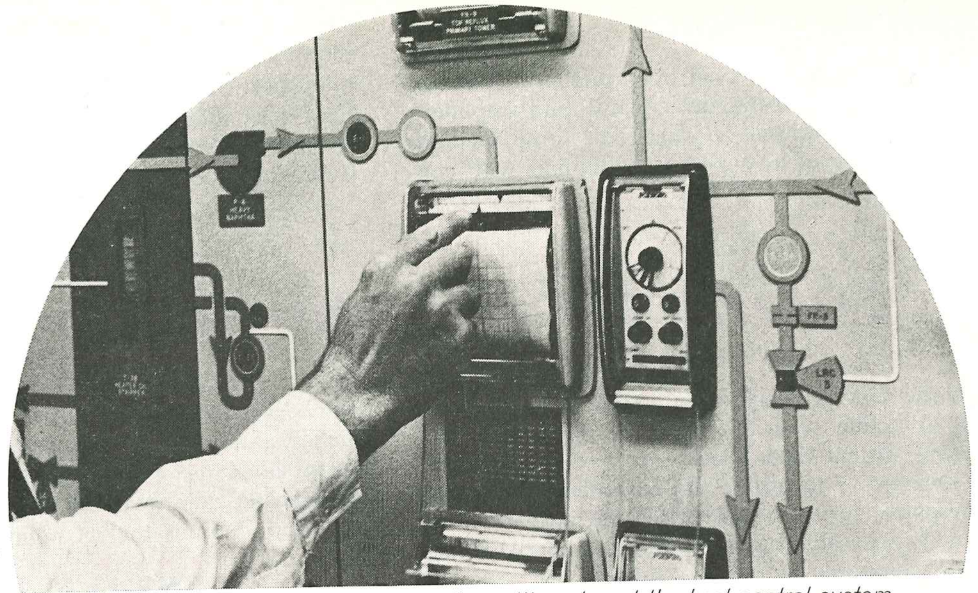

Consider the interaction of controller settings to get the best control system

CONTROLLERS
SET THEM RIGHT

An HP/PR Special Report

A controller is a special purpose analog computer within a control loop. How it works and how it is adjusted should get special attention if proper control is to be achieved

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CONVENTIONAL FEEDBACK CONTROLLERS can be applied to almost all hydrocarbon processing techniques to give proper dynamic characteristics. However, one main difficulty is encountered in tuning controllers to produce the desired results. Five methods are presented here to facilitate controller tuning. These are followed by a consideration of controller operating principles.

Although the tuning methods attempt to give the optimum combination of settings for a controller, the answers will vary. Some of the methods are easy to use, while some require time and effort to understand and apply. Generally, the accuracy of the answers is proportional to the time spent, and the user must decide if he wishes to make the additional effort to obtain the increased accuracy. Nevertheless, the controller will be tuned closer to its optimum combination of settings in much less time by any of these techniques than by the hit-or-miss procedures which often are used.

To tune controllers it is essential that the engineer understand the basis of the technique he is using. To this end, the assumptions and underlying principles of each method are covered, along with a discussion of the operation of the more common types of industrial controllers. This last aspect is particularly important because the characteristics of the controller will have pronounced effects on the control obtained. Pneumatic controllers are considered in detail in order to illustrate their operating characteristics, and the observed operating principles are extended to include electronic devices and fluid amplifiers.

In this article, emphasis is placed on those methods that can be easily used by a person unfamiliar with control theory. For this reason, methods based on frequency response techniques are omitted, although an excellent discussion can be found in the literature.[1] Since it is desired that those unfamiliar with control theory be able to use the techniques presented, a review of tuning criteria is probably an appropriate beginning.

Criteria for Adjusting Controllers

Perhaps the first problem encountered in tuning controllers is to define what is "good" control, and this, unfortunately, differs from process to process. The most common criterion is to adjust the controller so that the system's response curve has an amplitude ratio or decay ratio of 1:4, i.e., the ratio of the overshoot of the second peak compared to the overshoot of the first peak is 1 to 4 as shown in Figure 1.

There is no direct mathematical justification for requiring a decay ratio of 1:4. It is a compromise between a rapid initial response and a short line-out time. In many cases, this is not sufficient to specify a unique combination of controller settings. There may be an infinite number of settings that yield a decay ratio of 1:4. In such situations, it is often helpful to require that either the steady state offset (difference between actual output and the set point) or the control area (time integral of the output minus the set point) be minimized.

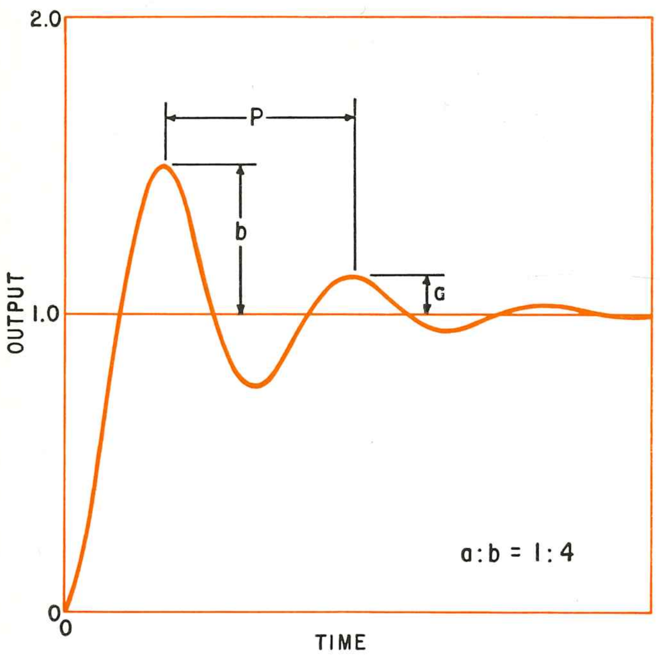

Fig. 1—There can be many controller settings which will yield a decay ratio of 1:4.

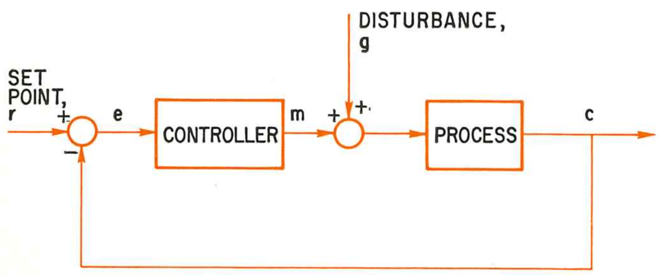

Fig. 2—The term controller will be used to denote only the analog computer element of the control loop.

MODES OF CONTROL

The control loop in Figure 2 is the main object of attention. The set point r is compared to the value of the controlled variable c which is the feedback variable. As a result of this comparison, an error signal e is generated, which the various modes in the controller use to generate the controller output m.

In an industrial instrument, the comparator and an "analog computer" containing the various types of control are housed usually within the same casing. However, it is more convenient when using block diagrams to use the term "controller" to denote only the analog elements devoted to the types of control—and not the entire instrument case. The various types of control are called "modes," and they determine the type of response obtained. Thus, they form the heart of the controller.

The output m of the controller in Figure 2 is combined with a disturbance signal g before being applied to the process. In most processes there are several disturbances, each of which can enter the process at a different point. Since it is not known how many disturbances exist nor where they enter the process, it is convenient to lump them together and show them entering the process along with the control signal.

The response to a change in the disturbance signal is different from the response to the same change in the set point. Furthermore, the damping ratios of these two responses are often different, but the difference is usually not too large. These points are mentioned primarily to acquaint the reader with some of the assumptions necessary to develop methods for obtaining controller settings.

Before discussing various methods to tune controllers, an understanding of the equations describing a controller and definitions of the various terms to describe the modes are necessary. In this article, only "throttling" controllers are considered; i.e., controllers whose output can vary continuously from the minimum value of the output signal to its maximum value. Although there is a large number of on-off (bang-bang) controllers in use, they are not discussed. Even though three different modes are common in throttling controllers, there is a variety of terms used to describe each.

Proportional. In describing the proportional mode, the setting can be prescribed in at least three ways. Electrical engineers typically prefer to specify the "gain" of the proportional mode, although process engineers like the term "sensitivity." In reality, the meanings are identical. However, the calibration in the controller is usually in proportional band (once called throttling range) expressed in percent. This is related to the controller gain or sensitivity by the following relationship.

$$\text{Proportional Band} = \frac{(100)\ (\text{output range of controller})}{(\text{Controller gain})\ (\text{span of controller chart})} \qquad (1)$$

The equation describing the proportional mode is

$$m_p = K_c e \qquad (2)$$

In this equation, m_p is the output of the proportional mode only. Thus, the output of this mode is directly proportional to the error signal e; i.e., the difference between the set point r and the value of the controlled variable c.

Reset. Generally, there is one main shortcoming to a controller with proportional mode only. The value of the output c may be different from the set point r even at steady state. This difference is called offset and is inversely proportional to the gain or sensitivity of the controller. To eliminate this shortcoming, an additional mode, called the reset or integral mode, is added. This mode acts to adjust the output c so that it equals r at steady state; i.e., it "resets" the value of the output—hence the origin of one of the terms: reset. To do this, the output of the reset mode is a constant multiplied by the time integral of the error signal; hence, the origin of the other term: integral. The equation for the output of this mode is

$$m_r = (K_c/T_r) \int e\,dt = (K_c/T_r p)\, e = (K_r/p)\, e \qquad (3)$$

The reader is referred to an accompanying box for a discussion of the differential operator p.

In the construction of an industrial controller, it is convenient to have each mode multiplied by the gain K_c of the proportional mode. Thus, when the reset adjustment is varied, the term T_r, the reset time, in the foregoing equation is varied. The unit of T_r is typically minutes, and it is the number of minutes required for the reset output to duplicate the proportional output for a constant error signal as shown in Figure 3.

In some controllers the reset adjustment is the reset rate, which is the reciprocal of the reset time. Furthermore, it is possible, although not common, to control a process using a controller with only a reset mode, called floating control. Since there is no proportional mode, the term K_c does not exist. Thus, it is necessary to define a gain K_r for this type of controller.

Derivative (Rate). Similarly, several terms also apply to a third mode, which is commonly called either pre-act, derivative, or rate action. Each of these terms are descriptive of the mode whose output is given by the following equation.

$$m_d = K_c T_d\, de/dt = K_c T_d\, pe \qquad (4)$$

Since the output m_d of this mode is proportional to the derivative of the error signal, the term derivative mode is used. The derivative is a measure of the rate of change of the error signal, and thus the term rate action.

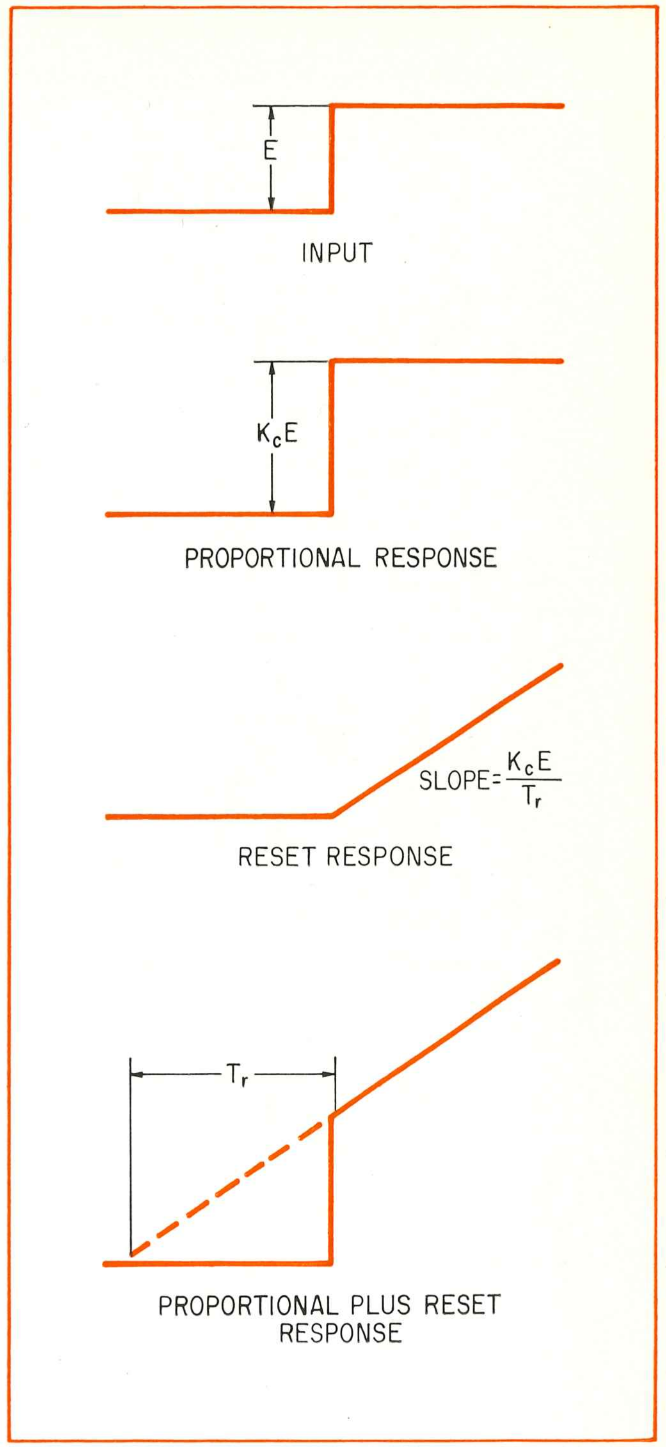

Fig. 3—Reset time is the time required for the reset output to duplicate the proportional output.

This mode gives the "impression" that it "anticipates" changes, and thus the term pre-act. Note that the proportional gain K_c is multiplied by the output of the derivative mode. The adjustment of the derivative mode is typically the "derivative time" in minutes. To define this term, consider the output of the proportional and derivative modes in Figure 4 to a steady increase in the error signal. The derivative time is the time required for the output of the proportional mode to duplicate that of the derivative mode and it is a direct measure of the amount of "anticipation" given to e.

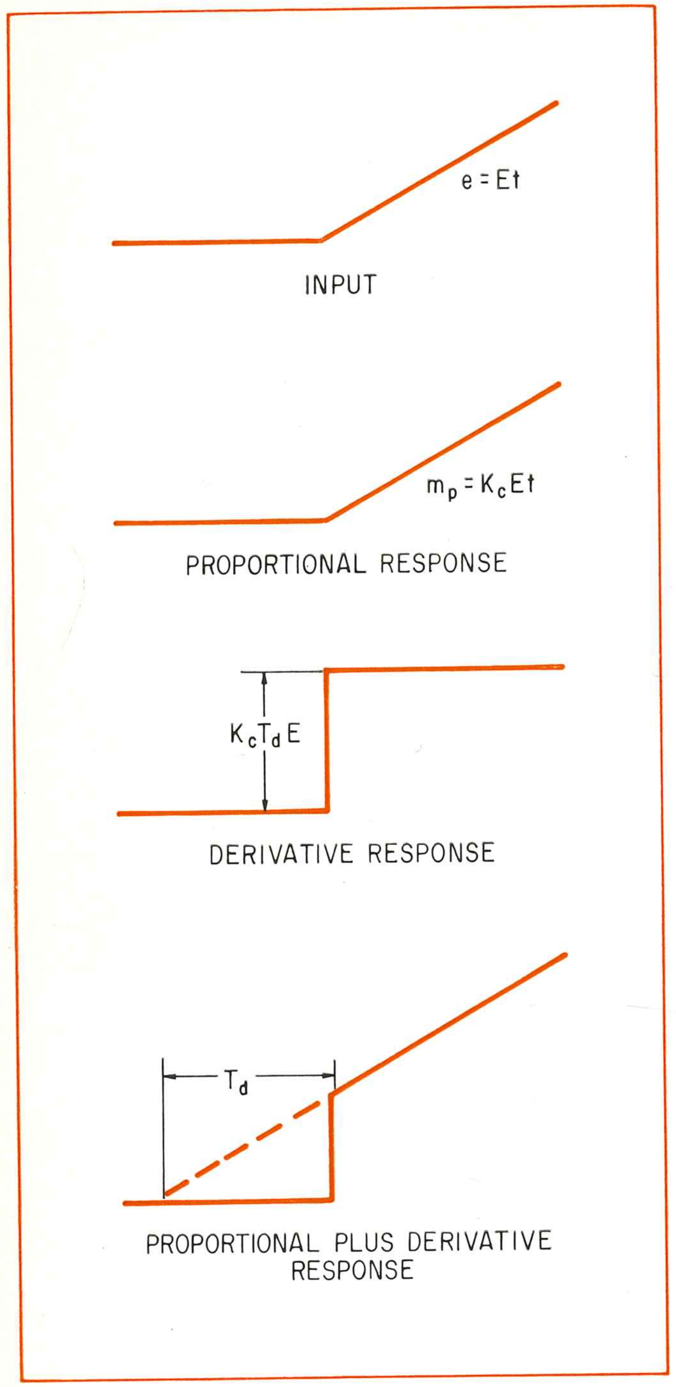

Fig. 4—Derivative response is added to compensate for a change in error signal with time.

Combined. For a three term controller, the total output m is the sum of the outputs from each mode; namely,

$$m = K_c \left[1 + (1/T_r p) + T_d p\right] e \qquad (5)$$

Unfortunately, this is an oversimplification. The equation describes an "ideal" controller; i.e., a controller with perfect gain, perfect reset, and perfect derivative modes. The modes are not independent of each other, i.e., adjusting T_r, for example, usually affects K_c and T_d. In addition, there is often a time lag associated with the proportional mode, as well as similar effects in the other modes.

HOW SETTINGS ARE DETERMINED

Techniques for adjusting controllers generally fall into one of two classes. First, there are a few methods based upon values determined from the closed loop response of the system, i.e., with the controller on "automatic." Second, a variety of methods are based upon variables determined from the open loop response curve, commonly called the process reaction curve. In using the open loop methods, the controller does not even have to be installed before the settings can be determined.

Closed Loop Methods. In this category are two common methods, one originally proposed by Ziegler and Nichols[2] and a second, called the damped oscillation method.[3] Probably the first method proposed was the so-called "ultimate" method submitted by Ziegler and Nichols in 1942. The term "ultimate" was attached to this method because determination of the ultimate gain or sensitivity and ultimate period are required. The ultimate sensitivity, S_u, is the maximum allowable value of gain (for a controller with only a proportional mode) for which the system is stable.

To determine the ultimate gain and the ultimate period, the gain of the controller (with all reset and derivative action tuned out) is adjusted until the process cycles continuously. To do this, the following steps are recommended:

1. Tune all reset and derivative action out of the controller, leaving only the proportional mode; i.e., set $T_d = 0$ and $T_r = \infty$.

2. Place the controller on automatic, if not so already.

3. With the gain at some arbitrary value, impose an upset on the process and observe the response. One easy method for imposing the upset is to move the set point for a few seconds, and then return it to its original value.

4. If the response curve in Step 3 does not damp out (as in the unstable curve of Figure 5), the gain is too high (proportional band setting too low). Thus, the proportional band setting is increased, and Step 3 repeated.

5. If the response curve in Step 3 damps out (as in the stable curve of Figure 5), the gain is too low (proportional band is too high). Thus, the proportional band setting is decreased, and Step 3 repeated.

6. When a response similar to the continuous-cycling curve of Figure 5 is obtained, the values of the proportional band setting and the period of the response are noted.

In reality, there are a few exceptions to Steps 4 and 5; i.e., decreasing the gain makes the process more unstable. In these cases, the "ultimate" method will probably not give good settings. Usually in cases of this type, the system is stable at high and low values of the gain, but unstable at intermediate values. Thus, the ultimate gain for systems of this type is defined slightly differently. To use this method, the lower value of the ultimate gain is sought; i.e., this method applies only when increasing the gain decreases the stability, which is the usual case.

To use the ultimate gain and the ultimate period to obtain controller settings, Ziegler and Nichols correlated, in the case of the proportional controllers, the decay ratio obtained vs. the gain in the controller expressed as a fraction of the ultimate gain. After doing this for a variety of processes they concluded that a value of gain equal to one half the ultimate gain would give a decay ratio of 1:4; i.e.,

$$K_c = 0.5\, S_u. \qquad (6)$$

By analogous procedures, the following equations were found to give good settings for more complex controllers:

Proportional plus reset:

$$K_c = 0.45\, S_u \qquad (7)$$

$$T_r = P_u/1.2 \qquad (8)$$

Proportional plus derivative[1]**:**

$$K_c = 0.6\, S_u \qquad (9)$$

$$T_d = P_u/8 \qquad (10)$$

Three Modes (Proportional plus reset plus derivative):

$$K_c = 0.6\, S_u \qquad (11)$$

$$T_r = 0.5\, P_u \qquad (12)$$

$$T_d = P_u/8 \qquad (13)$$

Again, it should be noted that the foregoing equations were found empirically to give good settings for most processes. Thus, exceptions inherently exist. An example using this technique follows:

Example: For a temperature control system whose ultimate sensitivity S_u is 0.4 psi/°C and ultimate period is 2 minutes, determine settings for proportional, proportional plus reset, proportional plus derivative, and three mode controllers so that the response has a decay ratio of 1:4.

Proportional: Using Equation (6),

$$K_c = 0.5\, S_u = (0.5)(0.4 \text{ psi/° C}) = 0.2 \text{ psi/° C}$$

Proportional plus reset: Using Equations (7) and (8),

$$K_c = 0.45\, S_u = (0.45)(0.4 \text{ psi/° C}) = 0.18 \text{ psi/° C}$$

$$T_r = P_u/1.2 = 2 \text{ minutes}/1.2 = 1.67 \text{ minutes}$$

Proportional plus derivative: Using Equations (9) and (10),

$$K_c = 0.6\, S_u = (0.6)(0.4 \text{ psi/° C}) = 0.24 \text{ psi/° C}$$

$$T_d = P_u/8 = (2 \text{ min.})/8 = 0.25 \text{ minutes}$$

Three modes: Using Equations (11) through (13),

$$K_c = 0.6\, S_u = (0.6)(0.4 \text{ psi/° C} = 0.24 \text{ psi/° C}$$

$$T_r = 0.5\, P_u = (0.5)(2 \text{ minutes}) = 1.0 \text{ minutes}$$

$$T_d = P_u/8 = (2 \text{ min.})/8 = 0.25 \text{ minutes}$$

A slight modification of the above procedure has also been proposed.[3] For some processes it is not feasible to allow sustained oscillations, and thus the "ultimate" method cannot be used. In this method, the gain (proportional control only) is adjusted, using steps analogous to those used in the "ultimate" method, until a response curve with a decay ratio of 1:4 is obtained. However, it is necessary to note only the period, P, of the response. With this value, the reset and derivative modes are set as follows:

$$T_r = P/6 \qquad (14)$$

$$T_d = P/1.5 \qquad (15)$$

After setting these modes, the gain is again adjusted until a response curve with a decay ratio of 1:4 is obtained.

Example: Suppose a process controlled presently by a proportional controller has a response whose period is 3 minutes when the decay ratio is 1:4. If reset and rate (derivative) action are added to the controller, what settings would be recommended?

Reset mode: Using Equation (14),

$$T_r = P/6 = 3 \text{ minutes}/6 = 0.5 \text{ minutes}$$

Derivative mode: Equation (15) indicates

$$T_d = P/1.5 = 3 \text{ minutes}/1.5 = 2 \text{ minutes}$$

In general, there are two obvious disadvantages of these methods. First, both are essentially trial and error, since several values of gain must be tested before the ultimate gain, or the gain to give a 1:4 decay ratio, is

The differential operator shortens mathematical expressions . . .

To show a change in a variable, Y, with time, t, a symbol, p, can be defined so that

$$pY = dY/dt$$

Thus

$$p = d/dt$$

and is called a differential operator. It is used to shorten mathematical expressions. For example, the second derivative of Y is:

$$d^2Y/dt^2 = p^2 Y$$

It can be shown also that

$$\int Y\, dt = (1/p)\, Y$$

and

$$\iint Y\, dt^2 = (1/p^2)\, Y$$

The differential operator follows the distributive law for multiplication:

$$p(X + Y) = pX + pY$$

The operator follows the commutative law for multiplication:

$$(p + a)(p + b) = (p + b)(p + a)$$

where a and b are constants.

The operator also follows the law of exponents:

$$p^n p^m Y = p^{n+m}/Y$$

where n and m are positive real numbers. The operator follows, in general, all algebraic laws except it does not follows cancellation since

$$(1/p)(p)[f(t)] = \int p[f(t)]\,dt = f(t) + C_1$$

Simple cancellation would have given $f(t)$ which is only true if the initial conditions yield $C_1 = O$. This particular problem concerning cancellation is not a serious one, however, because the description of physical problems seldom brings about this situation.

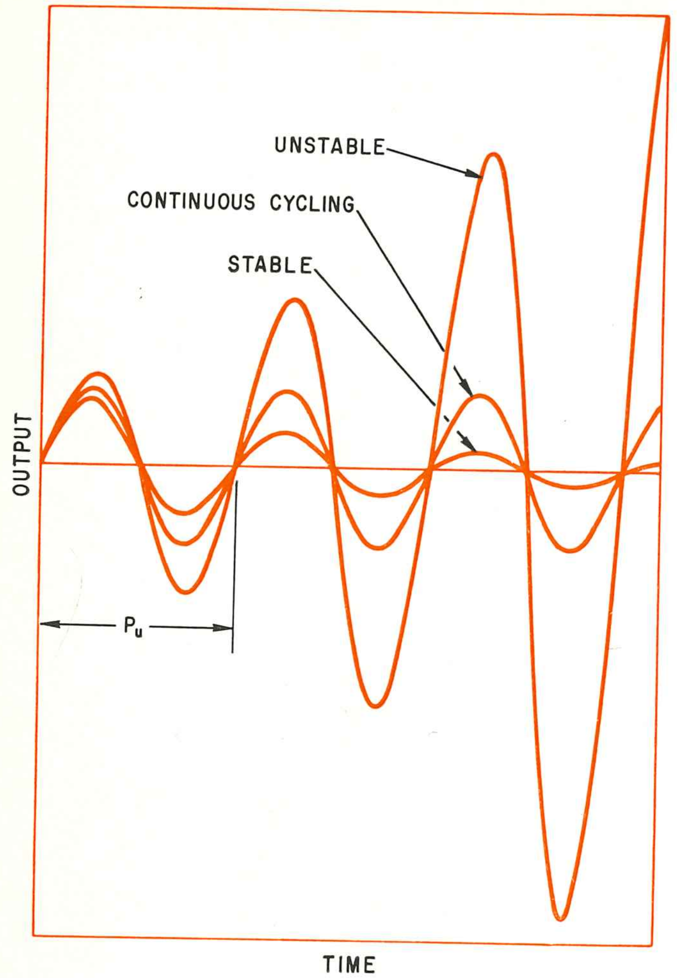

Fig. 5—The "ultimate method" determines the settings which will give continuous cycling without being unstable.

determined. To make one test especially at values of gain near the ultimate gain, it is necessary to wait for the completion of several oscillations before it can be determined whether the trial value of gain is the ultimate gain. Second, while one loop is being tested in this manner, its output may affect several other loops, thus possibly upsetting an entire plant. While all methods require that some change be made in the control loop, some techniques require only one test, not several as in the above methods.

Open Loop Methods. In contrast to the closed loop methods, the open loop methods require that only one upset be imposed on the process. Actually the controller is not in the loop when the process is tested. Thus, these methods seek to characterize the process, and then determine controller settings from the process characteristics.

In general, it is not possible to completely analyze a typical process; hence, approximation techniques are employed. Most of these techniques apply to the process reaction curve, which is simply the response of the process to a unit step change in the manipulated variable; i.e., the output of the controller. To determine the process reaction curve, the following steps are recommended:

1. Let the system come to steady state

2. Place the controller on manual operation; i.e., remove it from automatic operation.

3. Manually set the output of the controller at the value at which it was operating automatically.

4. Allow the system to reach steady state.

5. With the controller still in manual operation, impose a step change in the output of the controller; e.g., air to valve.

6. Record the response of the controlled variable. Although the response is usually being recorded by the controller itself, it is often desirable to have a supplementary recorder or a faster chart drive for the existing controller to insure greater accuracy.

7. Return the controller output to its previous value, and return the controller to automatic operation.

A typical curve resulting from the above procedure is shown in Figure 6. It is undoubtedly easier to obtain the process reaction curve than to obtain the ultimate gain, which is in turn faster than the frequency response methods.

Most open loop methods are based on approximating the process reaction curve by a simpler system, and several techniques[4,5] are available for doing this. By far the most common approximation is that of a pure time delay plus a first order lag.

The main reason for the popularity of this method is that a real time delay of any duration can only be represented by a pure time delay; i.e., there is no adequate approximation. Although it is theoretically possible to use systems of higher than first order in conjunction with a pure time delay, the approximations are difficult to obtain with accuracy for higher systems. Thus, the general system is approximated by a system with a pure time delay plus a first order lag. This approximation is easy to obtain, and is sufficiently accurate for most purposes.

The procedure for approximating the process reaction curve by a first order lag plus time delay is illustrated. The first step is to draw a straight line tangent to the process reaction curve at its point of maximum rate of ascent as shown in Figure 6. Although this is easy in principle, it is quite difficult to do in practice. This is one of the main difficulties in this procedure, and considerable error can be introduced at this point. The slope of this line is termed the reaction rate, R_r, and the time at which this line intersects the initial condition from which the process reaction curve originates is the time delay, L_r. In Figure 6, the determination of these values for a 1.0 psi change in the controller output to a temperature control process is illustrated.

If a different change in controller output were used, the value of L_r would not change significantly. However, the value of R_r is essentially directly proportional to the magnitude of the change in controller output; i.e., if a 2 psi change in output were used instead of 1 psi, the value of R_r would be approximately twice as large. For this reason, the value of R_r used in the equations presented later must be the value that would be obtained for a 1 psi change in controller output. In addition, the

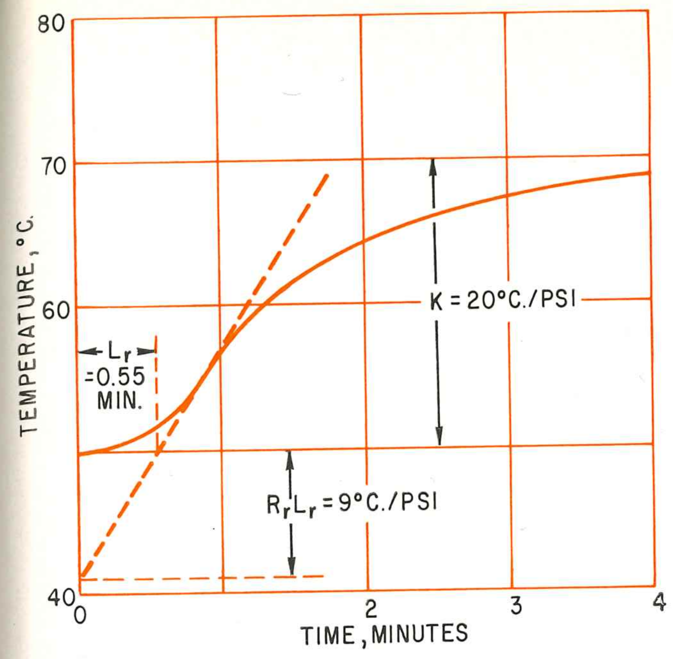

Fig. 6—This response curve is for a step change of 1 psi in output for a controller of a temperature control loop.

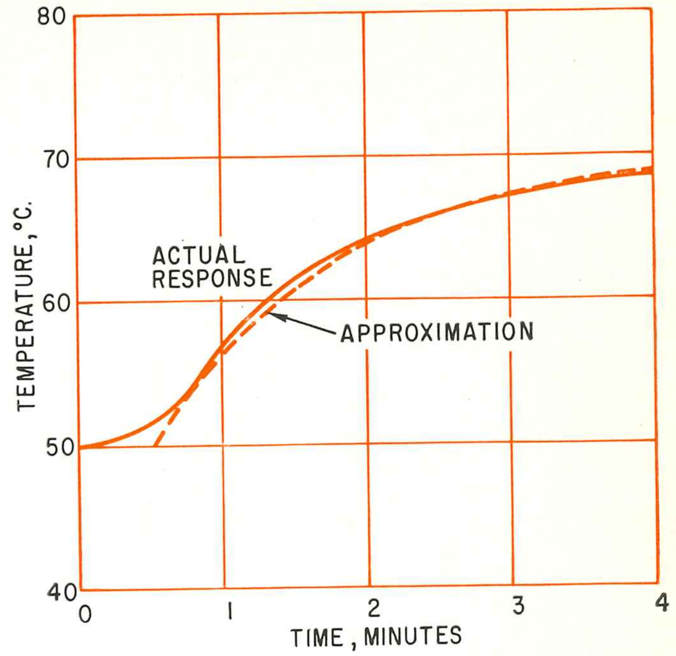

Fig. 7—The actual response of Figure 6 is approximated by finding the values which will satisfy Equation (32).

value of K, the process gain, must be determined, and is defined as follows:

$$K = \frac{\text{final steady state change in controlled variable}}{\text{change in controller output}} \quad (16)$$

One of the first methods using the process reaction curve was also originally proposed by Ziegler and Nichols.[2] To use this method only R_r and L_r must be determined. Using these parameters, the following equations are used to predict controller settings:

Proportional only:

$$K_c = 1/L_rR_r \quad (17)$$

Proportional plus reset:

$$K_c = 0.9/R_rL_r \quad (18)$$

$$T_r = 3.33\,L_r \quad (19)$$

Three modes:

$$K_c = 1.2/R_rL_r \quad (20)$$

$$T_r = 2.0\,L_r \quad (21)$$

$$T_d = 0.5\,L_r \quad (22)$$

Use of these equations is illustrated by the following example:

Example: For the process reaction curve in Figure 6, use the Ziegler-Nichols method to estimate controller settings for a 1:4 decay ratio for proportional, proportional plus reset, and three mode controllers.

From Figure 6,

$$L_r = 0.55 \text{ min}$$

$$R_rL_r = 9° \text{ C/psi}$$

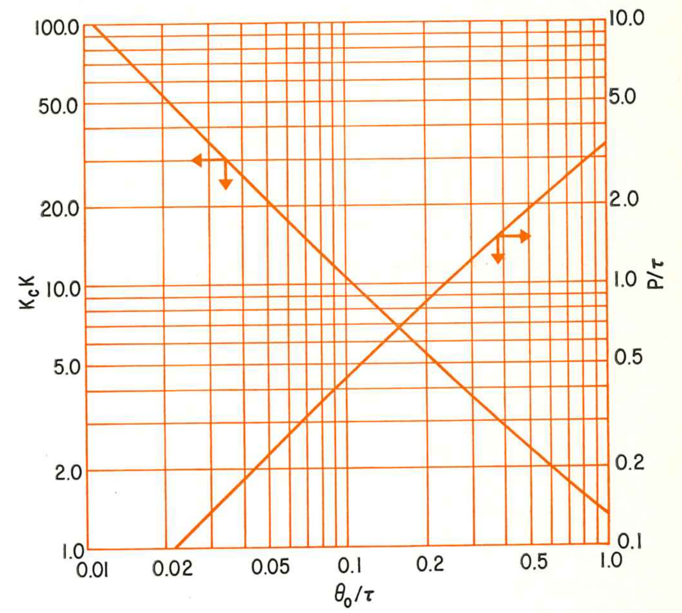

Fig. 8—The constants for proportional control and the frequency of the response can be determined from these curves.

Proportional only:

By Equation (17),

$$K_c = 1/R_rL_r = 1/(9° \text{ C/psi}) = 0.111 \text{ psi/° C}$$

Proportional plus reset:

By Equations (18) and (19),

$$K_c = 0.9/R_rL_r = 0.9/(9° \text{ C/psi}) = 0.1 \text{ psi/° C}$$

$$T_r = 3.33\,L_r = (3.33)(0.55 \text{ min}) = 1.83 \text{ min}$$

Three modes:

By Equations (20) through (22),

$$K_c = 1.2/R_rL_r = 1.2/(9° \text{ C/psi}) = 0.133 \text{ psi/° C}$$

$$T_r = 2.0\, L_r = (2.0)(0.55 \text{ min}) = 1.10 \text{ min}$$

$$T_d = 0.5\, L_r = (0.5)(0.55 \text{ min}) = 0.275 \text{ min}$$

In developing the above equations, Ziegler and Nichols considered processes that were not "self-regulating." To illustrate, consider level control of a tank with a constant rate of liquid removal, initially operating such that the level is constant. If a step change is made in the inlet valve opening, the level in the tank would rise until it overflows. This process is not "self-regulating." On the other hand, if the outlet valve opening and outlet pressure are constant, the rate of liquid removal increases as the liquid level increases. Hence in this case, the level in the tank will rise to some new position, but would not increase indefinitely. Thus, the system is self-regulating. To include this phenomenon, Cohen and Coon[6] introduced an index of self regulation, μ, defined as

$$\mu = R_rL_r/K \qquad (23)$$

For processes originally considered by Ziegler and Nichols, $\mu = 0$; i.e., no self regulation. To account for variation in μ, Cohen and Coon suggest the following equations.

Proportional control:

$$K_c = [1 + (\mu/3)]/R_rL_r \qquad (24)$$

Proportional plus reset control:

$$K_c = 0.9\,[1 + (\mu/11)]/R_rL_r \qquad (25)$$

$$T_r = 3.33\, L_r\, [1 + (\mu/11)]/[1 + (11\,\mu/5)] \qquad (26)$$

Proportional plus derivative:

$$K_c = 1.2\,[1 + (\mu/8)]/R_rL_r \qquad (27)$$

$$T_d = 0.27\, L_r\, [1 - (\mu/3)]/[1 + (\mu/8)] \qquad (28)$$

Three modes:

$$K_c = 1.35\,[1 + (\mu/5)]/R_rL_r \qquad (29)$$

$$T_r = 2.5\, L_r\, [1 + (\mu/5)]/[1 + (3\mu/5)] \qquad (30)$$

$$T_d = 0.37\, L_r/[1 + (\mu/5)] \qquad (31)$$

Since these equations contain an additional parameter, they should be more accurate than those originally proposed by Ziegler and Nichols. An example of this method is illustrated below.

Example: Determine optimum settings for proportional, proportional plus reset, proportional plus derivative, and three term controllers for the process whose reaction curve is given in Figure 6.

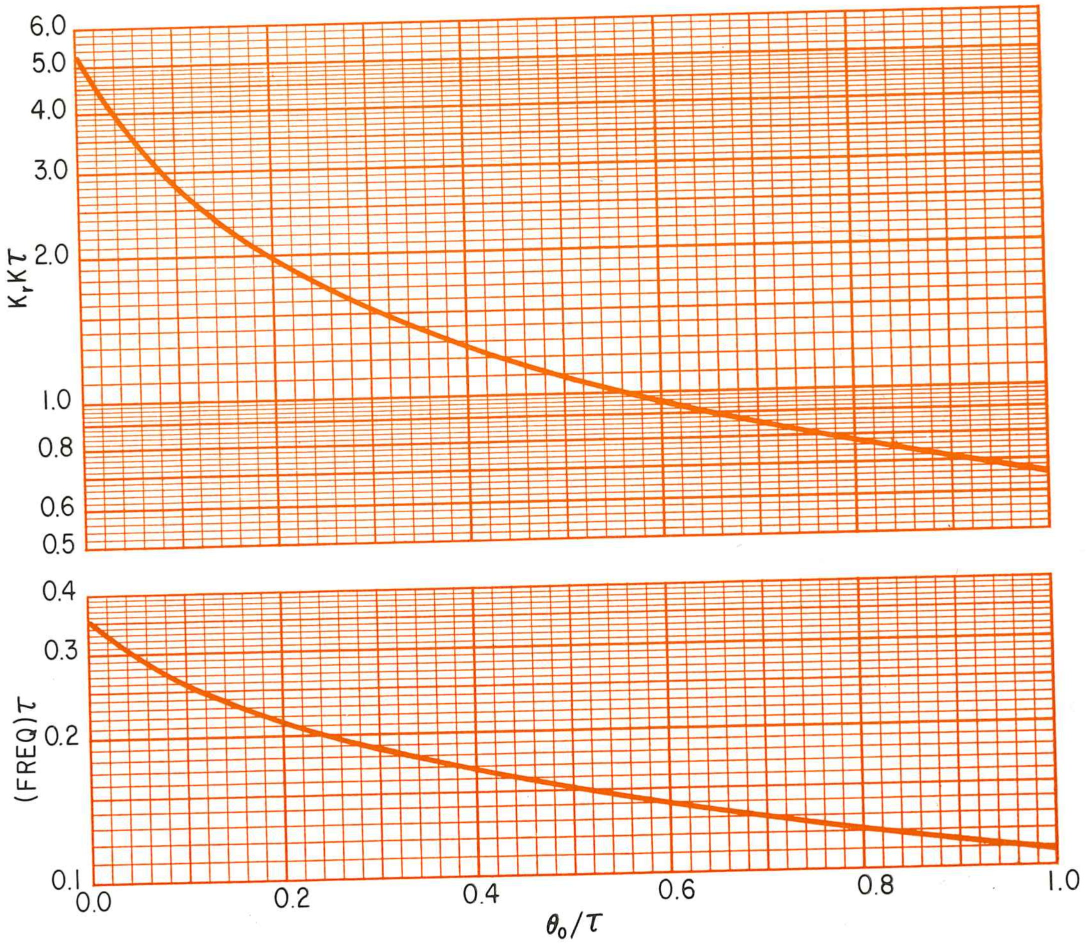

Fig. 9—The constants for reset control and the frequency of the response is obtained from these curves.

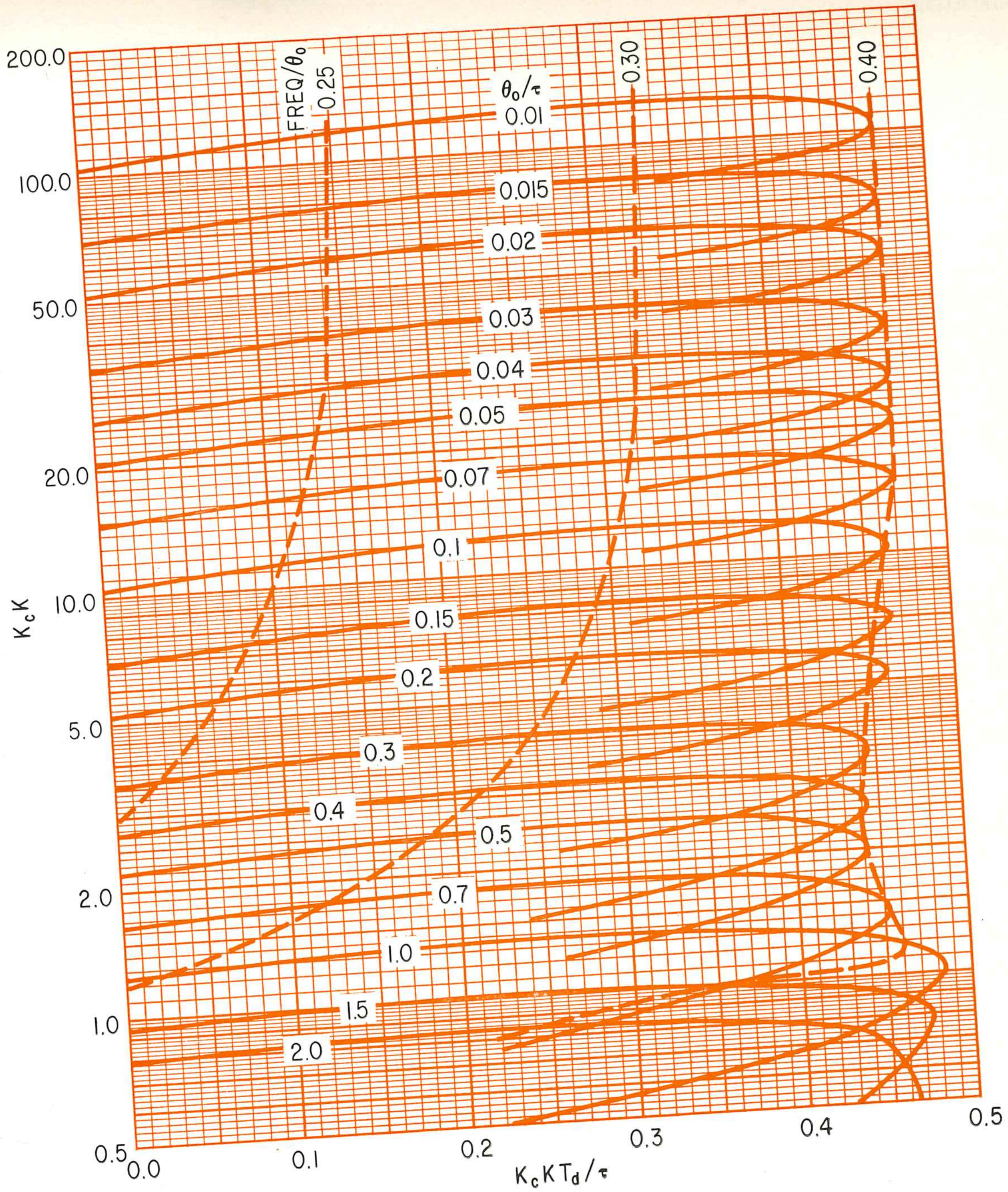

Fig. 10—The constants for a proportional plus derivative controller are read from these curves.

$$L_r = 0.55 \text{ minutes}$$

$$R_rL_r = 9° \text{ C/psi}$$

$$K = 20° \text{ C/psi}$$

$$\mu = R_rL_r/K = (9° \text{ C/psi})/(20° \text{ C/psi}) = 0.45$$

Proportional Controller:

Using Equation (24),

$$K_c = [1 + (\mu/3)]/R_rL_r = 1.15/(9° \text{ C/psi}) = 0.128 \text{ psi/° C}$$

Proportional plus reset:

Using Equations (25) and (26),

$$K_c = 0.9\,[1 + (\mu/11)]/R_rL_r = (0.9)(1.04)/(9.0° \text{ C/psi}) = 0.104 \text{ psi/° C}$$

$$T_r = 3.33\, L_r\, [1 + (\mu/11)]/[1 + (11\mu/5)] = (3.33)\,(0.55 \text{ min})\,(1.04)/(1.99) = 0.956 \text{ minutes}$$

Proportional plus derivative:

Using Equations (27) and (28),

$$K_c = 1.2\,[1 + (\mu/8)]/R_rL_r = (1.2)(1.056)/(9 \text{ °C/psi}) = 0.141 \text{ psi/° C}$$

$$T_d = 0.27\, L_r\, [1 - (\mu/3)]/[1 + \mu/8] = (0.27)(0.55 \text{ min})(0.85)/(1.056) = 0.120 \text{ minutes}$$

These methods tune controllers faster than the more oft[illegible]d hit-or-miss methods

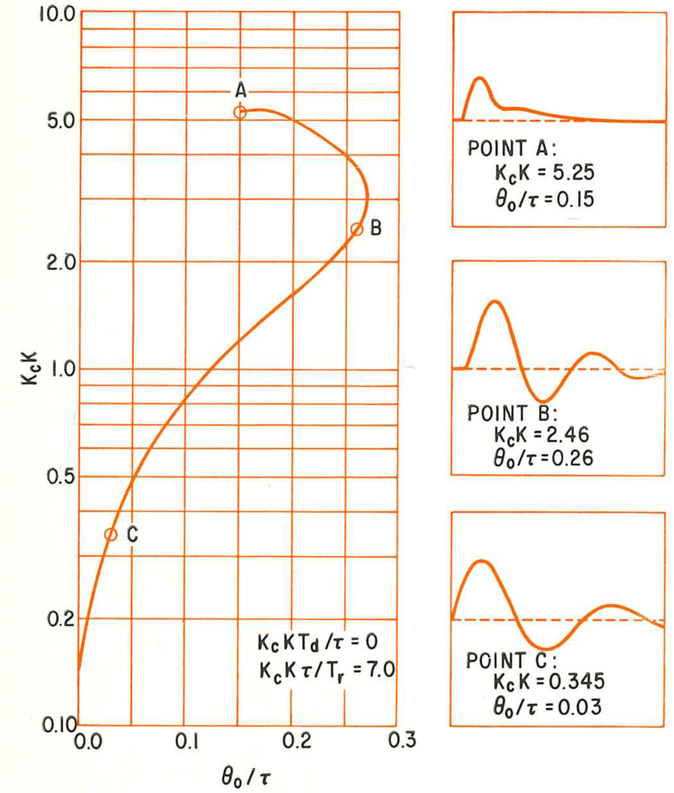

Fig. 11—A typical contour for a proportional plus reset controller with time response at various points shown at the right.

Three modes:

Using Equations (29) through (31),

$$K_c = 1.35\ [1 + (\mu/5)]/R_rL_r = (1.35)(1.09)/(9° \text{ C/psi})$$
$$= 0.163 \text{ psi/°C}$$
$$T_r = 2.5\ L_r\ [1 + (\mu/5)]/[1 + (3\mu/5)]$$
$$= (2.5)(0.55 \text{ min})(1.09)/(1.27) = 1.18 \text{ minutes}$$
$$T_d = 0.37\ L_r/[1 + (\mu/5)] = (0.37)(0.55 \text{ min})/(1.09)$$
$$= 0.187 \text{ minutes}$$

In addition to the above equations, Cohen and Coon[6] also presented graphs from which many possible combinations of settings to give a decay ratio of 1:4 can be obtained. However, the 1:4 decay ratio used in constructing the graphs was only for the dominant term in the complex numerical expression for the actual response. It was assumed that the decay ratio of the dominant term was approximately equal to the decay ratio of the actual response. Although this assumption is valid in most cases, there are numerous exceptions, and it is practically impossible to predict when this assumption is not valid. The main reason for making this assumption is that it shortens the calculations required in order to draw graphs and control parameters. With the advent of high-speed digital computers, the time required to perform the lengthy calculations is minimized, and is usually no longer an overly complex problem. Thus, in the graphs to be presented here, the decay ratio is computed for the actual transient response.

In developing the method to be presented next, an attempt is made to be as mathematically precise as possible. In essence, the graphs can be used to tune exactly and precisely an "ideal" controller for a process composed of a pure time delay plus a first order lag. An example of the response of this type of system to a unit step input is shown in Figure 7. The problem is to approximate the process reaction curve in Figure 6 by a system with a pure time delay plus a first order lag. The three terms required are the process gain, K, the time lag, θ_o, and the time constant, τ. If these terms are known, the Laplace transform expression relating the input to the output for the process is

$$\frac{\text{output}}{\text{input}} = \frac{K_c\ \varepsilon^{-\theta_o s}}{\tau s + 1} \qquad (32)$$

Thus, a technique for approximating the process reaction curve in Figure 6 by the above equation is required. Using the technique presented earlier, the values of R_r, L_r, and K can be determined. These are related to θ_o and τ as follows:

$$\theta_o = L_r \qquad (33)$$
$$\tau = K/R_r \qquad (34)$$

Thus, for Figure 6, K = same for both techniques.

$$\theta_o = 0.55 \text{ minutes,}$$
$$\tau = (20)(0.55)/9 = 1.222 \text{ minutes, and}$$
$$\mu = \theta_o/\tau = 0.45.$$

In the next method to be presented, only two major approximations remain. First, the controller is assumed to be ideal. Second, errors are introduced when the process reaction curve is approximated by a first order lag plus time delay. In most cases, the above method for obtaining this approximation is reasonably accurate; however, in a few cases, it is not very good. Undoubtedly, the accuracy of the results depend upon the accuracy of this approximation. Although not so obvious, this same shortcoming is present in the previous techniques for determining controller settings. Thus, any method that improves this approximation would consequently improve the accuracy of these methods also. Although it is possible to get a better fit by a least squares procedure, the relatively lengthy calculations can be justified only in exceptional cases.

In developing the graphs, four dimensionless groups of variables must be related. These groups are θ_o/τ, K_cK, $K_cK\tau/T_r$, and K_cKT_d/τ. Of these, *the group* θ_o/τ is completely specified from the process reaction curve. Thus, the other groups are presented as a function of θ_o/τ. Since K is specified from the process reaction curve, the value of K_c can be specified if the value of K_cK is known. Similarly, T_r and T_d are determined from the other two groups.

For proportional control only, $T_r = \infty$ and $T_d = 0$. Then, the groups, $K_cK\tau/T_r$ and $K_cK\ T_d/\tau$ are both zero. Thus, K_cK can be plotted vs. θ_o/τ as shown in Figure 8. In addition, the period or frequency of the response may also be determined from Figure 8.

In reset only, $K_c = T_d = 0$, which again eliminates two of the four dimensionless groups. Since there is only one mode in the controller, only one variable, K_r, must be determined for a floating controller. In order to facilitate this, Figure 9 can be used to determine the value of K_r to give a decay ratio of 1:4. Also, the frequency can be obtained from Figure 9.

Proportional plus derivative control is unlike the two foregoing cases where only one restraint—that the response have a decay ratio of 1:4—was placed on the system in order to completely specify the settings in the

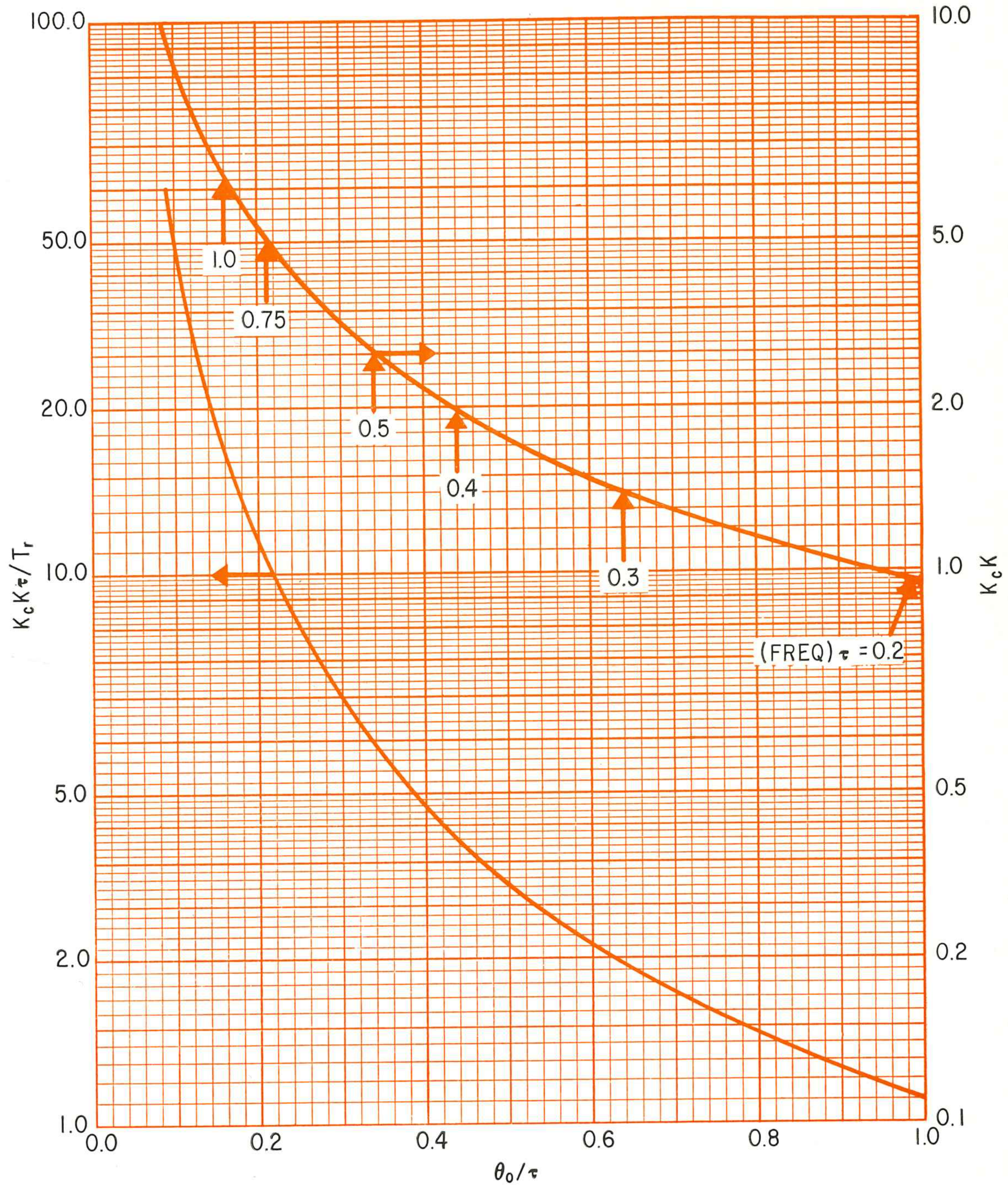

Fig. 12—The constants for a proportional plus reset controller to give a 1:4 decay ratio and minimum control area. $K_cK\ T_d/\tau = 0$.

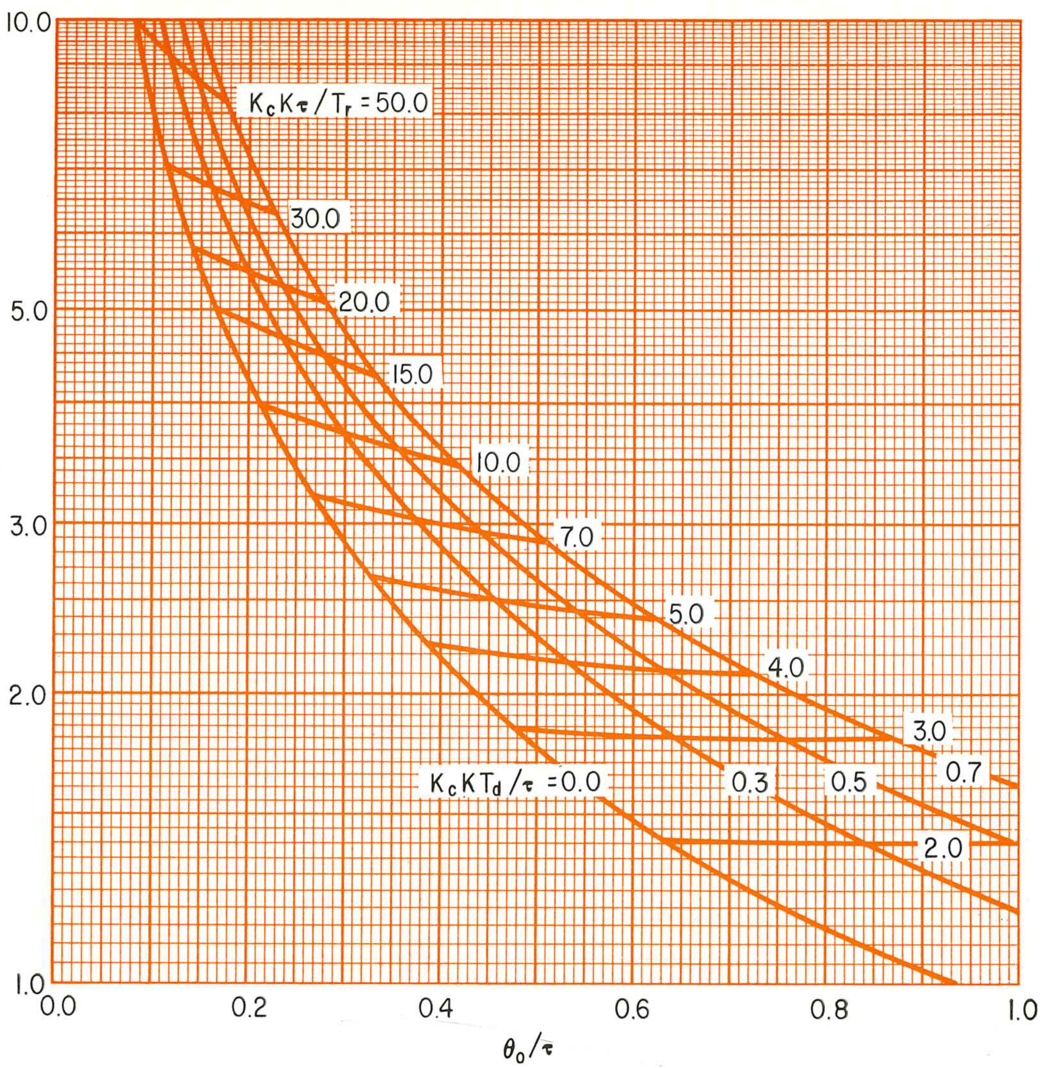

Fig. 13—The constants for a three term controller to give a 1:4 decay ratio are determined from these curves.

controller. When more than one mode is added to the controller, there is generally an infinite number of combinations of settings that will give a decay ratio of 1:4. In the case of a controller with two modes, one additional constraint is needed to determine a unique solution.

For a proportional plus derivative controller, all but one dimensionless group, namely $K_cK\tau/T_r$, are required. Since these groups are independent, there is an infinite number of values for a decay ratio of 1:4. From Figure 10 these values can be determined, along with the frequency. However, some other criterion is needed to specify which one of the infinite number of possible values is optimum. One popular criterion often used is for the system to have minimum offset at steady state. Using this criterion, the values can easily be determined from Figure 10. Since the value of θ_o/τ is known, the value of $K_cK\,T_d/\tau$ to be used is the one that gives the maximum value of K_cK.

For proportional plus reset, there are again two variables, K_c and T_r, that must be specified in order to tune the controller. Requiring that the decay ratio be 1:4 is not sufficient to insure a unique solution, and another constraint is needed. One useful constraint, in addition to the decay ratio constraint, is to require that the control area of the response be a maximum; i.e., the area under the response curve be a minimum. This can be shown to occur at the minimum value of reset time. Although this may not be obvious at first it can be shown by the following reasoning:

For a controller with a reset mode, the steady state error signal e is zero. Hence, the output from the proportional and derivative modes is zero at steady state. However, if a unit step disturbance is imposed upon the process, when the process again reaches steady state, the output of the controller cannot be zero, even if there is no error signal. In Figure 2 the final steady state values of e and c are both zero for a step change in disturbance. Since c is zero, the input to the process block is also zero. A balance around the summer at the disturbance signal input reveals that

$$1 + m = 0 \qquad (35)$$

or

$$m = -1 \qquad (36)$$

However, at steady state, m equals the output of the reset mode, since the output of all other modes is zero. Hence,

$$m = (K_c/T_r)\int e\,dt = -1 \qquad (37)$$

However, the integral of $c_t - r_t$ is the control area. From Figure 2 it is seen that

$$-e = c - r \qquad (38)$$

Substituting into Equation (37)

$$(\text{control area})\,(K_c/T_r) = 1 \qquad (39)$$

$$\text{control area} = T_r/K_c \qquad (40)$$

To minimize the control area, then the group $K_cK\tau/T_r$ must be a maximum, since K *and* τ are constant for a given system.

One value of reset is shown in Figure 11, along with responses at three points on the curve. To the right of this curve, the value of reset rate decreases, and it increases to the left. For $\theta_o/\tau = 0.27$, the maximum value of $K_cK\tau/T_r$ is 7.0, the value for which the curve is drawn. Although it is possible to draw a family of these curves, in Figure 12 both restraints (i.e., 1:4 decay ratio and minimum control area) are included in order that a unique combination of settings are specified. From Figure 12, the values of K_cK and $K_cK\tau/T_r$ can be determined if the value of θ_o/τ is known.

For a three term controller, graphs similar to that for the proportional plus reset control can be drawn for specified values of derivative time, as in Figure 13. Thus, the criteria of 1:4 decay ratio and minimum control area are both included. However, given values of θ_o, τ, and K, there is still an infinite number of solutions. In the case of the proportional plus derivative controller, maximum gain is desired to give minimum offset. But when the reset mode is present, the offset at steady state is zero for all values of gain. However, as the gain is increased, the frequency increases and the control area decreases (reset rate increases), both of which are desirable.

Although the gain cannot be increased indefinitely in practice, no satisfactory practical constraint is available to specify the optimum value of gain. As the gain increases, the decay ratio becomes more sensitive to the settings, and the response curve may deviate considerably from the smooth sinusoidal. Cohen and Coon[7] recommend a value of 0.5 for K_cKT_d/τ, and from the experience in drawing these graphs, this seems to be a reasonable value, but it is not the optimum value for all cases. If a higher value of derivative time can be used, a better response will be obtained.

In practice, the optimum derivative time could probably be obtained more readily by the following procedure:

1. Select the value of K_cKT_d/τ equal to 0.5 and determine the reset time and again using the graph in Figure 13. Thus, for this particular value of derivative time, the control area will be a minimum.

2. Test this value on the actual system.

3. If the response is satisfactory or is too slow the derivative time may be increased, and the gain and reset time adjusted according to Figure 13. This can be continued as long as the characteristics of the response improve; however, a point will usually be reached beyond which increasing the derivative time is not advantageous.

4. If the response curve obtained in Step 2 is unsatisfactory, then the derivative time should be decreased (adjusting the reset time and gain accordingly) until a satisfactory response is obtained.

The following example illustrates the use of these graphs.

Example. The preceding graphs can be used to tune a proportional, floating, proportional plus derivative, proportional plus reset, or three mode as follows: From Figure 6,

$K = 20°$ C/psi

$\tau = 1.222$ min

$\theta_o/\tau = 0.45$ min

Proportional only:

From Figure 8, the value of K_cK is 2.5. Thus,

$$K_c = 2.5/K = 2.5/(20°\ \text{C/psi}) = 0.125\ \text{psi/° C}$$

Floating controller:

From Figure 9, $K_rK\tau = 1.18$. Thus,

$$K_r = 1.18/(K\tau) = 1.18/(20°\ \text{C/psi})\,(1.222\ \text{min}) = 0.0480\ \text{psi/° C-min.}$$

Proportional plus derivative:

From Figure 10, the value of K_cK to give the minimum offset for $\theta_o/\tau = 0.45$ is 2.8, at which $K_cKT_d/\tau = 0.3$. Thus,

$$K_c = 2.8/K = 2.8/(20°\ \text{C/psi}) = 0.140\ \text{psi/° C}$$
$$T_d = 0.3\,\tau/K_c\,K$$
$$= (0.3)\ (1.222\ \text{min})/(0.140\ \text{psi/° C})\ (20°\ \text{C/psi})$$
$$= 0.131\ \text{min}$$

However, upon examination of the graph in Figure 10, it can be seen that the derivative time can be increased considerably above the value of 0.131 min, thereby increasing the frequency without increasing the offset significantly.

Proportional plus reset:

From Figure 12, the values of K_cK and $K_cK\tau/T_r$, for minimum control area are 1.93 and 3.18 respectively. Thus,

$$K_c = 1.93/K = 1.93/(20°\ \text{C/psi}) = 0.0965\ \text{psi/° C}$$
$$T_d = 0.3\tau/K_cK$$
$$T_r = K_cK\,\tau/3.18$$
$$= (0.0965\ \text{psi/° C})\ (20°\ \text{C/psi})\ (1.222\ \text{min})/3.18$$
$$= 0.742\ \text{min}$$

Three mode:

Arbitrarily setting K_cKT_d/τ equal to 0.5 as discussed earlier, the values of K_cK *and* $K_cK\tau/T_r$ for minimum control area are 2.90 and 7.0 respectively, as determined from Figure 13. Thus,

$$K_c = 2.90/K = 2.90/(20°\ \text{C/psi}) = 0.145\ \text{psi/° C}$$
$$T_r = K_cK\tau/7.0 = (0.145\ \text{psi/° C})\,(20°\ \text{C/psi})\,(1.222\ \text{min})/7.0$$
$$= 0.507\ \text{minutes}$$
$$T_d = 0.5\,\tau/K_cK$$
$$= (0.5)\ (1.222\ \text{min})/(0.145\ \text{psi/° C})\ (20°\ \text{C/psi})$$
$$= 0.211\ \text{minutes}$$

Basic Characteristics of Controllers

The controller is a special purpose analog computer which takes the difference between the actual value of a controlled variable and its desired value, and uses this difference to manipulate the control system. In general, process controllers can be classified as:

- Pneumatic
- Electronic
- Hydraulic

Hydraulic control systems constitute, by a wide margin, the smallest category and the hardware associated with hydraulic systems is very similar to that found in pneumatic systems.

The great bulk of control instrumentation for hydrocarbon processing is pneumatic or electronic in nature. Of these two, pneumatic components are probably the larger category, but the percentage use of electronic equipment is increasing. Each type has advantages and disadvantages so that there are some applications for which each is best suited.

Electronic components in general are best suited where the following situations exist.[8]

- Extreme accuracy is necessary.
- Fast control loops are encountered.
- Remote transmission is necessary.
- Computer control, control data procesing, and similar undertakings are under consideration.

Pneumatic equipment should be considered in situations where factors are just the opposite of those above; i.e.,

- Extreme accuracy is not necessary. Electronic instruments are capable of ¼ of one percent, whereas, pneumatic instruments can offer approximately ½ of one percent.
- Extreme speed is not necessary. Many process control loops contain such large time constants that the speed of electronic equipment is not at all necessary.
- Transmission distances are short. Pneumatic and electronic transmission systems are generally equal up to about 250 to 300 feet. Above this distance, electronic systems begin to offer savings.
- Computer control, control data processing, et cetera are not being considered. If these systems are under consideration, the interface between the pneumatic component and the electronic system would introduce additional costs.
- Maintenance men have not had sufficient training and background in electronics. Basically, pneumatic equipment is simpler.
- Operating personnel are reluctant to accept electronic instruments. Many operating personnel have more experience and confidence in pneumatic components and are unfamiliar with electronics.

Costs are not considered in the foregoing items. In a published cost comparison of a 100 loop installation, costs were tabulated for the necessary transmitters, receivers and controllers, and the cost of the electronic equipment was 56 percent greater than the pneumatic equipment.[8] Control valves are not normally included in cost comparisons because they are universally pneumatic. As far as construction costs are concerned, installation costs are roughly the same for either pneumatic or electronic (except above 250 to 300 feet of transmission lines), but check-out, calibration, and start-up costs are roughly four times more for electronic equipment. A good discussion of this basic question of comparison between pneumatic and electronic equipment is given in the literature.[8]

The hydraulic systems have the following advantages:

- Large positive output forces are readily available
- Hydraulic motors are smaller than equivalent electrical or pneumatic motors
- Components are quick acting
- Components are rugged, dependable.

The disadvantages of a hydraulic system are:

- Return lines are needed for the hydraulic fluid
- Most hydraulic fluids are flammable
- Temperature effects on fluid viscosity affect performance.

For typical process control applications, the disadvantages of hydraulic control components will generally outweigh the advantages, and very few pure hydraulic control systems are used. The biggest advantage of hydraulic components is their ability to position accurately large, heavy loads. Then a control system uses a hydraulic component as the final control element. The remainder of the control elements may be pneumatic or electronic or both.

Hydraulic controllers generally fall into one of two major categories: the four-way valve type or the flapper-nozzle type. The flapper-nozzle type operates exactly like the pneumatic flapper-nozzle controllers to be discussed later, except that the working fluid is incompressible. Hydraulic components of a third general type might be considered, and those are the non-moving-parts fluid amplifier also discussed later in the section about pneumatic controllers.

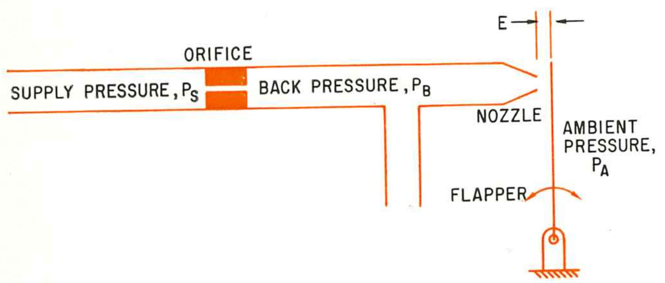

Fig. 14—This motion balance type of pneumatic controller uses a flapper-nozzle arrangement.

PNEUMATIC CONTROLLERS

Flapper-Nozzle. As the first class of controller equipment to analyze, a motion balance type of flapper-nozzle pneumatic controller will be considered.[9,10,11] This is so named because the input to the controller is basically a change in a displacement, i.e., a motion. The central component of the controller is a flapper-nozzle arrangement such as is shown in Figure 14.

The back pressure P_B in the nozzle chamber is controlled by the position of the flapper with respect to the nozzle. If the flapper is fully closed, i.e., if $E = 0$, the back pressure P_B will be equal to the supply pressure P_S. If the flapper is fully removed from the nozzle, the back pressure P_B will fall relatively close to P_A. As the flapper moves from fully closed to fully open, P_B varies from P_S to P_A. Relative orders of magnitude for these variables are $P_S = 20$ psig and nozzle diameter equal to 0.02 to 0.03 inches. The baffle position E is established by the controlled variable (through the feedback elements) and the chamber back pressure P_B can be used to establish the controller output.

In order to establish the equation describing this flapper-nozzle arrangement, its operation will be assumed isothermal and the air will be considered as an ideal gas. Flow of air into the nozzle chamber will be a function of the pressure drop available across the orifice.

W_i = weight rate of air flow into nozzle chamber
= a function of P_B (since P_S is constant)

Treating this relationship as linear in the region of interest:

$$w_i = -K_1 p_B \qquad (41)$$

where w_i = the variation in W_i

$$K_i = \frac{dW_i}{dP_B}$$

p_B = the variation in P_B

The flow of air out of the nozzle chamber will be a function of P_B and E. Linearizing this in the operating region:

$$W_o = f(P_B, E)$$

and,

$$w_o = K_2 p_B + K_3 e \qquad (42)$$

where $K_2 = \left.\frac{\partial W_o}{\partial P_B}\right|_i$, $K_3 = \left.\frac{\partial W_o}{\partial E}\right|_i$, e = the variation in E

If the mass of air M in the nozzle chamber varies, it will do so because of the difference between the flow into and out of the nozzle chamber.

$$W_i - W_o = dM/dt$$

and,

$$w_i - w_o = pm \qquad (43)$$

Substituting Equations (41) and (42):

$$-K_1 p_B - K_2 p_B - K_3 e = pm \qquad (44)$$

Using the ideal gas law and the molecular weight of air as 29, and assuming the chamber temperature and volume are constant.

$$M = 29 P_B V/RT$$

and

$$m = K_4 p_B \text{ where } K_4 = 29V/RT \qquad (45)$$

Substituting Equation (45) into Equation (44):

$$-(K_i + K_2)\, p_B - K_3 e = pK_4\, p_B$$

$$p_B = \frac{-K_3 e}{K_1 + K_2 + K_4 p}$$

$$= \frac{-K_3 e/(K_1 + K_2)}{1 + K_4 p/(K_1 + K_2)}$$

$$= \frac{-K'e}{1 + \tau p} \qquad (46)$$

where $K' = K_3/(K_1 + K_2)$

$\tau = K_4/(K_1 + K_2)$

The relationship between a flapper movement and a change in back pressure is, therefore, a first order lag. Considering the relatively small volume of the nozzle chamber, it is certainly probable the time constant τ will be negligible with respect to typical system time constants. The chamber back pressure may be used to charge a bellows or a diaphragm chamber (thus, a output displacement is achieved) and the volume of the chamber will vary with P_B. This will increase τ but it will still probably be very negligible with respect to other lags in all but the very fastest control loops. Therefore:

$$p_B = -K'e \qquad (47)$$

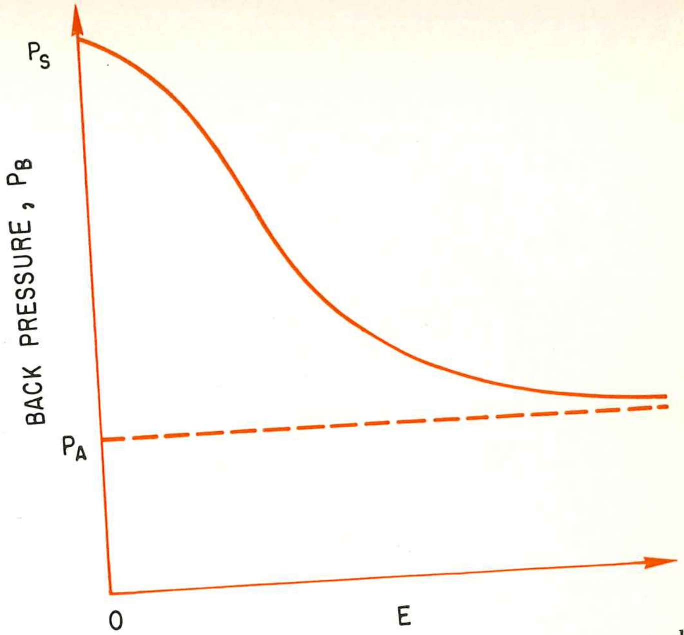

Fig. 15—The relationship between flapper movement and changes in back pressure is equivalent to a first order lag.

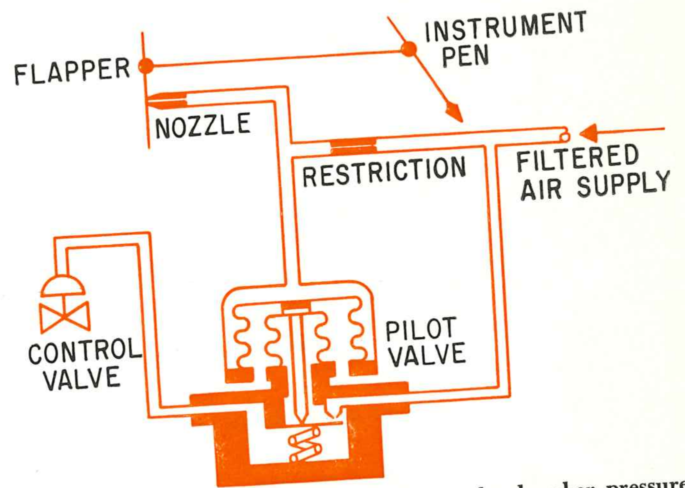

Fig. 16—The relay uses a change in nozzle chamber pressure to change air supplied to the control valve (Minneapolis-Honeywell).

In the typical flapper-nozzle arrangement K' is a very large number and to cover the P_B range of 3-15 psig (which is standard for all pneumatic instruments) may require a movement of the flapper position of less than 0.002 inch. The general shape of the P_B vs. E curve is as shown in Figure 15.

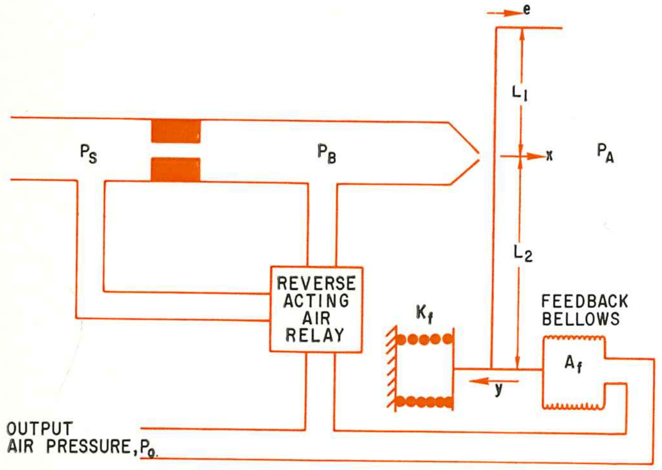

Fig. 17—A feedback arrangement reduces the sensitivity and gives a more linear relationship between nozzle position and P_o.

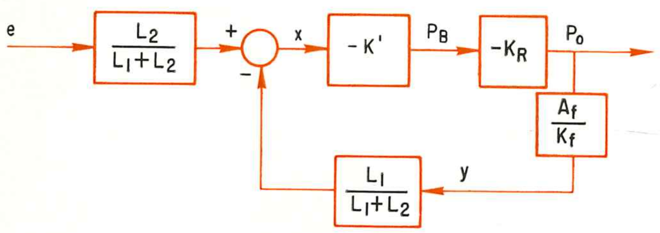

Fig. 18—Controller equations can be shown in block diagrams.

A guide for setting controllers . . .

In making corrective adjustments, an understanding of the effect of each mode upon the response is essential. Thus, the following *generalities* about the effects of adjustments in each mode are usually correct.

Proportional band. Decreasing the proportional band (increasing the gain) increases the decay ratio, thus making the system less stable. However, the frequency of the response is also increased, which is usually desirable. Increasing the proportional band has an opposite effect.

Reset mode. When the reset time is increased, the decay ratio is decreased, thus making the system more stable. Simultaneously, the frequency increases. Decreasing the reset time has an opposite effect. Recall that when the reset time is at its maximum value, this mode has been turned out of the controller.

Derivative mode. Of all the modes, the effect of this mode is the most difficult to predict. Starting at a derivative of zero, increasing the derivative time usually is beneficial, but not always. However, in almost all practical cases, there is a point beyond which increasing the derivative time will prove detrimental. Thus, about all that one can do is try a change in the derivative time and see what happens.

Two very important disadvantages prevent the back presure P_B from being used to position a final control valve in a system. First, the control valve's diaphragm chamber and the intermediate transmission lines have so much capacity with respect to the flow capacity of the nozzle arrangement that the lag of the controller would become intolerable. Second, the flapper-nozzle arrangement is so very sensitive that the control valve would either be open or shut; i.e., the mechanism would effectively be two-position control.

These two disadvantages can be overcome by the addition of separate mechanisms to the flapper-nozzle arrangement. The first problem, low output capacity for the flapper-nozzle system, can be overcome by amplification of the output signal. This is done through pneumatic "relays" which use a change in nozzle chamber back pressure to effect a change in the air supplied (from a source other than the nozzle chamber) to the valve diaphragm. An industrial example of the flapper-nozzle arrangement used as an on-off controller through the aid of a pneumatic relay is shown in Figure 16.

The second problem encountered in flapper-nozzle systems, their very high sensitivity, can be overcome by some means of reducing the tremendous increase in back pressure produced by a small movement of the flapper toward the nozzle. The most satisfactory solution of this is the incorporation of a negative feedback arrangement between the back pressure and the flapper position. A typical arrangement is shown in Figure 17. In this arrangement the flapper position with respect to the nozzle is x and Equation (47) becomes:

$$p_B = -K'x \qquad (48)$$

The back pressure operates a reverse acting air relay for amplification. This relay is termed "reverse acting" because for an increase in P_B there will be a decrease in its output pressure P_o.

$$p_o = -K_R\, p_B$$

or

$$p_o = -K_R\, p_B \qquad (49)$$

It might be noted that variation in K_R can be controlled throughout the range of P_B so that K_R varies inversely as K' so that a more linear relationship between nozzle position and output pressure is achieved over a wider operating range. The feedback bellows has a cross-sectional area A_f and a restoring force from a spring constant K_f. A force balance on this bellows can be written:

$$A_f P_o = K_f Y$$

or in terms of linear variations:

$$A_f p_o = K_f y \qquad (50)$$

The posiiton of the flapper with respect to the nozzle depends on E and Y.

$$X = f(E, Y)$$

or in terms of linear variations:

$$x = (\partial x/\partial E)e + (\partial x/\partial Y)y$$

and using the lever arms defined in Figure 17:

$$x = L_2 e/(L_1 + L_2) - L_1 y/(L_1 + L_2) \qquad (51)$$

These equations can be combined in block diagram form as shown in Figure 18. The equation relating e and p_o is:

$$p_o = \frac{[L_2/(L_1+L_2)](K')(K_R)\,e}{1 + (K')(K_R)(A_f/K_f)[L_1/(L_1+L_2)]} \qquad (52)$$

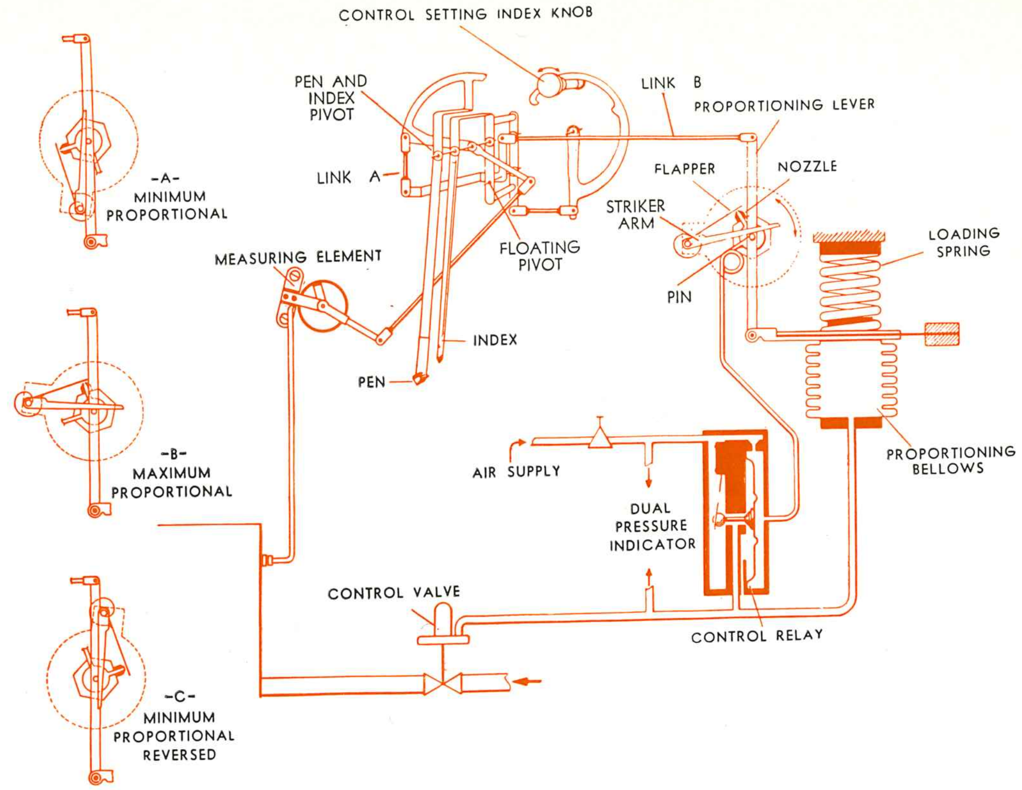

Fig. 19—One industrial example of a proportional controller uses this arrangement of parts (The Foxboro Co.).

Since K' is so large, the 1 in the denominator is negligible. Therefore:

$$p_o = \frac{L_2 K_f e}{L_1 A_f} \qquad (53)$$

There is a very important difference between Equation (47) and Equation (53). In Equation (47) the sensitivity was that of the flapper-nozzle arrangement, i.e., K' which was shown to be a very large (too large, in fact) sensitivity. By the addition of the internal feedback mechanism, the sensitivity of the relationship is the ratio of two lever arms, L_1/L_2, and the reciprocal of the feedback sensitivity $A_f K_f$. This brings the controller mechanism sensitivity down to a range that is satisfactory for proportional control, and it also provides some very convenient parameters to use in adjusting the sensitivity when the controller is to be tuned. An industrial example of a controller such as the one just described is shown in Figure 19.

Additional Flapper-Nozzle Controllers. With very slight modifications additional modes of operation can be incorporated into flapper-nozzle controllers.[9,11] Consider the addition of a variable restriction in the line leading to the feedback bellows. This is illustrated in Figure 20. The analysis of this nozzle arrangement is:

$$p_B = -K'x \qquad (48)$$

and,

$$p_o = -K_R\, p_B \qquad (49)$$

The weight rate of flow of air into the feedback bellows will be a function of the pressure drop across the restriction in the line leading into the bellows:

W_i = weight rate of flow of air into the bellows

$= f(P_o - P_f)$

Considering a linearization of this expression:

$$w_i = K_5(p_o - p_f) \qquad (54)$$

If the mass of air in the bellows is given in terms of the ideal gas law:

$$M = 29 P_f V/RT$$

where P_f = feedback pressure in the bellows

V = volume of the bellows

T = temperaure in the bellows, a consant.

The linearization of this yields:

$$m = K_6\, p_f + K_7\, v \qquad (55)$$

where $K_6 = 29V_i/RT$ and $K_7 = 29P_{fi}/RT$

The variation in volume of the feedback bellows may be given as:

$$v = A_f y \qquad (56)$$

The force balance on the feedback bellows and its restoring spring is:

$$p_f A_f = K_f y \qquad (57)$$

The weight rate of flow of air into the feedback bellows is the rate of change of the mass of air in the bellows:

$$w_i = pm \qquad (58)$$

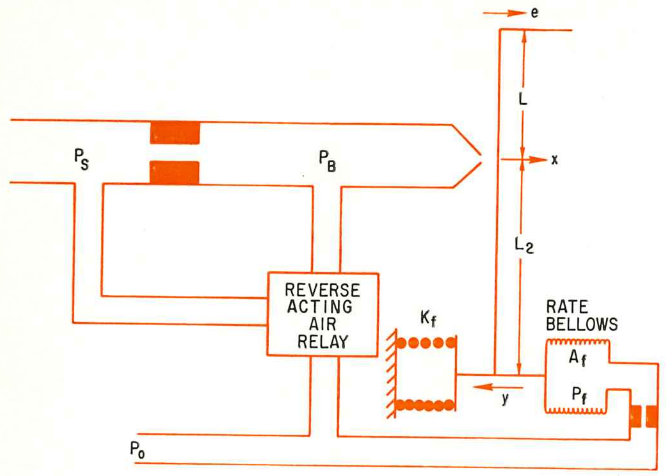

Fig. 20—Adding an adjustable restriction in the feedback line gives a proportional plus rate controller.

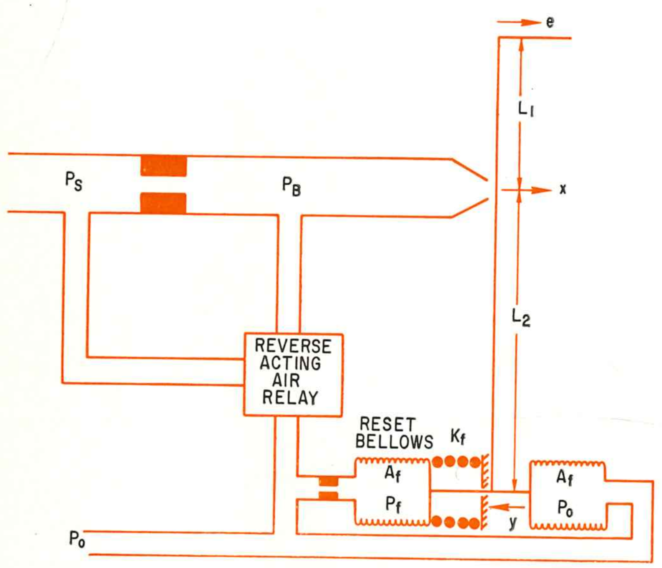

Fig. 21.—An additional bellows can be added to form a proportional plus reset controller.

Combining Equations (54) through (58):

$$y = \frac{(A_f/K_f)\,p_o}{1+\tau_f p} \qquad (59)$$

where $\tau_f = (K_6/K_5) + (K_7 A_f^2/K_5 K_f)$

As before:

$$x = L_2 e/(L_1+L_2) - L_1 y/(L_1+L_2) \qquad (51)$$

Equations (48), (49), (59) and (51) can be combined as follows:

$$p_o = \frac{[L_2/(L_1+L_2)]\,K'\,K_R\,e}{1+[L_1/(L_1+L_2)]\,[(A_f/K_f)(1+\tau_f p)]\,K'\,K_R} \qquad (60)$$

Since K' is large, the 1 in the denominator of Equation (60) is negligible and the equation reduces to:

$$p_o = (L_2/L_1)\,(K_f/A_f)\,(1+\tau_f p)\,e \qquad (61)$$

When compared to the more standard forms of controller equations it can be seen that this is proportional plus derivative control:

$$P_o = K_c\,(1+T_d p)\,e$$

where

$$K_c = L_2 K_f/L_1 A_f$$

$$T_d = \tau_f$$

It is seen that the derivative time τ_f may be adjusted by adjusting the restriction in the feedback bellows. If this restriction were a needle valve, for example, an adjustment of the needle valve would vary the rate or pre-act time of the controller.

For proportional plus reset operation, an additional bellows may be incorporated as shown in Figure 21. The equations which describe this controller are Equations (48), (49), (51), (54), (55), (56), (58) and a force balance which is:

$$p_o A_f - p_f A_f - K_f y = 0 \qquad (62)$$

Combining Equations (54), (55), (56), (58) and (62) gives:

$$y = \frac{K_8 p}{1+\tau_f' p}\,p_o \qquad (63)$$

where $K_8 = A_f K_6/K_f K_5$

$$\tau_f' = (K_6 K_f - K_7 A_f^2)/K_5 K_7$$

Equations (48), (49), (51) and (63) can be combined as follows:

$$p_o = \frac{[L_2/(L_1+L_2)]\,K'\,K_R\,e}{1+[L_1/(L_1+L_2)]\,[K_8 p/(1+\tau_f' p)]\,K'\,K_R} \qquad (64)$$

Since K' is so large, the 1 in the denominator of Equation (64) is negligible and the equation reduces to:

$$p_o = \frac{L_2\,(1+\tau_f' p)\,e}{L_1\,K_8\,p}$$

$$= \frac{L_2 \tau_f'}{L_1 K_8}\left(1+\frac{1}{\tau_f' p}\right)e \qquad (65)$$

This is proportional plus reset control as can be seen by comparison to the more standard form of the controller equation:

$$p_o = K_c\,[1+(1/T_r p)]\,e$$

where $K_c = L_2 \tau_f'/L_1 K_8$

$$T_r = \tau_f'$$

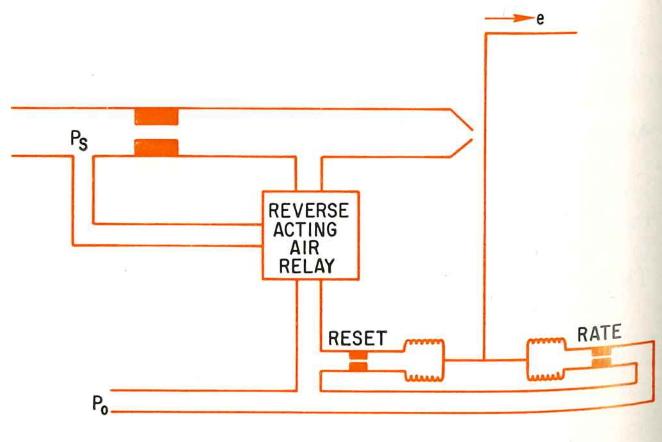

Fig. 22—The reset and rate bellows in this arrangement of a three mode controller can be inadvertently adjusted to cancel.

The reset time T_r can be adjusted by adjusting the restriction in the line leading into the feedback bellows.

One point of interest concerning the reset bellows is that for an increase in p_f there will be a decrease in y, an increase in x, a decrease in p_B, and an increase in p_o. This feedback loop is, therefore, a positive feedback loop. The unrestricted bellows is, of course, negative feedback as was shown earlier, and since it is unrestricted, it leads the positive feedback of the reset bellows.

For a combination of proportional, rate, and reset modes of control, the logical scheme to assume is as shown in Figure 22. The difficulty with a scheme such as this is that both the restriction in the reset and rate bellows can be inadvertently adjusted to the same value and the two modes of control will exactly cancel one another. To avoid this possibility a more practical scheme might be as shown in Figure 23.

Force Balance Controllers. In all of the controllers shown so far the input has been in the form of a motion, and all of the feedback signals were expressed as a motion; i.e., the movement of a bellows. These controllers are generally referred to as "motion balance" controllers although "position balance" might be more accurate. There is also a very large class of controllers in which the input is a force; e.g., a pressure exerted in a bellows. In these controllers a common arrangement is one in which the measured or controlled variable is fed back to the controller as a pressure and is compared to a pressure representing the set point of the controller. Any difference between these two pressures will be manifest as a force which will be counterbalanced by a feedback force from the controller output. The flapper position in these controllers is determined by the balance of these two forces, and the term "force balance" controllers is often applied to them. A schematic representation of the distinction between force balance and motion balance controllers is shown in Figure 24.

Stack Controllers. One variety of the force balance pneumatic controllers[9,11,12], is the so-called "stack controllers." Although a discussion of these will be deleted for brevity, an analysis of 12 different types of stack controllers are given in the literature.[12]

Fluid Amplifiers. Before leaving pneumatic controllers it would be wise to mention a special type of "force balance" device which has no moving parts, is inexpensive, is very small, and may become more and more widespread in its control applications. This device is the fluid amplifier.[13,14] There are two basic types of fluid amplifiers, examples of which are shown in Figure 25. The three terminal modulator, shown in the upper part of Figure 25, is a momentum exchange device in which a control jet is used to bias or deflect a power jet. Amplification of 10 to 1 is possible.

The impact modulator shown in the lower part of Figure 25 is a pressure balancing device in which the balance is achieved between two directly opposing jets. A radial jet is produced whose axial position is determined by the relative pressures in two opposing jets. Amplification of 100 to 1 is practical.

These fluid amplifier devices are the fluid counterparts of vacuum tubes and transistors, and they can be used as controller components just as electronic or more conven-

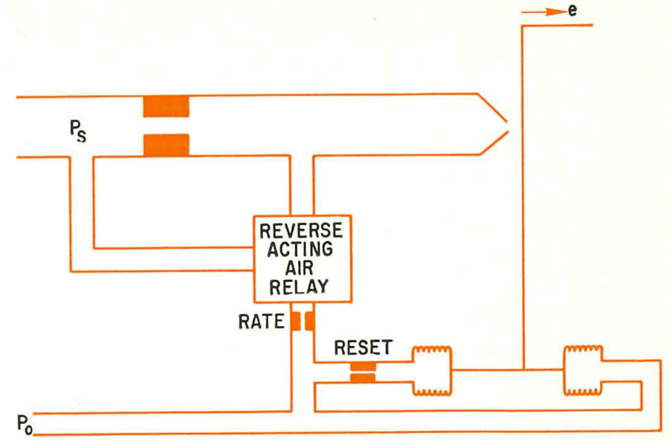

Fig. 23—A more practical arrangement of a three mode controller is shown here.

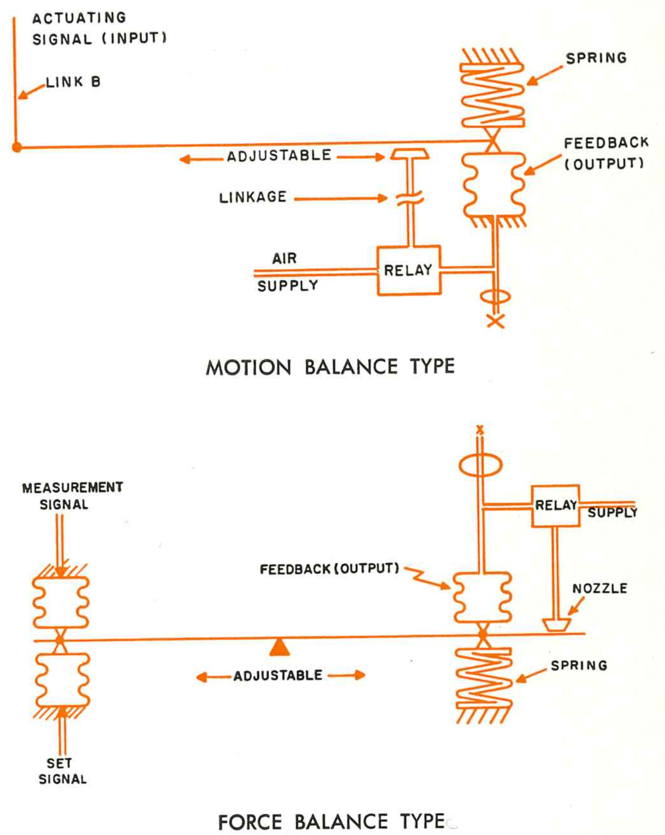

Fig. 24—Most of this discussion has been for motion balance controllers, although there are also force balance types.

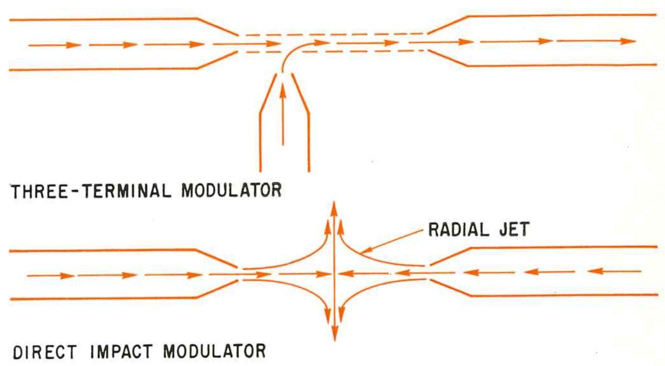

Fig. 25—A fluid amplifier is a special type of force balance controller that is finding more widespread applications.

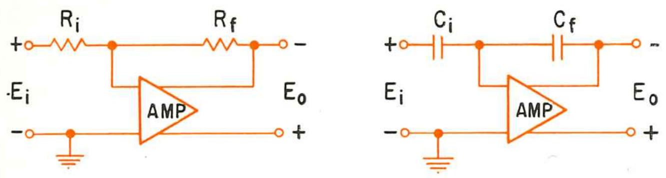

Fig. 26—Electrical circuits can be used as controllers.

tional pneumatic components might be used. The result is a very small, inexpensive, and rugged controller.

ELECTRONIC CONTROLLERS

There are controllers[3] which are the electronic and electric version of the hydraulic and pneumatic devices already discussed. The more general control devices such as digital computers will not be discussed here. The central component of the electronic controllers is a high gain operational amplifier which is connected between input and feedback networks. These networks are formed by using two resistors or two capacitors as shown in Figure 26. The description for these circuits is:

$$-E_o/E_i = R_f/R_i \qquad (66)$$

or

$$-E_o/E_i = C_i/C_f \qquad (67)$$

The proportional band or gain is adjusted by varying a resistance, capacitance, or by attenuating a feedback voltage with a potentiometer. To obtain reset action a resistor is used in parallel with an input capacitor, and to obtain derivative action, a separate circuit is added.

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Electric controllers use standard electrical circuits whose descriptive equations are those of the desired modes of control. A typical controller might use a null-balance Wheatstone bridge with either resistance or voltage balance devices for proportional control and a current balance device for proportional plus reset control.

The substance of this report will also be included as part of a forthcoming book entitled "Automatic Control of Processes" by the authors for publication through International Textbook Co.

SYMBOLS USED

(Small letters usually refer to small variations or differences in the value of the capital letter quantity.)

C, c	controlled variable
E, e	baffle position, error signal
G, g	disturbance
K	process gain
K_c	proportional gain of a controller
K_R	relay gain
K_r	floating controller gain associated with reset mode
$K_{1,2,...}$	constants
L	lever arm length
L_r	open-loop time delay
M, m	mass of air, controller output
m_d	controller output for derivative or rate mode
m_p	controller output for proportional mode
m_r	controller output for reset mode
P	period
P_A	ambient pressure
P_B, p_B	back pressure
P_o, p_o	output air pressure
P_S	supply air pressure
P_u	ultimate period
p	differential operator, d/dt
R	gas constant
R_r	open-loop reaction rate
R, r	set point adjustment
S_u	ultimate sensitivity
T_d	derivative or rate time
T_r	reset time
t	time
V, v	volume
W_i, w_i	weight rate of air flow into nozzle chamber
W_o, w_o	weight rate of air flow out of nozzle chamber
X, Y, x, y	distances
ε	exponential base
θ_o	delay time
μ	index of self regulation
τ	time constant

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How to Clean and Dry Compressed Air

Operating conditions determine what type of equipment should be used to clean and dry air. Here are the factors to consider

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AIR LEAVING A COMPRESSOR, regardless of whether it is for plant, process or instrument use, requires treatment before it can be put to work. Oil removal units are needed for oil-lubricated compressors; solids and moisture removal systems are needed for almost any type compressor system. Here are major factors to consider in obtaining clean oil-free dry air.

WHY CLEAN DRY AIR

The justification for clean, dry compressed air becomes evident when the low cost of adsorbers, filters, and dryers are compared to the maintenance and replacement costs incurred when such equipment is omitted.

Oil Contamination. Consider what oil contamination in compressed air may cause: erratic operation of pneumatic controllers, valves and other instrumentation; contamination of intermediate or end product when air is used for agitating a liquid or conveying solids; softening of hose line, causing leaks and decreasing service life; and, contamination of spray paint equipment.

Oil removal also protects air drying equipment, keeping water vapor adsorbents at high level of activity and preventing oil fouling of heat exchangers in refrigerated dryers. Solid particle contamination, such as pipe scale and metal chips from welded or threaded connections also inhibit optimum equipment operation.

Water Contamination. Moisture-containing air causes corrosion in pneumatic devices, and pipe breakage or flow interruptions will occur when condensed moisture freezes in pipeline low spots. Liquid water can flood pneumatic controls, cause imbalance, false readings and erratic operation. Air-line moisture washes lubrication from air cylinders and similar devices. Of course, paint spraying, sand blasting and most process air demands freedom from oil, solids and moisture contamination.

OIL REMOVAL

Oil in compressed air can be classified in two distinct phases: one a vapor and the other a liquid. The liquid phase can be considered in two particle size ranges: 0.01 to 5 microns as a mist, and above 5 microns as a spray. The vapor has sufficient Brownian motion to be effectively adsorbed by activated carbon or alumina. Particles larger than 0.01 micron are to be removed by filtration.

For each phase and size distribution, different means of removal should be employed. Before choosing specific equipment, though, it is necessary to consider why the oil appears, how much of each phase is estimated to be present, and finally, what type of equipment can do the required job.

During the compression cycle of the typical compressor, the air temperature climbs to between 400° F and 700° F. Some of the lubricating oil is vaporized and a portion may react with the compressed air to form partially oxygenated carbon and nitrogen compounds. Upon cooling, part of the vaporized oil and these newly formed compounds condense to various size droplets.

Liquid Oil Removal. Over 99 percent of the liquid oil in compressed air consists of droplets in the size range