

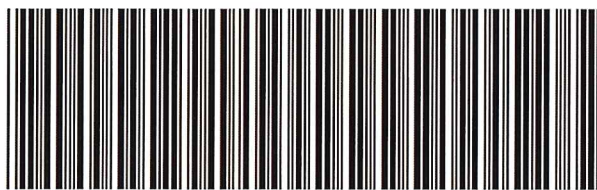
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## **Resource Request Slip Letter**

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**Please note:** The following manual description has been specified in this request - 1968, 41(6), s.77-84: Dahlin, EB: Designing and tuning digital controllers



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This paper, to be published in two parts, presents a Direct Synthesis Method of digital controller design, applicable to single and cascade loop configurations. One parameter is selected per loop to achieve desired response speed and disturbance suppression. Part I describes the theory, and is illustrated with simulation results. Part II will describe a multivariable control application.

### Part I

**P**OPULARITY OF proportional, integral, and derivative control modes results largely from economy of implementation with analog hardware and familiarity among designers and operators. Systems controlling many process variables normally depend on these controllers in independent and cascaded loops, or in feed-forward or ratio connections. The classical algorithms, however, may be inefficient or unsuitable in many process control situations (e.g., those including transport delays).

In most computer applications, there is no constraint imposed by prefabricated controllers with fixed algorithms. The ideal algorithm for a particular portion of a system considers design ease and parameter tuning. Absence of other simple algorithms often leads designers to assign *PI* to less critical loops without considering alternatives.

The Direct Synthesis design method features individual loop sampling rates, analytic simplicity, design of integrated control systems by loop cascades, simple inclusion of smoothing filters, and good loop behavior regardless of process transport delays.

Individual loop sampling rates result in economical computer utilization. For example, a controller pro-

Table I—Nomenclature.

|           |                                                                                        |
|-----------|----------------------------------------------------------------------------------------|
| <b>C</b>  | = process output or controlled variable                                                |
| <b>D</b>  | = controller transfer function algorithm                                               |
| <b>G</b>  | = process transfer function                                                            |
| <b>K'</b> | = closed loop transfer function                                                        |
| <b>N</b>  | = nearest integer of $\tau/T$                                                          |
| <b>n</b>  | = sample number                                                                        |
| <b>R</b>  | = setpoint                                                                             |
| <b>S</b>  | = Laplace transform operator                                                           |
| <b>T</b>  | = sampling interval                                                                    |
| <b>x</b>  | = controller input                                                                     |
| <b>y</b>  | = controller output                                                                    |
| <b>Z</b>  | = Z transform operator                                                                 |
| $\lambda$ | = tuning parameter defining closed loop characteristics                                |
| $\tau$    | = transport delay                                                                      |
| $\Omega$  | = $[1 - e^{-\lambda T}] / [1 - e^{-\lambda T} Z^{-1} - (1 - e^{-\lambda T}) Z^{-N-1}]$ |

# Designing and Tuning Digital Controllers

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gram with low execution frequency may be kept in bulk storage (saving space in working storage), while algorithms requiring frequent execution (e.g., flow regulator loops) are kept in working storage. Execution time for programs is minimized if loops are executed as infrequently as dynamics permit.

Totally integrated digital controllers have filtering and cascade control strategies included in the algorithms. These demand more involved tuning than classical systems (for which filters and cascaded loops operate on local disturbances at lower system structural levels).

In control of paper-making machines, frequent system retuning (to counteract changes in dynamics) ordinarily necessitates process identification and controller tuning programs (Ref. 2). The Direct Synthesis method, in which parameters (computed from process data) were inserted automatically in the control algorithms, eliminated manual tuning.

### General Synthesis Formula

Consider a control loop (Fig. 1) with no distinction between continuous (analog) or sampled (digital) data operation. For the latter, sampling and holding devices are included in the process. Laplace transforms are used for continuous, and Z-transforms for sampled transfer functions. The closed loop transfer function,  $K'$ , (process output/setpoint) is given in Eq. 1; if  $K'$  is known, the required algorithm is the *general synthesis formula* of Eq. 2.

$$K' = \frac{C}{R} = \frac{D G}{1 + D G} \quad (1)$$

$$D = \frac{1}{G} \frac{K'}{1 - K'} \quad (2)$$

This formula is the origin of many controller designs (Ref. 1). Defined by system structure, it is valid for any system with the configuration shown.

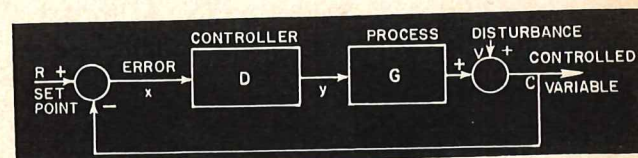


FIG. 1. CONTROL system structure.



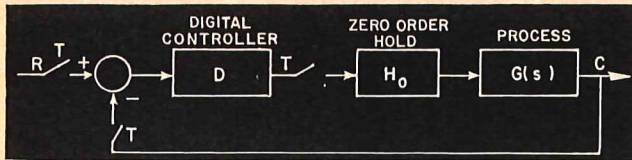


FIG. 2. DIGITAL control system for first-order process with transport delay.

The transfer function must be selected such that the controller is physically possible (e.g., at  $K' = 1$ ,  $D$  is infinite and the controller is nonexistent). The algorithm must have no predictor elements (positive powers of  $Z$ ; terms of the form  $e^{sV}$ ,  $V > 0$ ). In addition,  $D$  must not cancel a non-minimum phase singularity\* of  $G$  (if  $G$  has a zero outside the unit circle in the  $Z$ -domain, a pole of  $D$  cancelling this zero promotes instability).

Although the model is assumed minimum phase, process singularities may approach non-minimum conditions, necessitating special precautions (to be shown).

### Controller Synthesis

The general synthesis formula, Eq. 2, will be used to develop an analog controller, and an equivalent digital algorithm. The latter, a special case of the controller to be derived by Direct Synthesis, is useful in multi-loop systems to transfer algorithms between continuous and sampled domains, and to illuminate the origin of the "ringing" phenomenon.

### Analog Controller

Transport delay,  $\tau$ , in a control loop must be cancelled with transport delay in the closed loop response function,  $K'(S)$ , to avoid a predictor element in  $D$ . If  $K'(S)$  is selected to have first order exponential response, as in Eq. 3, the control algorithm becomes that of Eq. 4. Although the delay in  $G(S)$  cancels the exponential in the numerator of the algorithm, the denominator still contains  $e^{-s\tau}$ , complicating analog implementation. If  $\tau = 0$  (no transport delay), the first order process, and corresponding algorithm, are found as in Eqs. 5 and 6. This represents a proportional plus reset controller with gain  $(\lambda/K_A)$  and reset rate  $A$  repeats/sec.

$$K'(S) = e^{-s\tau} \frac{\lambda}{S + \lambda} \quad (3)$$

$$D(S) = \frac{e^{-s\tau}}{G(S)} = \frac{\lambda}{S + \lambda(1 - e^{-s\tau})} \quad (4)$$

$$G(S) = K \frac{A}{S + A} \quad (5)$$

$$D(S) = \frac{\lambda}{KA} \left( 1 + \frac{A}{S} \right) \quad (6)$$

### Digital Equivalent of Analog Controller

If a digital controller with sampling devices and zero order hold is substituted for the analog controller (Fig. 2), a high sampling rate can result in nearly identical loop behavior. Algorithms can be found from

\* Non-minimum phase singularity: A pole or zero in the right half  $S$ -plane or outside the unit circle in the  $Z$ -plane.

certain difference approximations of Eq. 4. For example, the process transfer function, Eq. 7, corresponds to the differential equation, Eq. 8.

$$G(S) = K e^{-s\tau} \frac{A}{S + A} \quad (7)$$

$$\frac{dy}{dt} + \lambda y + \lambda y(t - \tau) = \frac{\lambda}{KA} \left( \frac{dx}{dt} + Ax \right) \quad (8)$$

A sampled data equation is formed by backward differences as in Eq. 9. Solving this for  $y_n$  yields the control algorithm of Eq. 10. In the  $Z$ -domain, the desired digital control algorithm becomes that of Eq. 11. This algorithm works well under the conditions of Eq. 12, because response equals that of the original analog loop when the sampling interval,  $T$ , approaches zero.

$$\frac{y_n - y_{n-1}}{T} + \lambda(y_{n-1} + y_{n-N-1}) = \frac{\lambda}{KA} \left( \frac{x_n - x_{n-1}}{T} + Ax_{n-1} \right) \quad (9)$$

$$y_n = (1 - \lambda T) y_{n-1} - \lambda y_{n-N-1} + \frac{\lambda}{KA} [x_n - (1 - AT)x_{n-1}] \quad (10)$$

$$D(Z) = \frac{\lambda}{KA} \frac{1 - (1 - AT)Z^{-1}}{1 - (1 - \lambda T)Z^{-1} - \lambda Z^{-N-1}} \quad (11)$$

$$T \leq \frac{0.2}{A}; \lambda \leq \frac{0.4}{T} \quad (12)$$

### Direct Synthesis of Digital Controller

The general synthesis formula can be used with direct specification of a sampled data function,  $K'(Z)$ . An exponential first order closed loop has the difference equation given in Eq. 13, where tuning parameter  $a = e^{-\lambda T}$ . The  $Z$ -transform of  $K'(Z)$  is found as Eq. 14.

$$C_n = a C_{n-1} + (1 - a) R_{n-N-1} \quad (13)$$

$$K'(Z) = \frac{C(Z)}{R(Z)} = \frac{1 - a}{(1 - a)Z^{-1}} \quad (14)$$

The general control algorithm for the Direct Synthesis method, which will be used subsequently as a synthesis formula, may then be found as Eq. 15. Substituting  $\lambda$  for  $a$  in this algorithm makes the tuning parameter invariant to sampling interval changes.

$$D(Z) = \frac{Z^{-N-1}}{G(Z)} = \frac{1 - e^{-\lambda T}}{1 - e^{-\lambda T}Z^{-1} - (1 - e^{-\lambda T})Z^{-N-1}} \quad (15)$$

The process of Eq. 7 (sample and zero order hold) yields the sampled data transfer function of Eq. 16. The control algorithm for this case is expressed as in Eq. 17 where  $\Omega = [1 - e^{-\lambda T}] / [1 - e^{-\lambda T}Z^{-1} - (1 - e^{-\lambda T})Z^{-N-1}]$ .

$$G(Z) = KZ^{-N-1} \frac{1 - e^{-AT}}{1 - e^{-AT}Z^{-1}} \quad (16)$$

$$D(Z) = \frac{\Omega(1 - e^{-AT}Z^{-1})}{K(1 - e^{-AT})} \quad (17)$$

Note that this algorithm is more general than Eq. 11 (Eq. 11 is a special case of Eq. 17, valid for  $AT \ll 1$ ,  $\lambda T \ll 1$ ).  $\lambda$  is physically similar in both methods; closed loop behavior (defined by Eq. 3), resembles that of Eq. 14 for small  $T$  (useful in cascade control loop synthesis); this demonstrates that analog controller synthesis is a workable intermediate step for small sampling intervals.



Eq. 13 implies zero steady state error for step input to the closed loop. Inherent in controllers having at least one integration, the specification automatically guarantees a reset algorithm.

### Ringling

The synthesis methods described, applied to the first order process, produce no difficulties. Higher order processes may introduce two complications: non-minimum process phase characteristics and controller ringling. Techniques to cope with these are similar.

A minimum phase continuous process, with zero order holding, may produce a non-minimum phase process at some sampling frequency. Ringling occurs when singularities approach (but have not reached) the non-minimum phase region.

Consider the general transfer function, Eq. 18, with  $N$  a non-negative delay integer. Ringling amplitude, Eq. 19, is the difference between the first and second amplitude of the step response.  $M$  defines system order.

$$D(Z) = K_c Z^{-N} \frac{1 + \sum_{r=1}^M a_r Z^{-r}}{1 + \sum_{r=1}^M b_r Z^{-r}} \quad (18)$$

$$RA = b_1 - a_1 \quad (19)$$

The significance is shown in Table 2. The undamped oscillator (example 1) has  $RA = 1$ . The controller behaves in an oscillatory fashion, with ringling frequency half the sampling frequency. The point at  $Z = -1$  which produces undamped oscillation is the ring-

ing node. Complex as well as single poles near the ringling node ( $Z = -0.5$ ) give rise to ringling (examples 2, 3, and 4). Transfer function zeros in the right half plane aggravate ringling (example 4) while poles in the right half plane reduce it (example 3).

For the first order system previously analyzed, the algorithm  $RA$  is  $(\lambda - A)T$  for indirect synthesis, and  $(e^{AT} - e^{-\lambda T})$  for direct. If  $\lambda \leq A$ , then  $RA \leq 0$  (no ringling). Higher order processes require special attention.

For a double pole process (Fig. 2),  $G(S)$  is given as in Eq. 20. The  $Z$ -transform, with zero order holding, is found as Eq. 21, where variables  $f_1$  and  $f_2$  are defined in Eq. 22. The direct method synthesis (Eq. 15) gives the control algorithm of Eq. 23, with the corresponding ringling amplitude found in Eq. 24.

$$G(S) = e^{-sT} \frac{KA}{S+A} \frac{B}{S+B} \quad (20)$$

$$G(Z) = K f_1 Z^{-N-1} \frac{1 + \frac{f_2}{f_1} Z^{-1}}{(1 - e^{-AT} Z^{-1})(1 - e^{-BT} Z^{-1})} \quad (21)$$

$$f_1 = 1 + \frac{1}{A-B} (Be^{-AT} - Ae^{-BT})$$

$$f_2 = e^{-(A+B)T} + \frac{1}{A-B} (Be^{-BT} - Ae^{-AT}) \quad (22)$$

$$D(Z) = \Omega \frac{1 - (e^{-AT} + e^{-BT}) Z^{-1} + e^{-(A+B)T} Z^{-2}}{K f_1 \left(1 + \frac{f_2}{f_1} Z^{-1}\right)} \quad (23)$$

$$RA = \frac{f_2}{f_1} - e^{-\lambda T} + e^{-AT} + e^{-BT} \quad (24)$$

Table 2—Ringling Characteristics of Representative Systems.

|   | D (Z)                                   | Impulse Response                  | Step Response                  | RA | Time Plots |
|---|-----------------------------------------|-----------------------------------|--------------------------------|----|------------|
| 1 | $\frac{1}{1 + 1/Z}$                     | 1<br>-1<br>1<br>-1<br>1           | 1<br>0<br>1<br>0<br>1          | 1  |            |
| 2 | $\frac{1}{1 + .5/Z}$                    | 1<br>-.5<br>.25<br>-.125          | 1<br>.5<br>.75<br>.625         | .5 |            |
| 3 | $\frac{1}{(1 + .5/Z)(1 - .2/Z)}$        | 1<br>-.3<br>.19<br>-.087<br>.045  | 1<br>.7<br>.89<br>.803<br>.848 | .3 |            |
| 4 | $\frac{1 - .5/Z}{(1 + .5/Z)(1 - .2/Z)}$ | 1<br>-.8<br>.36<br>-.188<br>.0924 | 1<br>.2<br>.50<br>.37<br>.46   | .8 |            |



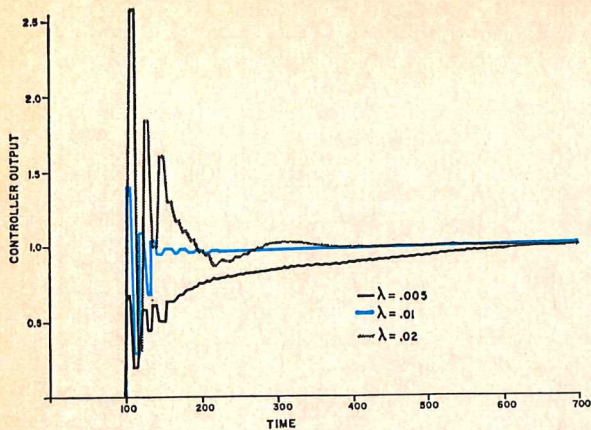


FIG. 3(a). STEP response of controller with double pole process for various  $\lambda$ .

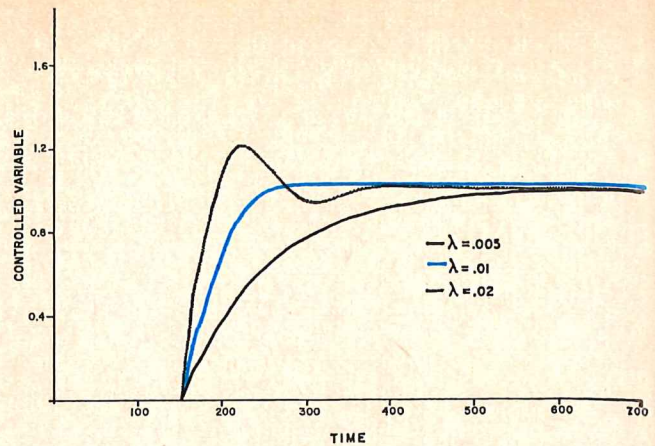


FIG. 3(b). STEP response of double pole process control loop with  $K = 1$ ,  $A = 0.04$ ,  $B = 0.1$ ,  $\tau = 50$ , and  $T = 10$ .

If  $T \rightarrow 0$  then  $f_2/f_1 \rightarrow 1$ ,  $e^{-\lambda T} \rightarrow 1$ ,  $e^{-AT} \rightarrow 1$ ,  $e^{-BT} \rightarrow 1$ , and  $RA \rightarrow 2$ . Thus the pole at  $Z = -f_2/f_1$  approaches the ringing node for short sampling times with a large ringing amplitude (Fig. 3a). The controlled variable is not affected by ringing since the ringing controller pole is cancelled by a process zero (Fig. 3b). However, ringing on the controller output may cause unnecessary equipment wear; it may reduce the transient capability of the control loop if the output is tightly clamped (the useful portion of the controller output is the average over successive ringing cycles; clamping prevents the process from receiving its proper input). In multivariable control systems, loop interaction may convert a damped ringing oscillation for a single loop into an undamped oscillation, making ringing a threat to stability.

To retain the useful portion of the controller output, yet eliminate the ringing, the problem pole is deleted and a constant added to assure long-term settling behavior. Since response for  $t \rightarrow \infty$  corresponds to  $Z \rightarrow 1$ ,  $[1 + (f_2/f_1) Z^{-1}]$  is replaced by  $[1 + f_2/f_1]$  in Eq. 23, which yields the algorithm of Eq. 25 for the ringing-free controller.

$$D(Z) = \Omega \frac{1 - (e^{-AT} + e^{-BT}) Z^{-1} - e^{-(A+B)T} Z^{-2}}{K(1 - e^{-AT} - e^{-BT} + e^{-(A+B)T})} \quad (25)$$

The simulation result for this process (Fig. 4) shows that ringing is essentially eliminated. Closed loop response is improved, particularly at higher  $\lambda$ , and no undesirable transient effects are introduced.

### Frequency Response Characteristics

The specification used in the algorithm synthesis is a response to setpoint change. Disturbance suppression is also of interest. Consider the disturbance to enter at the controlled variable of a high-rate (approximately continuous) sampled data system. The response of the controlled variable to a disturbance,  $V$ , can be derived as Eq. 26 by transfer function algebra. (Continuous closed loop behavior is specified in Eq. 3.)

$$\frac{C(S)}{V(S)} = 1 - e^{-S\tau} \frac{\lambda}{S + \lambda} \quad (26)$$

For  $\tau \neq 0$ , transfer function amplitude exceeds unity (the complex transfer locus will spiral about  $S=1$ ), and delays lead to disturbance amplification at some frequencies (Fig. 5). Longer transport delays have more pronounced peaks. Disturbance frequencies below a noise cut-off are attenuated by the loop, frequencies above may be amplified. If  $T_{63}$  is the time in sec for

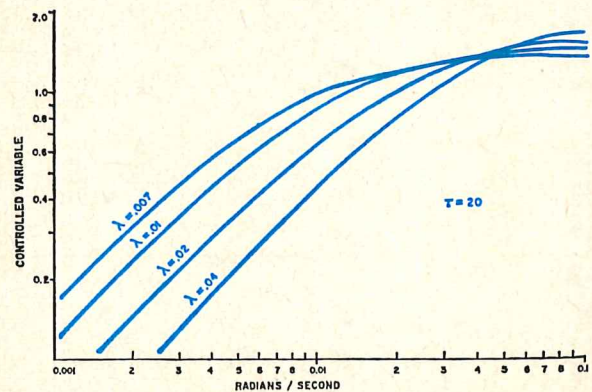
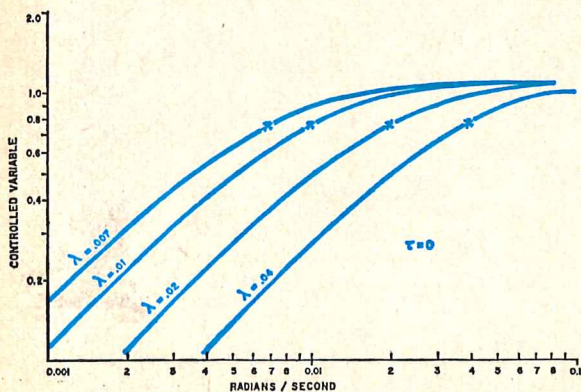


FIG. 5. AMPLITUDE of controlled variable for sinusoidal disturbance with unit amplitude for various  $\tau$ .



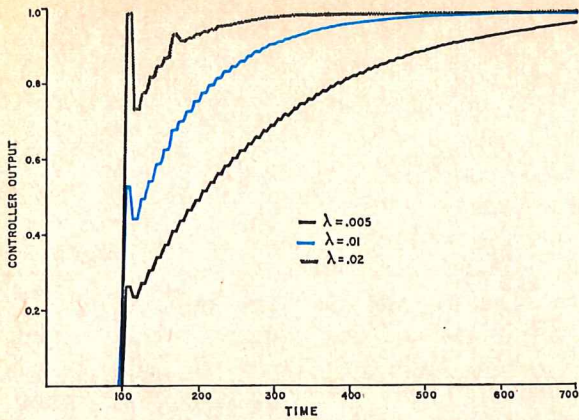


FIG. 4(a). STEP response of controller in double pole process with ringing pole removed.

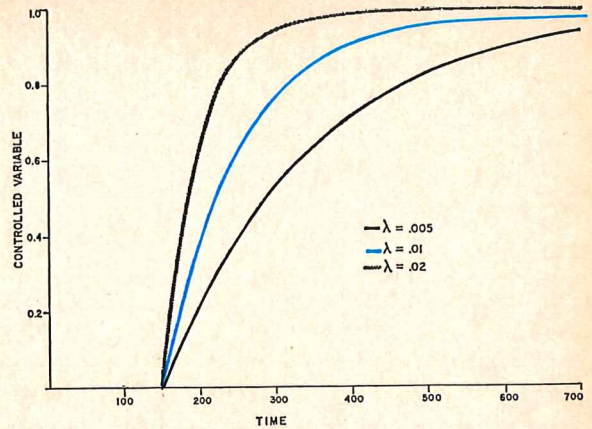


FIG. 4(b). STEP response of double pole process with ringing pole removed, for  $K = 1$ ,  $A = 0.04$ ,  $B = 0.1$ ,  $\tau = 50$ , and  $T = 10$ .

the step response to reach 0.63 of its final value, it is related to  $\tau$  (sec) and  $\lambda$  (radian/sec) by Eq. 27.

$$T_{63} = \tau + 1/\lambda \quad (27)$$

Noise cut-off frequency, NCOF, is  $1/T_{63}$  (radian/sec). As a rule of thumb, NCOF is the smaller of  $\lambda$  and  $1/\tau$ ; therefore either  $\lambda$  or  $\tau$  may dominate loop disturbance suppression.

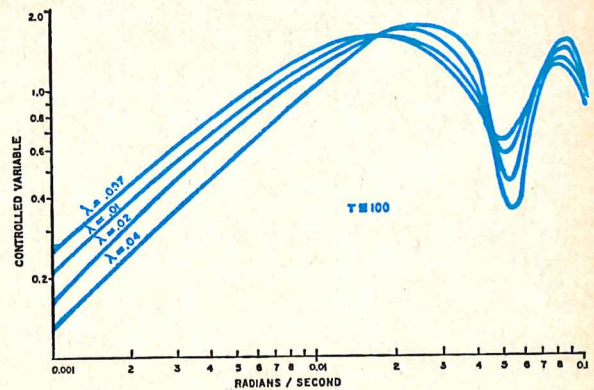
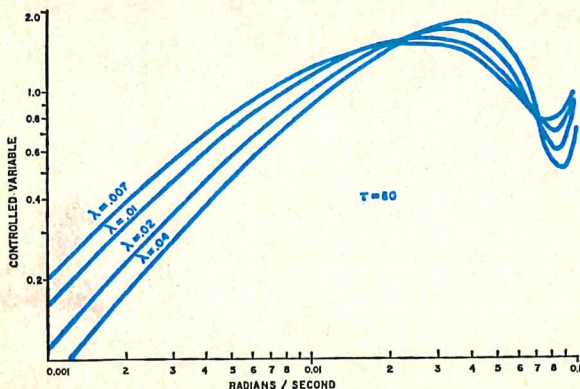
For the transients in the simulated system (Fig. 4) response times and cut-off frequencies are shown in Table 3. The relation between control loop frequency response (disturbance attenuation) and tuning parameter  $\lambda$  allows direct observation of tuning accuracy. During steady state operation a dominant disturbance cycle is often evident. If this is much below cut-off frequency, poor tuning or abnormal excitation is indicated. If frequency is much above cut-off, the control loop is not effective in the range, and amplification at certain disturbance frequencies may be possible. This is typical for long transport delays (Fig. 5). For long sampling times, characteristic response still prevails. Disturbance amplitude at 0.005 radian/sec is reduced by about 0.64 with  $\lambda = 0.02$ . A continuous, or a short sampling interval digital system would cause reduction to 0.53.

Table 3—Response Times and Cut-off Frequencies for Simulated System (Fig. 4).

|                   | $\lambda$ |      |     |
|-------------------|-----------|------|-----|
|                   | 0.005     | .01  | .02 |
| $T_{63}$          | 250       | 150  | 100 |
| Observed $T_{63}$ | 230       | 130  | 90  |
| NCOF (rad/sec)    | .004      | .006 | .01 |

### Cascade Control

Cascade loops eliminate effects of local disturbances and nonlinearities in major control loops. Direct synthesis permits simple tuning of cascaded loops by making external controllers look into first order closed loops with known response times.





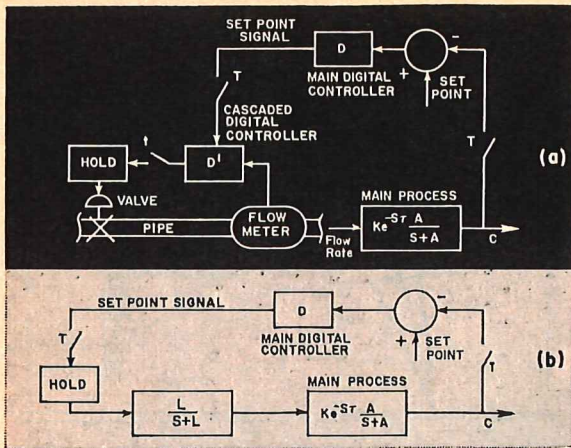


FIG. 6. (above) DIGITAL controller system with cascaded digital flow regulator loop; (below) approximate system model for controller synthesis.

As an example, it is desired to regulate output  $C$  with controller  $D(Z)$  using manipulation of a particular process feed rate (Fig. 6a). Due to upstream fluctuation of the head, a local cascaded loop regulates flow (using digital controller  $D'$ ). The setpoint of  $D'$  is manipulated by the main controller  $D$ .

Response in feed rate to change in valve signal is assumed given as Eq. 28. The cascaded loop is specified to have its tuning parameter  $\lambda = L$ . From the example of Eq. 17, with  $\tau = 0$ , and with  $t$  representing sampling rate in the cascaded loop,  $D'(Z)$  is found in Eq. 29. The dynamic equivalent of the control system (Fig. 6b) shows a double pole process, and from the example of Eq. 25, this has the algorithm of Eq. 30.  $Q$  is valve gain,  $R$  is valve pole.

$$\frac{F}{V} = Q \frac{R}{S + R} \quad (28)$$

$$D'(Z) = \frac{1 - e^{-L\tau}}{R(1 - e^{-R\tau})} \frac{1 - e^{-R\tau} Z^{-1}}{1 - Z^{-1}} \quad (29)$$

$$D(Z) = \Omega \frac{1 - (e^{-AT} + e^{-LT}) Z^{-1} + e^{-(R+L)T} Z^{-2}}{R(1 - e^{-RT} - e^{-LT} + e^{-(R+L)T})} \quad (30)$$

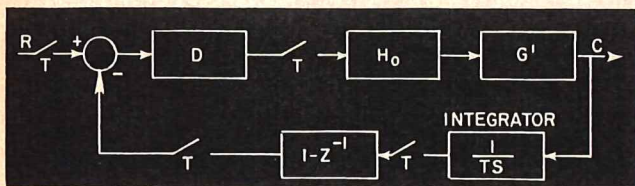


FIG. 8. MODEL of a single rate system.

Note that the response of the continuous transfer function  $\lambda/(S + \lambda)$  is equivalent to the specification of Eq. 13. The sampling rates in the cascade and outer loops determined by  $t$  and  $T$ , respectively, can be independently selected without changing tuning formulas. The cascade loops normally use more frequent sampling.

Simulation using the tuning formulas (Fig. 7) shows good loop behavior for all outer loop  $\lambda$  values chosen. The cascade loop parameter,  $L$  is equal to that previously used (Fig. 4), and system behavior is consequently similar. This verifies that the sampled data cascaded loop influences the outer loop approximately as

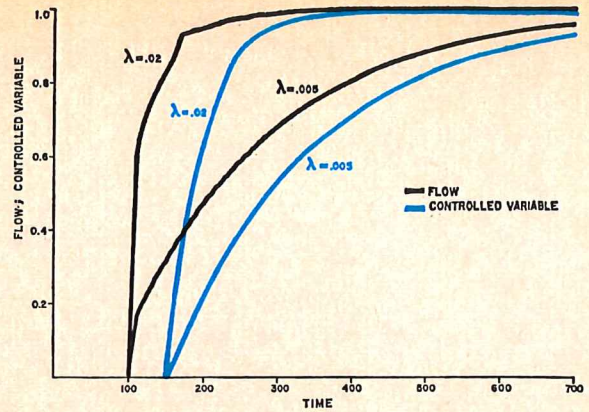


FIG. 7. SIMULATION of feed flow rate and step variable response for system of Fig. 6.

a continuous first order system with a pole equal to the  $\lambda$ -value of the sampled data loop.

### Digital Filtering

The signal representing the controlled variable may be smoothed to prevent a highly fluctuating output from increasing equipment wear and in processes with transport delays, to prevent control loop cycling at closed

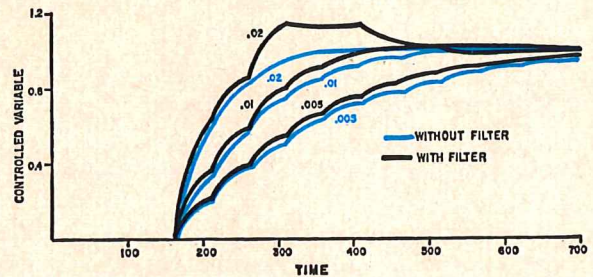


FIG. 9. RESPONSE of the controlled variable for a first order system with and without the average filter.

loop frequency response peaks. Heavy smoothing introduces filter effects on control loop dynamics. Including filter functions in the dynamic description of the system results in a multiple sampling rate situation, cumbersome to handle analytically, especially when there are large differences between sampling rates. The direct synthesis method considers only the execution rate of the control algorithm (lower than the sampling rate used for data filtering). In addition, the filter can be included in the system without appreciable deterioration of loop response. For the special smoothing filter to be discussed, the  $\lambda$ -tuning method is nearly independent of filter parameters.

The filter averages a given number,  $M$ , of samples, updates, and thus accumulates data values into a sum. The system has dual sampling rates—for data accumulation and filtering and, lower, for execution of control algorithm,  $D$ .

An actual process is represented by a single sampling rate system (Fig. 8), which uses an analog integrator and a difference operator  $(1 - Z^{-1})$  to represent the filter. The latter converts the integrator to fixed period with non-overlapping integrations.



The total process transfer function to be used in the synthesis formula is defined by the  $Z$ -transform of Eq. 31. If the transform contains multiple poles, it is helpful to approximate larger poles with an addition to the actual transport delay (Refs. 3, 4) or to combine several time constants. The first order process illustrates the method. Evaluating  $G(Z)$  and inserting into the synthesis formula gives the control algorithm of Eq. 32, where functions  $H_1$  and  $H_2$  are defined in Eq. 33.

$$G(Z) = (1 - Z^{-1}) Z \left( \frac{1 - e^{-ST}}{S} G'(S) \frac{1}{TS} \right) \quad (31)$$

$$D(Z) = \Omega \frac{1 - e^{-AT} Z^{-1}}{K H_1 \left( 1 + \frac{H_2}{H_1} Z^{-1} \right)} \quad (32)$$

$$H_1 = 1 - \frac{1 - e^{-AT}}{AT} \quad (33)$$

$$H_2 = -e^{-AT} + \frac{1 - e^{-AT}}{AT}$$

When  $AT \rightarrow 0$ ,  $H_2/H_1 \rightarrow -1$ , the ringing node. For small sampling times,  $T$ , severe ringing occurs with the amplitude of Eq. 34. When the ringing pole is removed at  $-H_2/H_1$ , the algorithm becomes that of Eq. 35, identical to Eq. 17, for the first order system without filtering.

$$RA = \frac{H_2}{H_1} - e^{-\lambda T} + e^{-AT} \quad (34)$$

$$D(Z) = \Omega \frac{1 - e^{-AT} Z^{-1}}{K (1 - e^{-AT})} \quad (35)$$

The filter thus adds a ringing node which does not contribute significantly to the response of the controlled variable, although it oscillates the controller output. Therefore, a filter can be added using the control algorithm developed without a filter, a conclusion found valid on higher order systems as well as in on-line multi-loop control.

The simulation using the control algorithm with a first order system shows the response to a step change in setpoint with and without a digital filter (Fig. 9). Sampling times are long compared with process transport delay and time constant. The responses are similar except for the largest  $\lambda$ -value (0.02 rad/sec) where some overshoot occurs with the filter. Tests with other sampling intervals verifies that the filter does not significantly alter the behavior of the controlled variable.

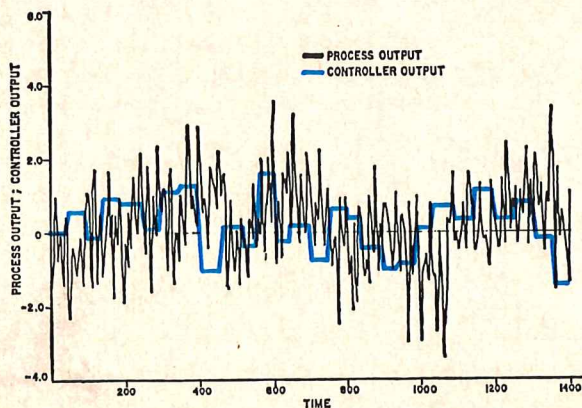


FIG. 10. CONTROLLER and process outputs for white noise inputs, with digital filter.

The filter also reduces the controller output. White noise is introduced directly on the controlled variable. The stock flow or manipulated variable without a filter (Fig. 10) has approximately 20 times the amplitude with the filter (Fig. 11). The times for control execution and transport delay are 50 and 60 sec. respectively. The unfiltered loop has low frequency cycling near cutoff which is reduced when the filter is used.

### Tuning Procedure

Tuning is based on dynamic process parameters. These may be estimated from observations of transients or calculated from physical principles, or can be determined automatically by computer (Ref. 2). Knowing the process parameters, only  $\lambda$  must be selected to compute control algorithm coefficients. One starting point for manual selection of  $\lambda$  is the dominant (smallest) pole. High  $\lambda$  corresponds to tight control and is desirable. Highest possible  $\lambda$  is recognized by overshoot caused by mismatch between actual and assumed parameters, nonlinearities and ignored dynamic characteristics. Gradually increasing  $\lambda$  and observing response to small setpoint changes, yields the optimum value. With no digital filtering, if disturbances have large amplitude at high frequency, the controller output amplitude typically defines the limit for  $\lambda$ . The operator can observe the controller output and increase  $\lambda$  until the peak to peak amplitude is near allowable limits. The system then has its best tuning. If amplitudes are too large,  $\lambda$  can be reduced in proportion to desired change.

In systems with cascaded loops, inner loops are first tuned to maximum  $\lambda$  values and then outer loops are set.

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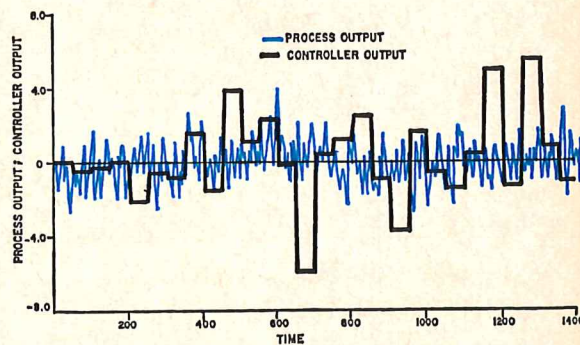


FIG. 11. CONTROLLER and process outputs for white noise inputs, with digital filter.