the selected towable lengths (and using the towing parameters summarized in Table I) was evaluated as summarized in Fig. 5 and Fig. 6.
The feasibility of the towing configurations are illustrated by shading the regions that are constrained by the minimum or maximum airspeed or by the load limited or airspeed limited minimum towing radius. Note that the y-axis is plotted using Logarithmic scale. The two plots clearly show that it is the two constraints that apply to the minimum turning radius which restricts the achievable performance of the towed system. To obtain the exact optimal configurations of these baseline cases the optimization problem is solved numerically as detailed in III.C and are summarized in Table V. Note that since the cable diameter is held fixed, the constraint on this parameter is not applied.

<table>
<thead>
<tr>
<th>Case Length</th>
<th>Cable Length</th>
<th>OPTIMIZED CONFIGURATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline 200 m</td>
<td>20.38</td>
<td>35.52</td>
</tr>
<tr>
<td>Baseline 600 m</td>
<td>20.38</td>
<td>35.52</td>
</tr>
</tbody>
</table>

The shape of the towcable associated with the 600 m long baseline case is shown in Fig. 7.

![Fig. 7. Towcable Shape for 600m Long Baseline Case](image)

The optimization problem is then solved numerically for a selection of design parameters while also optimizing for the cable diameter. Table VI below compares the baseline cases (now slightly improved due to the optimized cable diameter) to the optimized configurations achieved when varying key design variable(s) as specified in the first column. Note that since the full UAV dynamics is not included in the model, the degradation in performance that will result due to the maximum thrust limitation is not reflected in the data. Specifically, it is expected that increasing the overall weight of the towed system (so either the weight of towed cable and/or the towed object) as well as increasing the drag of the towed body will be subject to additional performance constraints.

It is of interest to note that no valid solution was obtained for a Nylon towline with the selected safety factor of 10 applied to the constraint on minimum allowable cable diameter (equation 33). If the safety factor is reduced to 4 the case involving a 600 m long towline displays greatly reduced performance, while the performance of the case involving a 200 m long towline is only slightly reduced. Even if the selected safety factor may be a bit excessive, this observation indicates that selecting a high strength to weight ratio towline and a suitable towing altitude is critical to the success of the circularly towed mission. As expected, it is seen that increasing the maximum bank angle, reduces the minimum UAV turn radius, hence leads to a small orbit formed by the towed object. Increasing the drag on the towed object has no effect mainly due to the very low airspeeds, absence of winds and the fact that UAV thrust is not included in the model.

<table>
<thead>
<tr>
<th>Case</th>
<th>Cable Length</th>
<th>OPTIMIZED CONFIGURATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline 200 m</td>
<td>20.38</td>
<td>35.52</td>
</tr>
<tr>
<td>Baseline 600 m</td>
<td>20.38</td>
<td>35.52</td>
</tr>
<tr>
<td>m = 1 kg</td>
<td>20.38</td>
<td>35.52</td>
</tr>
<tr>
<td>m = 3 kg</td>
<td>20.38</td>
<td>35.52</td>
</tr>
<tr>
<td>Nylon³ Tillone</td>
<td>20.38</td>
<td>35.52</td>
</tr>
<tr>
<td>Nylon³ Tillone, SF = 4</td>
<td>20.38</td>
<td>35.52</td>
</tr>
<tr>
<td>Nylon³ Tillone, SF = 2</td>
<td>20.38</td>
<td>35.52</td>
</tr>
<tr>
<td>Nylon³ Tillone, SF = 1</td>
<td>20.38</td>
<td>35.52</td>
</tr>
<tr>
<td>ϕmax = 40 deg</td>
<td>20.38</td>
<td>35.52</td>
</tr>
<tr>
<td>ϕmax = 60 deg</td>
<td>20.38</td>
<td>35.52</td>
</tr>
<tr>
<td>Streamline endbody Cdp = 0.05</td>
<td>20.38</td>
<td>35.52</td>
</tr>
<tr>
<td>Parachute endbody Cdp = 0.25</td>
<td>20.38</td>
<td>35.52</td>
</tr>
<tr>
<td>Parachute endbody Cdp = 1.3</td>
<td>20.38</td>
<td>35.52</td>
</tr>
</tbody>
</table>

V. CONCLUSIONS

A feasibility study of the use of UAV towed cable-body systems for use in endbody precision maneuvers has been performed and the results indicate that the concept is viable and warrants further study.

Analysis show that the minimum possible UAV turning radius results in the smallest achievable orbit for the towed body at a given towable length. With respect to UAV design, a low wing loading is required for the ability to perform small radius turns, as it allows generation of more lift at any speed. The lowest allowable speed in combination with the highest allowable bank angle will result in the smallest radius turn achievable for a particular UAV. Of course carrying a...
large payload results in high wing loading, effectively limiting the weight of the towed system. A few design strategies that may be considered in an attempt to get the "best of both worlds", i.e. low drag and higher cruise speeds for longer range missions with some amount of useful payload as well as the increased maneuverability offered by low wing load factor, includes variable sweep wings, Fowler flaps and to some extent blended wing-fuselage designs.

In addition to a small radius UAV turn, the length of the towcable and the towable material are important design parameters. A longer towcable results in a smaller orbit for the towed object as long as the cable diameter and ultimate strength allows the resulting increase in cable tension (due to the added weight).

The results from this study will be used to formulate and solve an OCP in order to analyze strategies to minimize the motion of the towed object when the towed UAV system is subjected to winds and other disturbances.

ACKNOWLEDGEMENT

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REFERENCES