AT-204

THERMO-MECHANICAL PROPERTIES OF MATERIAL

LABORATORY REPORT

Some Characteristics of the Sea Ice in the Van Mijen Fjord during the Spring 2000

MARCH 1st to 3rd, APRIL 12th to 14th and MAY 4th 2000

Students

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1. INTRODUCTION

The van Mijen Fjord is well shelter from the open sea by Akseløya. Further, the island hinders the easterly winds to empty the fjord for winter ice. Thus the ice conditions are stable throughout the year and very minor ridging is observed. Riding is mainly observed close to the mouth of the fjord (Løset, 1998; Høyland; 1999, 2000). The present paper discusses aspects related to the ice condition in the fjord and forces the ice may exert on coastal structures in the inner part of the fjord, more precisely at Kapp Amsterdam. The site is shown in Fig. 1.1.
The most important factors that governs the ice load are:

- Ice feature
- Ice properties
- Scenario of interaction
- Structure geometry.

The ice properties strongly affect the ice loads. The most important parameters are the ice strength and the ice thickness. The ice strength depends on the internal structure (granular or columnar type of ice), temperature, grain size, salinity, porosity.

With this in mind we collected the most relevant data and made laboratory analysis at UNIS for those parameters that we could not sample in the field.

While Chapter 2 of the report presents the ice characteristics, Section 2.6 elaborates on ice thickness growth. Chapter 3 gives a comparison of the theoretical estimated ice thickness and the actual measured ones. The reported is rounded off by a brief discussion of the obtained results and does also highlight them in view of earlier year's findings in this area.

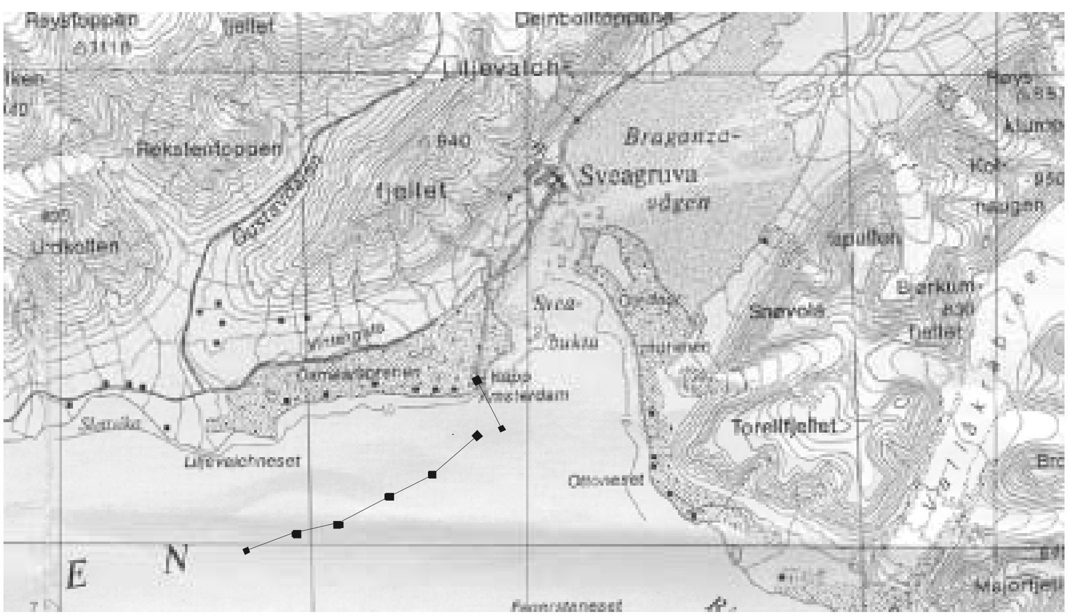

Fig. 1.1 Site map showing the location of the ice sampling in the Van Mijen fjord.

2. ICE PARAMETERS IN THE VAN MIJEN FJORD

2.1 Ice thickness and snow depth

SEA ICE INVESTIGATIONS 01 MARCH, 2000

The ice thickness was measured in two parallel profiles perpendicular to the qua at Kapp Amsterdam. The two lines were separated by about 50 m. Line 1 was located east of Line 2.

Table 2.1.1 and Fig. 2.1.1 show the variation in ice thickness, snow depth and freeboard with distance from the shore for Line 1.

Table 2.1.1 Ice thickness profiles 01.03.00 along Line 1, Kapp Amsterdam.

Distance from shore, m	Thickness, cm		
	Ice	Snow	Freeboard
25	59	9	2.5
50	60	6	5
75	69	11	6
100	60	5	4
125	60	18	3
150	62	8	3
175	62.5	20	1
200	78	12	5
225	69	10	6
250	70.5	6	5.5
275	69	30	4.5
300	76	6	7.5
325	63	13	5
350	69	11	5.5
375	54	10	6
400	64	17	3

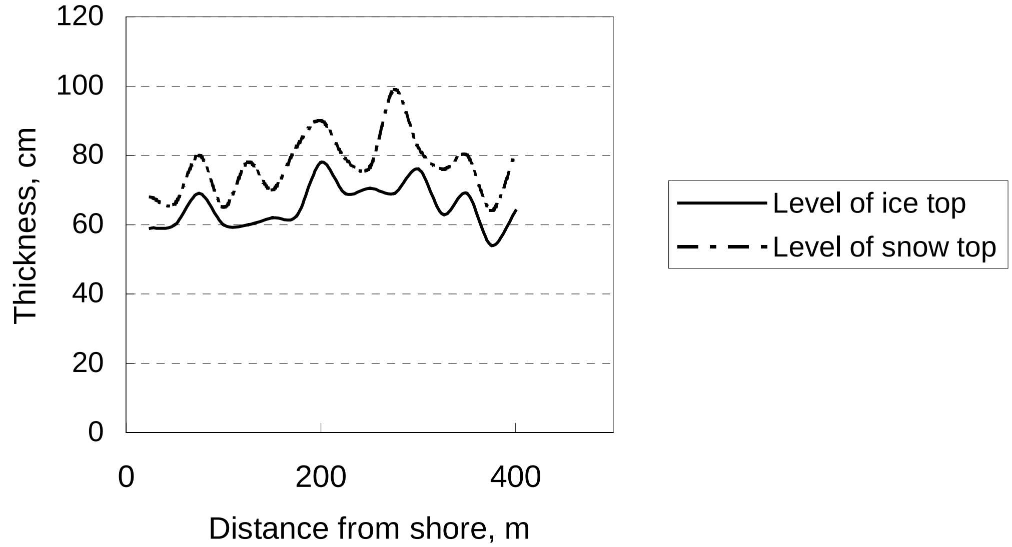

Fig. 2.1.1 Ice thickness, snow depth as measured 01 March 2000, off Kapp Amsterdam. Line 1

The ice thickness various from 59 cm to 78 cm. The ice thickness profile has non-linear distribution and contents some picks. The positive values for freeboard level were obtained. The behaviour of snow spreading has similar character.

The average ice thickness, snow depth and standard deviations for these values are:

$\bar{h} = 65.3 \pm 6.5$ cm, $\bar{S}_n = 12.0 \pm 6.5$ cm

Table 2.1.2 and Fig. 2.1.2 show the variation in ice thickness, snow depth and freeboard with distance from the shore for Line 2.

Table 2.1.2. Ice thickness profiles 01.03.00 along Line 2, Kapp Amsterdam.

Distance from shore, m	Thickness, cm		
	Ice	Snow	Freeboard
25	52	14	3
50	60	12	4
75	100	8	5
100	60	8	3
350	78	7	6
375	60	18	1
400	69	9	5

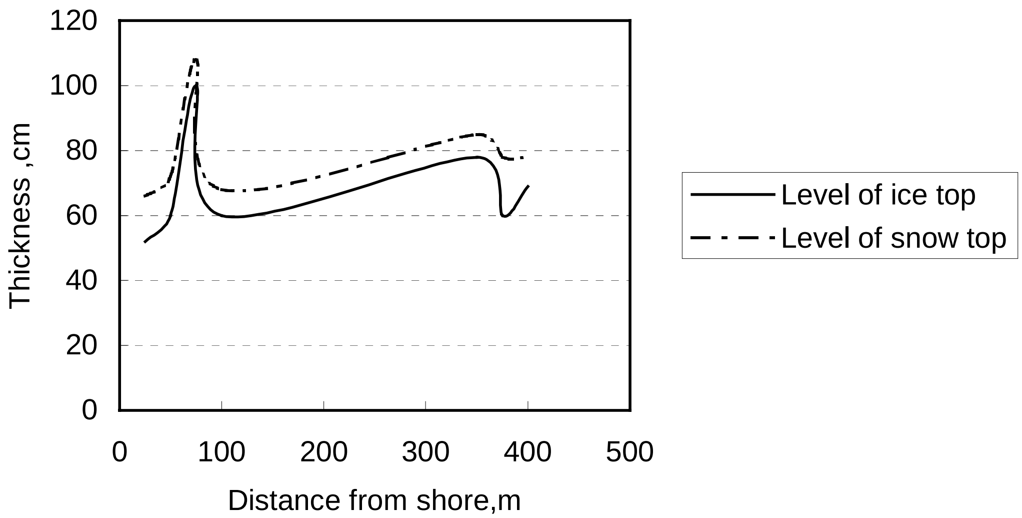

Fig. 2.1.2.Ice thickness, snow depth and as measured 01 March 2000, off Kapp Amsterdam.
Line 2

The ice thickness various from 52 cm to 100 cm. The ice thickness profile may be expressed as linear distribution. It has strong tendency to increase from the shore. The huge pick, which located at 75 m from the shore, is two times bigger that the average ice thickness along whole profile. It may be explain due to following assumption: some rafted ice inclusion takes place in this location. The rafted ice was appeared in this position due to vessel track, which may took place there some time before our observations. The positive values for freeboard level were obtained. The behaviour of snow spreading has similar character.

The average ice thickness, snow depth and standard deviations (huge pick is exclude) for these values are:

$\bar{h} = 63.2 \pm 9.0$ cm, $\bar{S}_n = 10.9 \pm 4.0$ cm

SEA ICE INVESTIGATIONS 12-13 APRIL, 20

The ice thickness was measured along Line 1 from the previous survey (01 March 2000). Table 2.1.3 and Fig. 2.1.3 show the variation in ice thickness, snow depth and freeboard with distance from the shore.

Table 2.1.3 Ice thickness profiles 12.04.00 along Line 1, Kapp Amsterdam.

Distance from shore. m	Thickness, cm		
	Ice	Snow	Freeboard
175	89	28	-1
200	82.5	24	-1
225	77	36	-2
250	84	29	0
275	82	36	-2
300	91	38	0.5
325	84	46	-0.5
350	77	40	-3
375	81	44	-2
400	74	37	-1
425	74	33	-1
450	81	36	-1
475	72	33	-4
500	77	32.5	-0.3
525	75.5	31.5	1.5
550	66	40	-4
575	81	41	-3

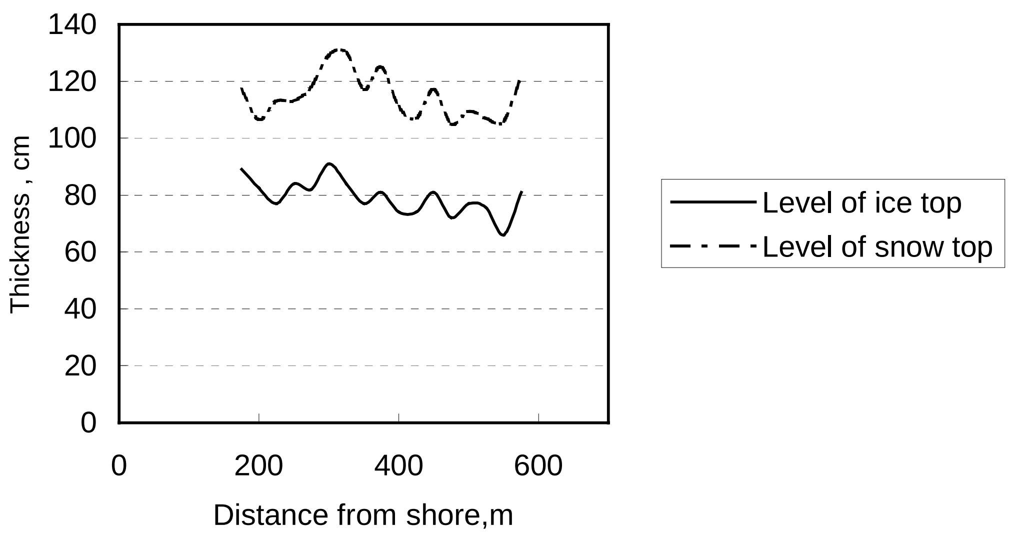

Fig. 2.1.3 Ice thickness, snow depth and as measured 12-13 April 2000, off Kapp Amsterdam.

Line 1

The ice thickness various from 92 cm to 66 cm. The ice thickness profile has non-linear distribution and contents some picks. Due to large amount of snow on the ice, which induced the

big pressure influences on the water masses, the negative freeboard level takes place. The behaviour of snow spreading has similar character, as ice thickness profile.
The average ice thickness, snow depth and standard deviations for these values are:

$\bar{h} = 79.3 \pm 6.2$ cm, $\bar{S}_n = 35.6 \pm 5.8$ cm

Two profiles for the Line 1, which measured at 01 March and 12 April, were represented on Fig 2.1.4

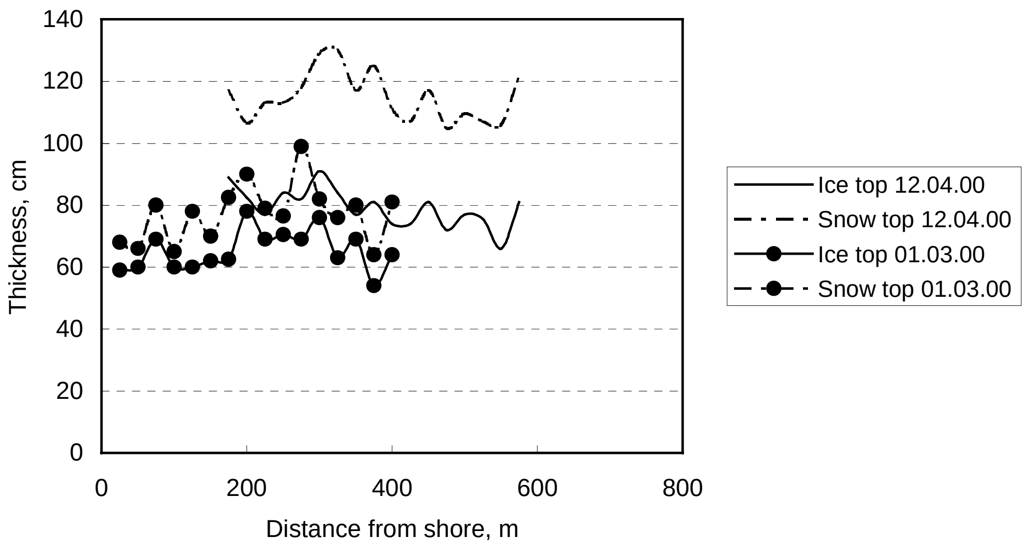

Fig. 2.1.4 Ice thickness, snow depth and as measured at 01 March and 12-13 April 2000, off Kapp Amsterdam, Line1.

Basic on the Fig. 2.1.4, the following suggestions were provided:

- The ice thickness was increasing from April to March
- The snow depth was increasing from April to March
- The freeboard level was positive in March and negative in April.

According these, the parameters, which obtains in April, have main importance for ice load design, as contents the extreme values for the ice thickness.

On 13 April we took a section of the ice thickness. Six different drillings were made, each separated by about 1 km. This section was south -west starting about 600 m off Kapp Amsterdam.

The results of measurements is represented in Table 2.1.4 and Fig. 2.1.5

Table 2.1.4 Ice thickness profiles 13.04.00.

Position, system Geographic coordinates	Position, system 33X / UTM	Thickness, cm		
		Ice	Snow	Freeboard
N77°50.798' E016°39.625'	0539020 / 8642057	76	33	-4
N77°50.590' E016°37.621'	0538105 / 8641644	83	29	-1
N77°50.330' E016°34.468'	0537024 / 86411131	89	24	2.5
N77°49.133' E016°32.118'	0536113 / 8640740	94	17	6
N77°49.942' E016°29.235'	0534992 / 8640356	89	26	-1
N77°49.760' E016°26.939'	0534100 / 8639995	80.5	29	-1.5

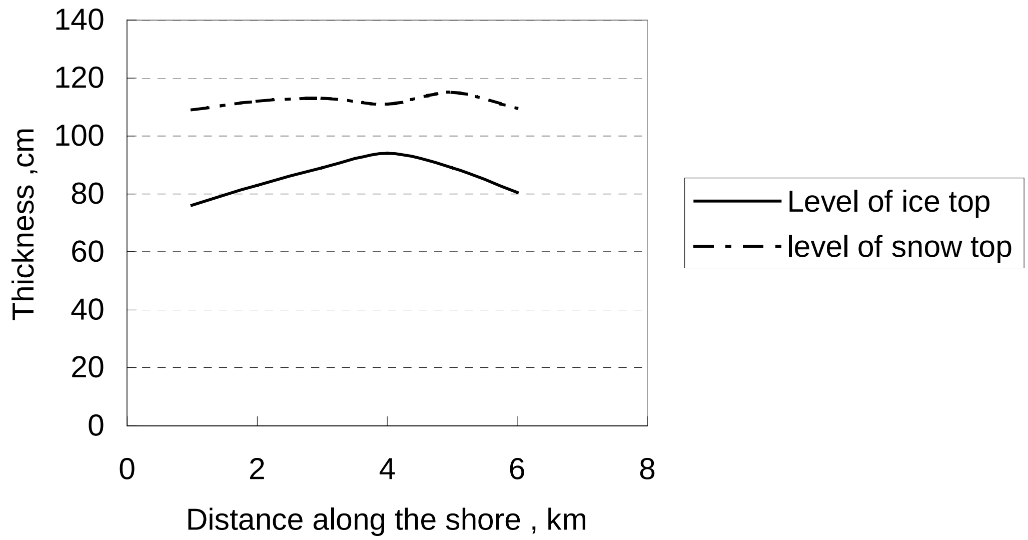

Fig. 2.1.5 Ice thickness profile along the shore as measured at 13 April 2000, off Kapp Amsterdam.

The average ice thickness, snow depth and standard deviations for these values are:

$\overline{h} = 85.3 \pm 6.6$ cm, $\overline{S}_n = 26.3 \pm 5.5$ cm

SEA ICE INVESTIGATIONS 04 MAY, 2000

On 4th of May we took the same section of the ice thickness, as on 13th of April. Six different drillings were made, each separated by about 1 km. This section was south -west starting about 600 m off Kapp Amsterdam.

The results of measurements is represented in Table 2.1.5 and Fig. 2.1.6

Table 2.1.5 Ice thickness profiles 04.05.00.

Position, system Geographic coordinates	Position, system 33X / UTM	Thickness, cm		
		Ice	Snow	Freeboard
N77°50.798' E016°39.625'	0539020 / 8642057	93	26	0
N77°50.590' E016°37.621'	0538105 / 8641644	79	25	3
N77°50.330' E016°34.468'	0537024 / 86411131	90	34	1
N77°49.133' E016°32.118'	0536113 / 8640740	85	33	-5
N77°49.942' E016°29.235'	0534992 / 8640356	84	33.5	-1
N77°49.760' E016°26.939'	0534100 / 8639995	81	13	0

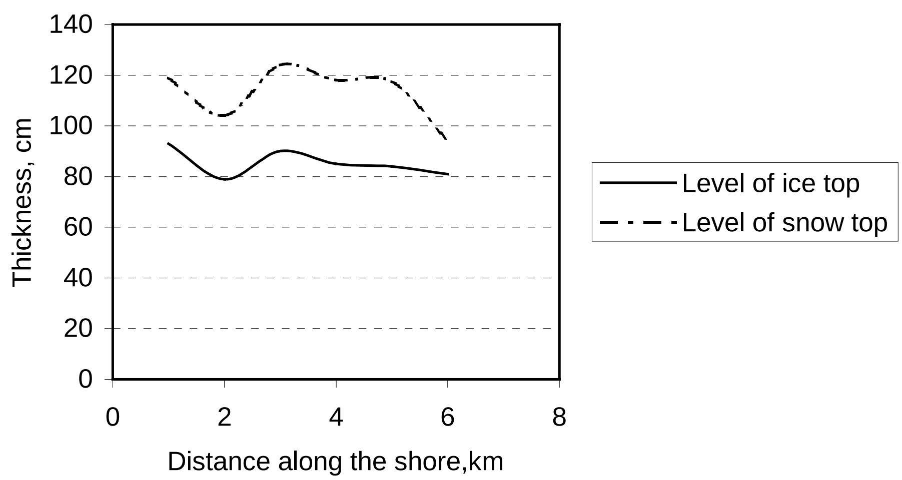

Fig. 2.1.6 Ice thickness profile along the shore as measured at 04 May 2000, off Kapp Amsterdam.

The average ice thickness, snow depth and standard deviations for these values are:

$\overline{h} = 85.3 \pm 5.3\,\text{cm}$, $\overline{S}_n = 27.4 \pm 8.1\,\text{cm}$

Two profiles for the section along the shore, which measured at 13 April and 04 May, were represented on Fig 2.1.7

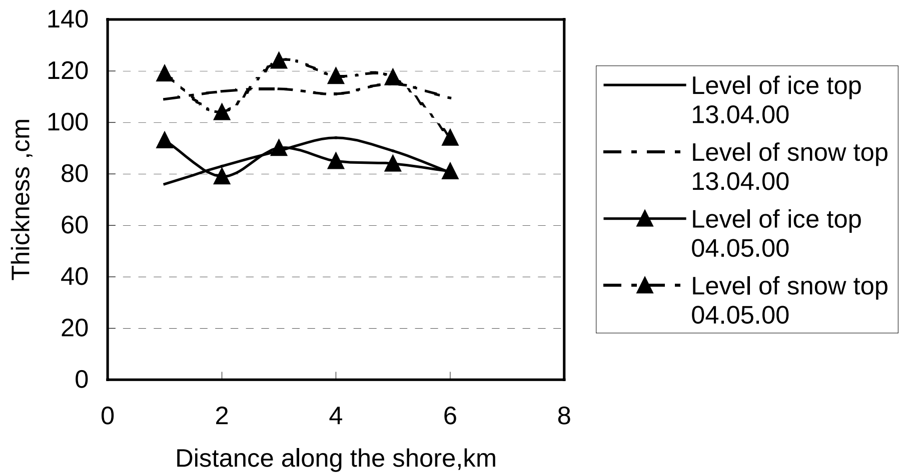

Fig. 2.1.7 Ice thickness, snow depth along the shoreline as measured at 13 April 2000 and 04 May 2000, off Kapp Amsterdam.

Basic on the Fig. 2.1.4, the following suggestions were provided:

- The ice thickness and snow depth various approximately in the same ranges, but the April measurements have the biggest values, except first point.
- The distributions of ice thickness and snow depth are more quite in April to compare with profiles in May, which contents some picks.
- The average ice thickness, snow depth and standard deviations are:

$\overline{h} = 85.3 \pm 6.6\,\text{cm}$, $\overline{S}_n = 26.3 \pm 5.5\,\text{cm}$ for April measurements, $\overline{h} = 85.3 \pm 5.3\,\text{cm}$, $\overline{S}_n = 27.4 \pm 8.1\,\text{cm}$ for May. It's possible to see that the values are the same.

- The values for freeboard various from positive to negative, which depends on ice thickness and snow depth.

The measurements in May, are agree with the following suggestions which is mentioned before, that the parameters, which obtains in April, have main importance for ice load design, as contents the extreme values for the ice thickness.

Also measurements at one point of Line 1 was investigated. All parameters were obtained for position 200m from shore.
Table 2.1.6 represents all results, which obtain during our investigations at position 200m from shore for Line1.

Table 2.1.6 Ice thickness, snow depth, freeboard for Line 1, 200 m off Kapp Amsterdam.

Data	Thickness, cm		
	Ice	Snow	Freeboard
01.03.00	78	12	5
12.04.00	82.5	24	-1
04.05.00	88	28	-2

2.2 Ice density

SEA ICE INVESTIGATIONS 12-13 APRIL, 2000

The weight –scale was using during this experiment. The mass of sample was measured. The volume of sample was obtaining basic on classical Archimed law: the mass of water, which was displaced out by drown down the sample, was weighed.

Core 1 was taken at 12 April 2000 175 m off Kapp Amsterdam. The core provided samples for thin sections and close by a separate core was used for density, temperature and salinity measurements. The results for density measurements are shown in Table 2.2.1 and Fig. 2.2.1

Table 2.2.1. Core 1, density profile, 12 April 2000 175 m off Kapp Amsterdam

Depth ,cm	Volume,cm^3	Mass, g	Density, g/cm^3
10-12	772	644	0.834
12-26	870	805	0.925
26-42	907	850	0.937
42-55	835	728	0.871

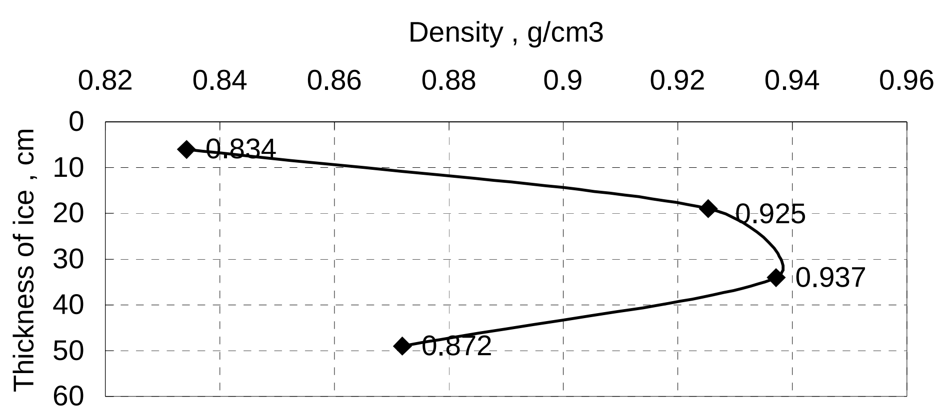

Fig. 2.2.1 Core 1, density profile, 12 April 2000 175 m off Kapp Amsterdam

The average density and standard deviation for this value are:

$\overline{\rho} = 0.892 \pm 0.048$ g/cm^3.

Core 3 was sampled 15 April 2000 200 m off Kapp Amsterdam.
Table 2.2.2 and Fig. 2.2.2 show the density profile

Table 2.2.2 Core 3, density profile, 15 April 2000 200 m off Kapp Amsterdam

Depth,cm	Volume,cm^3	Mass, g	Density, g/cm^3
10-12	607	540	0.890
12-22	620	580	0.935
22-36	834	774	0.928
36-49	840	788	0.938
49-70	1227	1119	0.911

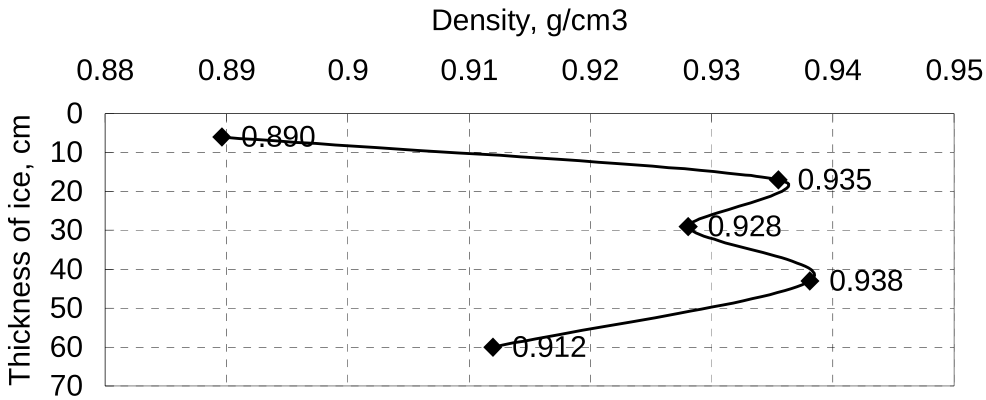

Fig. 2.2.2 Core 3, density profile, 15 April 2000 200 m off Kapp Amsterdam.
The average density and standard deviation for this value are:
$\overline{\rho} = 0.921 \pm 0.020\,g/cm^3$.

Core 4 was sampled 15 April 2000 250 m off Kapp Amsterdam.
Table 2.2.3 and Fig. 2.2.3 show the density profile

Table2.2.3. Core 4, density profile, 15 April 2000 250 m off Kapp Amsterdam

Depth,cm	Volume,cm^3	Mass, g	Density, g/cm^3
10-12	706	768	0.919
12-24	675	723	0.934
24-40	834	915	0.911
40-59	1057	1165	0.907
59-80	1148	1232	0.932

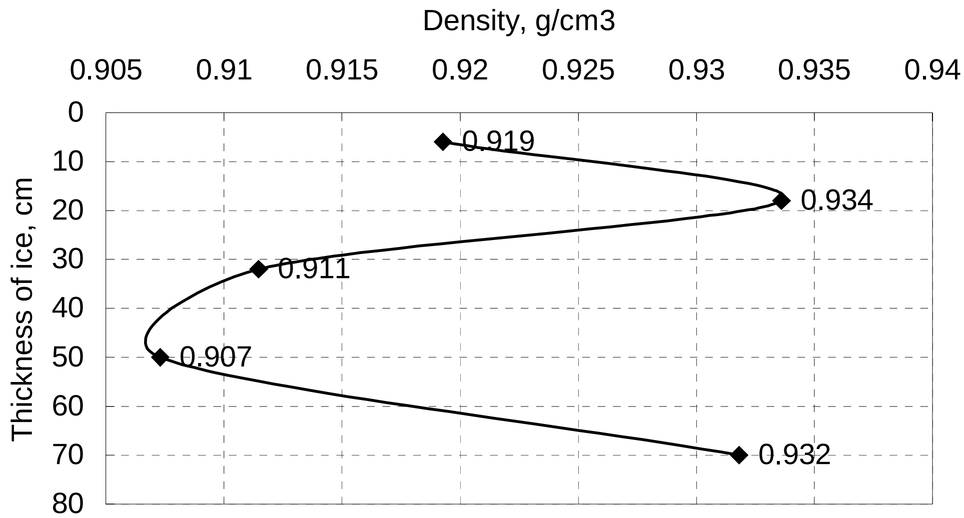

Fig. 2.2.3 Core 4, density profile, 15 April 2000 250 m off Kapp Amsterdam
The average density and standard deviation for this value are:
$\overline{\rho} = 0.921 \pm 0.011 \, \mathrm{g/cm^3}$.

The Fig 2.2.4 represents the all density profiles.

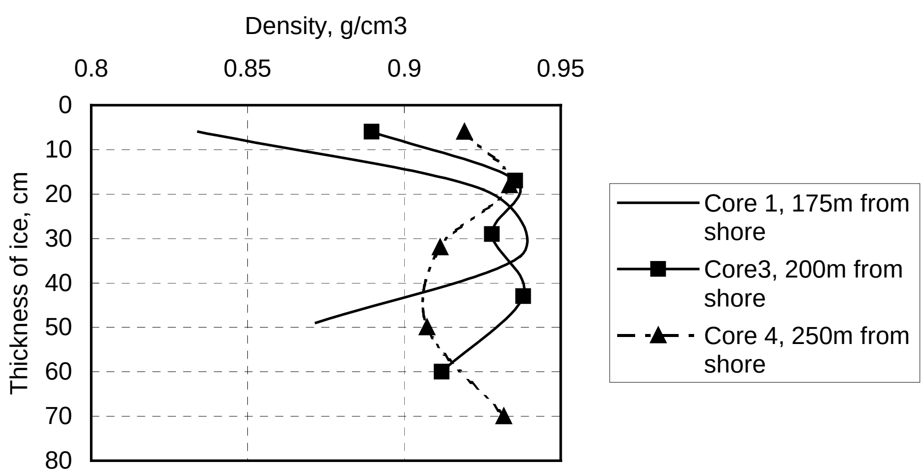

Fig. 2.2.4 Density profiles, April 2000 off Kapp Amsterdam

2.3 Salinity profile

SEA ICE INVESTIGATIONS 01 MARCH, 2000

The core was taken 2 March 2000 on Line 1, 200 m off Kapp Amsterdam. The salinity measurements were provided at 3 March in the laboratory of UNIS.

The conductivity measuring instrument “Testo 240”, which has possibility to measure the salinity directly without any additional calculation, were used. It contents the probe Typ (e) 07.
At first the sample melted in the oven, to water condition. After that, the samples were left for some time, until it should reached the inside temperature. During this time the instrument was tested. After those the measurements were provided.

Table 2.3.1 and Fig 2.3.1 show the results.

Table 2.3.1. Salinity profile 02.03.00 on Line 1, 200 m off Kapp Amsterdam

Depth, cm	Salinity, mg/l
0-12	5.53
12-24	3.64
24-35	2.86
35-50	6.86

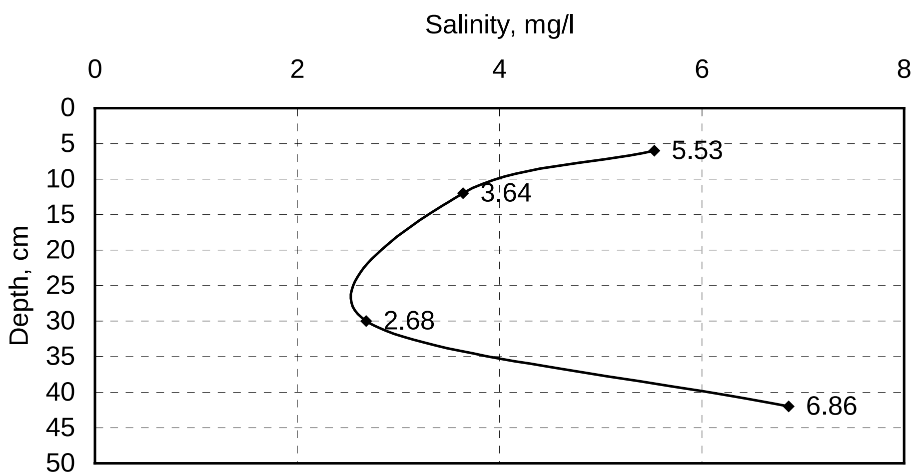

Fig. 2.3.1 Salinity profile 02.03.00 on Line 1, 200 m off Kapp Amsterdam

The average salinity and standard deviation for this value are:

$\overline{S} = 4.72 \pm 1.81$ g/cm^3.

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Two cores were taken at 13th of April 2000 in fjord near Kapp Amsterdam. The salinity measurements were provided at 14th of April in Laboratory at UNIS.
Core 5 and 6 were located at position 175 m and 250 m from shore respectively.

Core 5
The results are shown in Table 2.3.2 and Fig.2.3.2

Table 2.3.2 Core 5 Salinity profile 13.04.00, 175m off Kapp Amsterdam

Depth, m	Salinity, mg/l
0-10	2.26
10-20	2.33
20-35	2.25
35-65	2.36

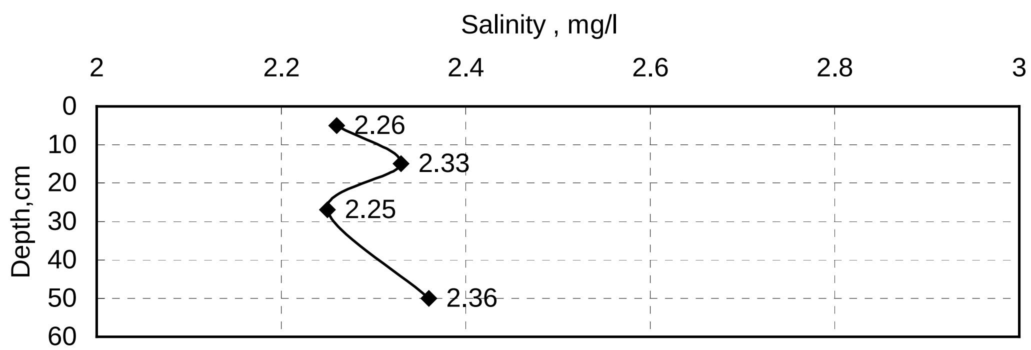

Fig. 2.3.2 Core5 Salinity profile 13.04.00, 175m off Kapp Amsterdam

The average salinity and standard deviation for this value are:

$\overline{S} = 2.30 \pm 0.05$ g/cm^3.

Core6

Table 2.3.3and Fig.2.3.3 show the results.

Table 2.3.3 Core6 Salinity profile 13.04.00, 250m off Kapp Amsterdam

Depth, m	Salinity, mg/l
0-12	2.79
12-22	1.89
22-32	2.68
32-43	2.67
43-54	3.06
54-71	3.44
71-81	4.19

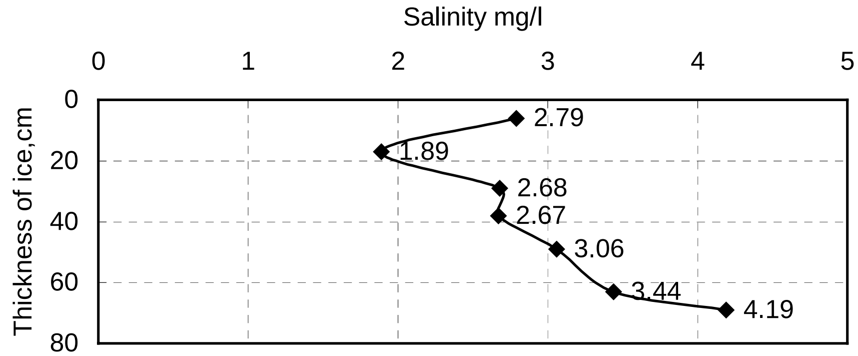

Fig.2.3.3 Core 6 Salinity profile 13.04.00, 250m off Kapp Amsterdam

The average salinity and standard deviation for this value are:

$\overline{S} = 2.96 \pm 0.71\,\mathrm{g/cm}^3$.

The Fig. 2.3.4 represents all salinity profiles.

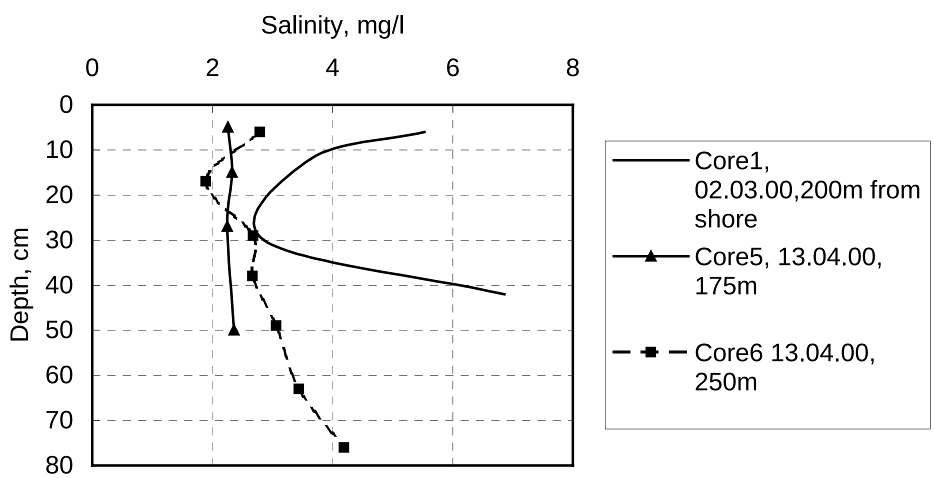

Fig. 2.3.4 Salinity profiles March, April 2000 off Kapp Amsterdam

2.4. Ice temperature

SEA ICE INVESTIGATIONS 12-13 APRIL, 2000

As mentioned before Core 1 was taken at 12 April 2000 175 m off Kapp Amsterdam. The core provided samples for thin sections and close by a separate core was used for density, temperature and salinity measurements.

The results of temperature measurements shown at Table 2.4.1 and Fig 2.4.1

Table 2.4.1 Core 1, temperature profile, 12 April 2000 175 m off Kapp Amsterdam

Depth ,cm	Temperature, °C
0	-7.1
10	-6.9
20	-7.1
30	-6.2
40	-4.7
50	-5.5
60	-4.8
70	-4.5
80	-4
90	-4.2

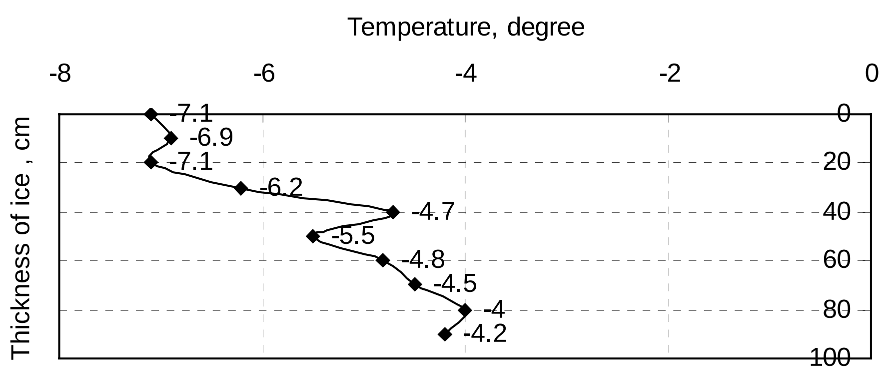

Table 2.4.1Core 1, temperature profile, 12 April 2000 175 m off Kapp Amsterdam

Core 2 was taken at 13 April 2000 175 m off Kapp Amsterdam.
The results of temperature measurements shown at Table 2.4.2 and Fig 2.4.2

Table 2.4.2 Core 2, temperature profile, 13 April 2000 175 m off Kapp Amsterdam

Depth ,cm	Temperature, °C
0	-5.7
10	-5.4
20	-5.0
30	-4.2
40	-3.3
50	-2.8
60	-2.5
70	-2.0

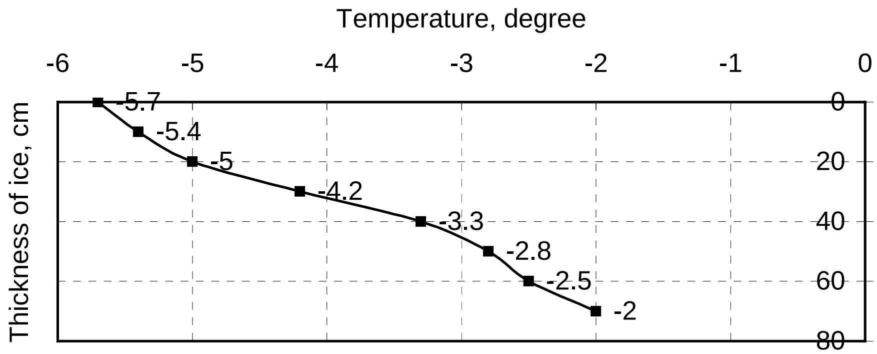

Fig. 2.4.2 Core 2, temperature profile, 13 April 2000 175 m off Kapp Amsterdam

Core 3 was sampled 15 April 2000 200 m off Kapp Amsterdam.
Table 2.4.3 and Fig. 2.4.3 show the temperature profile.

Table 2.4.3. Core 3, temperature profile, 13 April 2000 200 m off Kapp Amsterdam

Depth,cm	Temperature, °C
0	-5.3
15	-3.9
25	-4.1
35	-3.3
45	-2.8
55	-2.6
65	-2.1

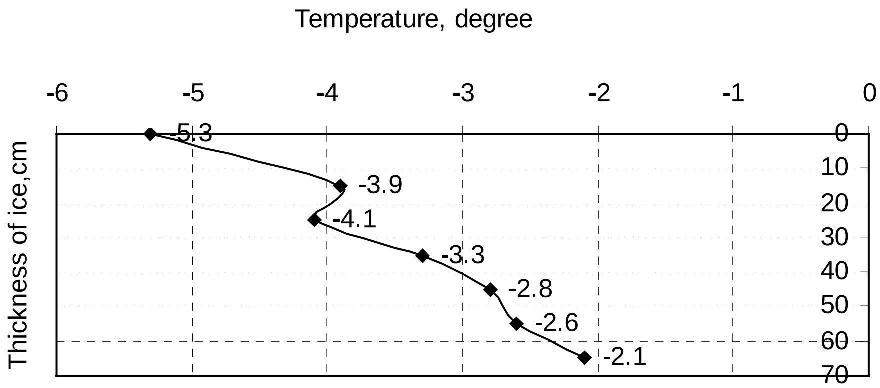

Fig. 2.4.3 Core 3, temperature profile, 13 April 2000 200 m off Kapp Amsterdam

Core 6 was taken at 13 April 2000 250 m off Kapp Amsterdam

Table 2.4.4 and Fig. 2.4.4 show the temperature profile.

Table 2.4.4 Core 6, temperature profile, 13 April 2000 250 m off Kapp Amsterdam

Depth, cm	Temperature, °C
0	-5.7
10	-6.3
20	-6.2
30	-4.1
40	-3.5
50	-4.1
60	-4.0
70	-2.5

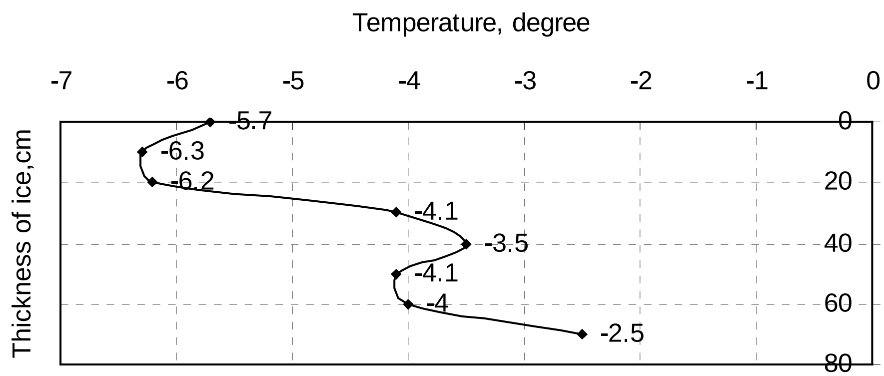

Fig. 2.4.4 Core 6, temperature profile, 13 April 2000 250 m off Kapp Amsterdam

Fig. 2.4.5 represents all temperature profiles.

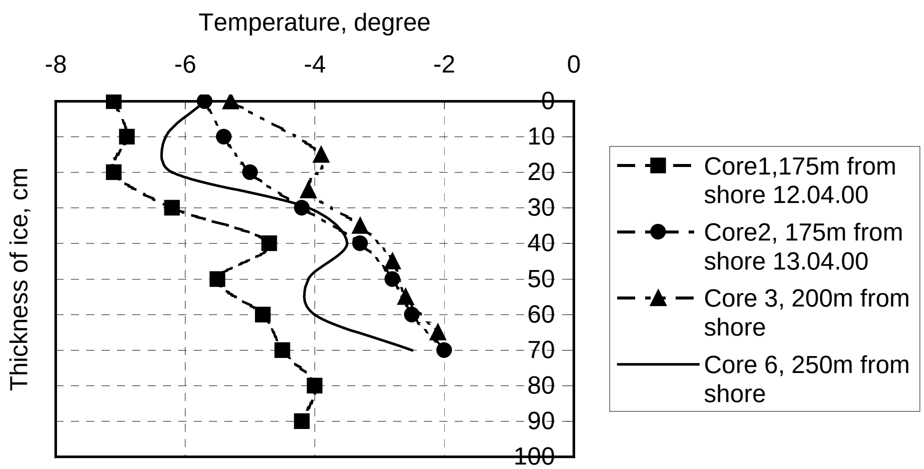

Fig.2.4.5 Temperature profiles, April 2000 off Kapp Amsterdam

2.5 Ice porosity

Core 6 was taken at 13th of April 2000 in fjord near Kapp Amsterdam. It was located at position 250 m from shore. The temperature profile measurements were done at that day. The salinity measurements were provided at 14th of April in Laboratory at UNIS.
Basis on these data the brine volume that is similar to ice porosity was calculated. The following equation (Sanderson, 1988) was used:

$$v_b = 0.001S(0.53 - 49.2/\theta) \qquad (2.4.1)$$

where S - salinity , ppt
θ - temperature , °C.

Table2.5.1 Core 6 brine volume profile, 13 April 2000 250 m off Kapp Amsterdam

Depth, cm	Salinity, mg/l	Temperature,°C	Brine volume ,%
10	3.12	-5.7	2.84
20	2.79	-6.3	2.33
30	1.89	-6.2	1.6
40	2.68	-4.1	3.36
50	2.67	-3.5	3.89
60	3.06	-4.1	3.83
70	3.44	-4	4.41
80	4.19	-2.5	8.47

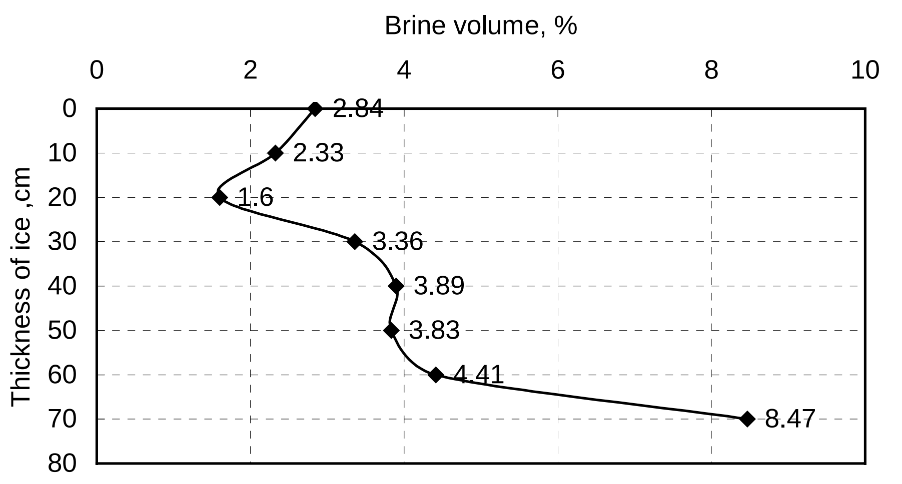

Fig. 2.5.1 Core 6, Brine volume profile, 13 April 2000,250 m off Kapp Amsterdam

2.6 Grain size, thin section

Ice cores were taken 3rd of March 2000 on line 1, 200m off Kapp Amsterdam in the Van Mijen fjord and 12th of April 2000 at the same site. They were taken back to lab and analysed by using thin section analysis. The samples were photographed with and without polarisation filter. Then the grain sizes were measured with a ruler.

The Table 2.6.1 represents the results of the grain size distribution and standard deviation, the Fig, 2.6.1 and Fig 2.6.2 show the profiles for these values respectively.

Table 2.6.1 Grain size distribution, March 2000 off Kapp Amsterdam

3/3/00	20 cm	42 cm	54 cm
	2.78	7.22	6.52
	1.11	10.00	9.57
	2.22	8.89	6.09
	1.67	3.89	6.09
	1.67	6.11	10.87
	1.67	5.56	12.17
	1.67	3.33	13.04
	2.22	9.44	3.91
	1.11	3.89	14.35
	2.78	2.78	3.04
	2.22	4.44	
	1.11	5.56	
	1.11	4.44	
	1.67		
	1.67		
	1.67		
	2.22		
	2.22		
	1.11		
	1.11		
	5.00		
	5.00		
	4.44		
	7.22		
	3.89		
	7.78		
	3.89		
Average	**2.67**	**5.81**	**8.57**
Standard deviation	**1.82338**	**2.396149**	**3.963973**

The Table 2.6.2 represents the results of the grain size distribution and standard deviation, the Fig, 2.6.1 and Fig 2.6.2 show the profiles for these values respectively.

Table 2.6.2 Grain size distribution, April 2000 off Kapp Amsterdam

12/4/00	3 cm	15 cm	30 cm	40 cm	57 cm
	2.50	1.25	1.25	4.12	29.41
	2.08	0.83	13.13	5.29	20.59
	1.67	1.67	3.13	2.94	17.65
	1.25	1.67	2.50	5.88	10.00
	1.67	1.25	10.63	5.29	7.65
	2.08	1.67	5.00	10.00	11.76
	2.50	1.25	4.38	2.35	24.71
	1.25	0.83	10.63	1.76	14.12
	1.25	0.83		11.18	
	0.83	0.83			
	0.83	1.67			
	0.42	1.67			
	0.83	2.08			
		1.67			
Average	**1.47**	**1.37**	**6.33**	**5.42**	**16.99**
Standard deviation	**0.672186**	**0.414371**	**4.462611**	**3.258975**	**7.549065**

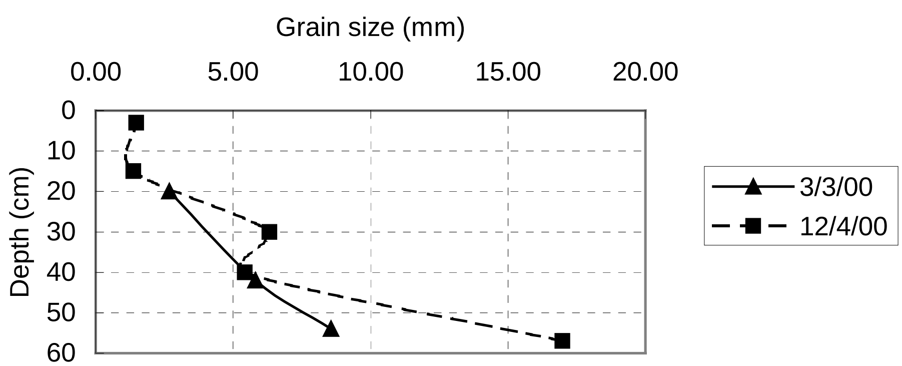

Fig 2.6.1 Grain size distribution, March, April 2000 off Kapp Amsterdam

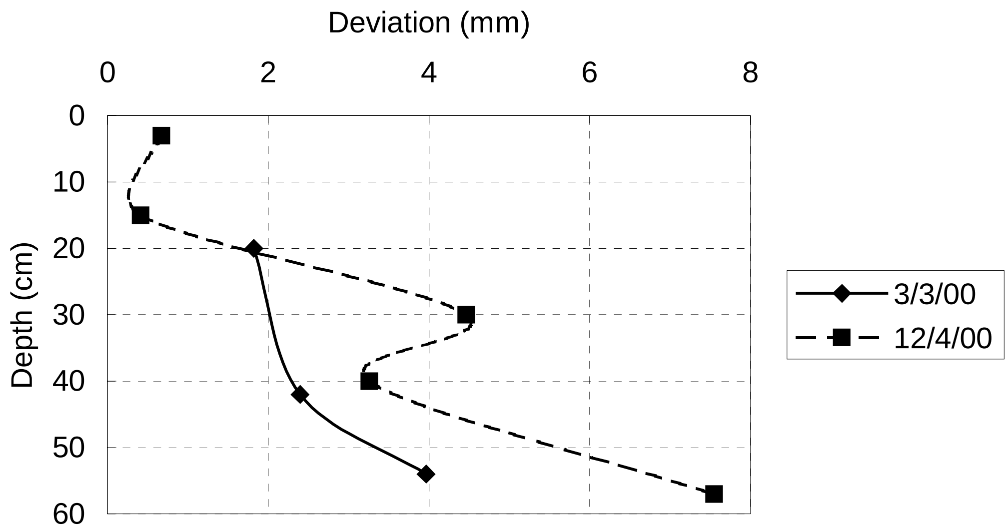

Fig 2.6.2 Deviation for grain size distribution, March, April 2000 off Kapp Amsterdam.

Graph #1 shows that grain size increases with increasing depth and time. The reason why this occurs is that when the grains are growing they grow together, forming bigger grains. According to graph #2 the grain size varies more with increased depth and time. This is occurs because some grains grow together and some don't.

3. ESTIMATED ICE GROWTH

3.1 Theory for ice growth

The seawater in the oceans has salinity at approximately 34.5 ppt. The freezing point temperature (T_f) is then a function of salinity with the assumption that there is a constant composition of the salty brine pockets left in the ice and that the salinity of the brine is the same as of the seawater.

$$T_f = -0.0539S \tag{3.1}$$

The temperature of maximum density also depends on the salinity

$$T_m = 3.98 - 0.2229S \tag{3.2}$$

T_m and T_f is equal at S=24.7 ppt, and when S>24.7 is T_f>T_m. This leads to an unstable vertical density distribution with lowest density at the top of the water column, which causes convective mixing. Therefore, the whole water column has to be cooled down to T_f to get a primary ice cover. When a primary ice cover is formed will the ice growth continue due to how much latent heat that can be released and on a water surface this will be done by conduct the heat through the already formed ice layer towards the air above. If T_a is the mean air temperature, T_m is the melting point temperature of ice, h_i is the ice thickness and k_i is the mean thermal conductivity of ice, then is the quantity of heat conducted upward through the ice per unit area in time dt given by

$$dq = -k_i \frac{T_a - T_m}{h_i} dt \tag{3.3}$$

dq has its source from the latent heat that is released by freezing an additional thickness dh_i of ice. The latent heat, dq, is then $-l_f \rho_i dh_i$, where l_f is the latent heat of fusion and ρ_i is the density of ice. This gives the following equation:

$$k_i \frac{T_m - T_a}{h_i} dt = l_f \rho_i dh_i \tag{3.4}$$

The assumption that h_i (t=0)=0 and integration of the equation above gives:

$$h_i^2 = \frac{2k_i}{l_f \rho_i} \int_0^t (T_m - T_a) dt = \frac{2k_i}{l_f \rho_i} FDD \tag{3.5}$$

FDD (degree-days of freezing) is the integrated days of freezing exposure. What is neglected in the equations above is that it usually is a snow cover on top of the ice and that there is heat input from the ocean. Both these factors reduce the ice growth. The ice growth when there is a snow cover on top of the ice can be obtained from the following equation:

$$\int_0^t (T_m - T_a)dt = \int_0^{h_i} l_f \rho_i \left(\frac{k_i h_s + k_s h_i}{k_i k_s}\right) dh_i \tag{3.6}$$

If constant snow cover is assumed, the equation above can be expressed as

$$\int_0^t (T_m - T_a)dt = \frac{l_f \rho_i}{2k_i} h_i^2 + \frac{l_f \rho_i h_s}{k_s} h_i \tag{3.7}$$

To make a more precise estimate of ice growth, the transient heat diffusion equation should be applied, which can be solved by finite difference method for known boundary and initial conditions.

There are also existing a number of empirical approximations for ice growth, Zubov (1943) proposed the following formula:

$$h_i^2 + 50h_i = 8FDD \tag{3.8}$$

where h_i is in cm. Lebedev (1938) suggested that

$$h_i = 1.33|FDD|^{0.58} \tag{3.9}$$

Both these equations are derived from data from Russian polar stations.